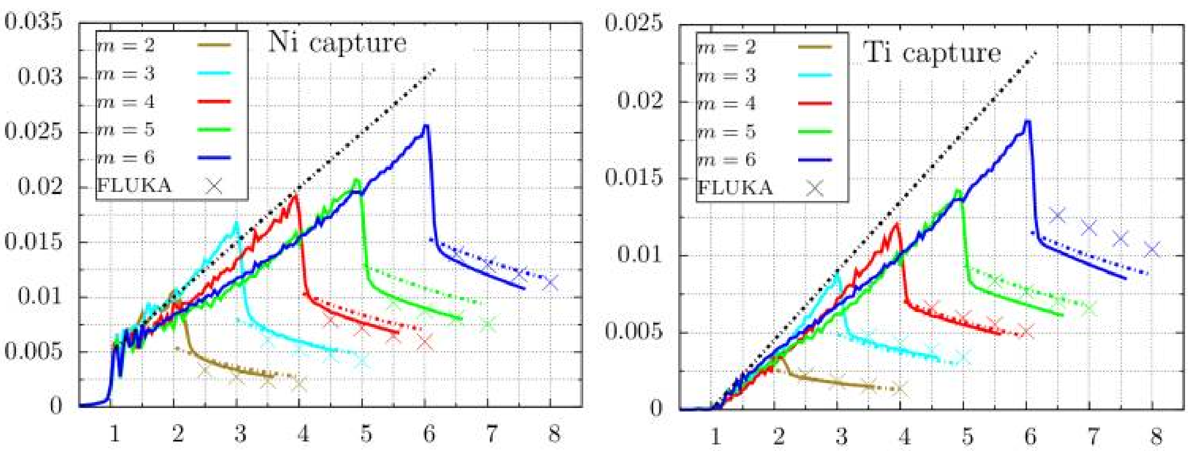
Shielding prompt gamma at the ESS long instruments

Neutron capture in the guide coating and prompt gamma radiation

Neutrons at the long instruments are transported via guide systems to the distances up to 160 meters from the source. During transport neutrons are specular reflected from the guide walls either coated by nickel or by a multilayer structure where layers have high neutron optical contrast. In the latter case the glancing angle at which the specular reflection is possible is increased.

Capture above $q_\perp > q_c^{Ni}$

Capture probabilities in the multilayer coatings have been calculated using multilayer compositions used in commercial production of supermirrors by SwissNeutronics. It was found that probability of absorption in the coating materials per incident neutron is a function of normal component of momentum of the incident neutron. It is convenient to express probability parameterization via ratio of normal momentum component to the critical momentum for nickel[1], $\mu = q_\perp / q_c^{Ni}$.Transverse momentum above which the neutron is not reflected expressed in units of critical momentum for nickel corresponds to the m-value of the coating.



The calculations were performed for m-values of the supermirror from m=1.2 to m=6, and the following conservative estimate for the capture rates in coating materials per incident neutron was suggested:

- Below supermirror cutoff m, $1 < \mu < m$
  - $f_a^{Ni} = 0.005 + 0.005 \cdot (\mu - 1)$
  - $f_a^{Ti} = 0.00027 + 0.00027 \cdot (\mu - 1)$
- Beyond cutoff, $\mu > m$

[1] Critical momentum is the momentum of a neutron below which the neutron is reflected from the material layer at normal incidence.

- f_a^{Ni} = 0.0025 · $\frac{(m+0.1)^2}{\mu}$
- f_a^{Ti} = 0.00225 · $\frac{(m-0.9)(m+0.1)}{\mu}$

The calculated dependences have a straightforward interpretation: the capture probability scales as path length of neutrons in the coating, the linear rise indicates reflection at deeper layers. The result is wavelength independent as the lower capture cross sections for lower wavelength are compensated exactly by increase in the pathlength when the transverse momentum component is kept constant.

The details of the calculations and extensive discussions may be found in (Rodion Kolevatov, Peter Boeni, Christian Schanzer)

Capture below q_c^{Ni}, effect of waviness.

The commonly accepted low-angle reflectivities of the guide coatings which are consistent with neutron beam loss while transport in the guide range from 0.99 or 0.995, that is from 0.5 to 1 % of neutrons are lost upon reflection. In contrast, the reflectivity of a polished nickel plate at a glancing angle below the critical value $\theta = 0.099\, m\, \lambda$ is close to unity. The reflection loss due to interaction in the nickel is typically below 0.01%. The origin of the discrepancy between the two numbers becomes of a special interest when the guide shielding problems are considered. Namely, attributing the 1% loss of the reflectivity at low angles to the interaction in the coating seems to be a clear overestimate which would lead to way too conservative estimates and unnecessary overshielding of the guide.

The origin for the 1% reflectivity loss at low angle for m=1 coating was explained in the note by Marton Marko. According to the study, the main reason for the low-angle reflectivity loss is the waviness of the reflecting surface which is typically $\lesssim 10^{-4}$ mrad [2]. Due to the waviness the reflection angle receives a random contribution, which for each individual reflection can lead to both increasing and decreasing divergence. However, on the average this random process leads to diffusion of neutrons to higher divergences. Finally, the neutrons appear beyond the supermirror cutoff where they can't be transported anymore but instead interact in the coating or are transmitted.

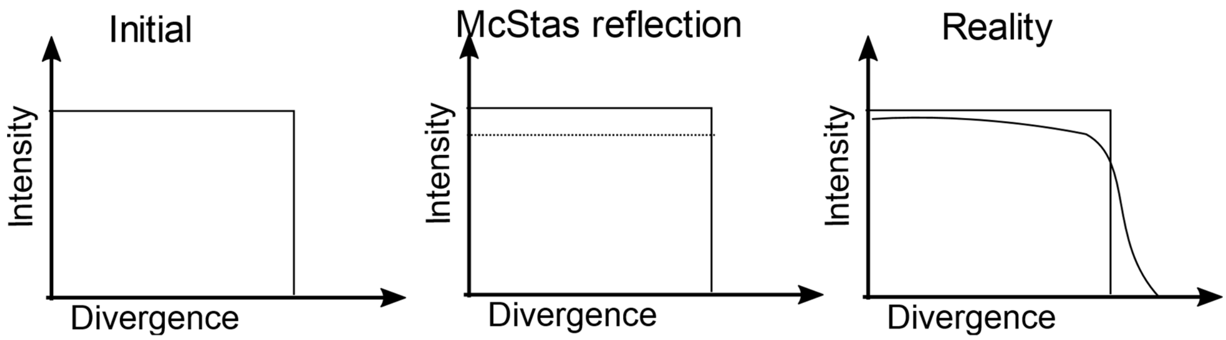
The waviness of 10^{-4} rad corresponds to approximately $0.5 \cdot 10^{-4}$ rad variation of the glancing angle at reflection[3]. At a wavelength λ= 5Å this corresponds to a $\delta q_\perp = 6.3 \cdot 10^{-5}$ Å^{-1} variation of the normal momentum component which constitutes around 0.6% of the critical momentum for nickel, q_c^{Ni}=0.0109 Å^{-1}. Assuming a sharp cutoff for the m=1 supermirror and a uniform divergence profile (uniform probability distribution for the normal momentum component at reflection from $q_\perp = 0$ up to $q_\perp = q_c^{Ni}$), a fraction of transported neutrons around $\sim\delta q_\perp / q_c^{Ni}$ will be pushed beyond the cutoff of the

[2] The waviness is defined as r.m.s. angle between the two vectors normal to the surface averaged over the surface area.

[3] The waviness according to the definition is $\langle \alpha_1^2 + \beta_1^2 + \alpha_2^2 + \beta_2^2 \rangle$ where α and β are variation of angles of the two normal vectors in two orthogonal planes with respect to a fixed direction, so $\langle 4\,\alpha^2 \rangle \lesssim (10^{-4})^2$, $\sqrt{\langle \alpha^2 \rangle} \lesssim 0.5 \cdot 10^{-4}$.

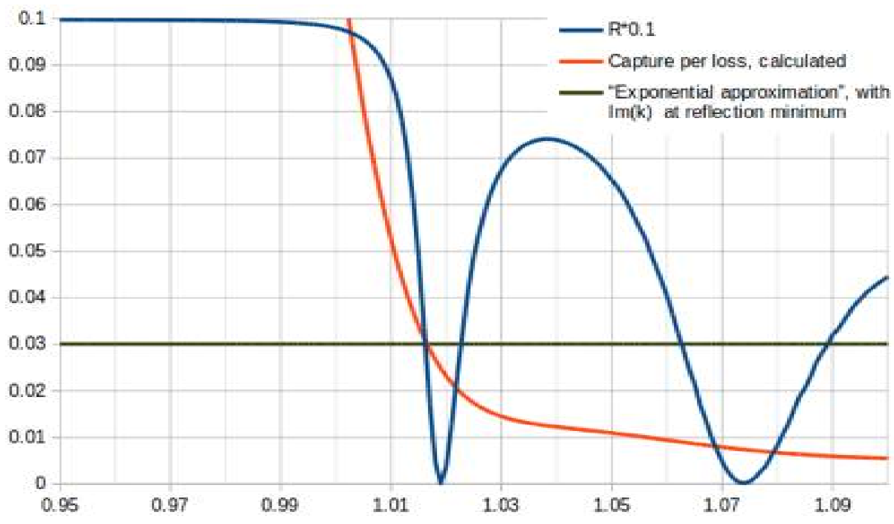
coating and thus effectively not reflected. The number is consistent with 0.5-1% loss of the reflectivity at low angles which is commonly accepted for the neutron optics calculations.

The figure below illustrates this mechanism. Here a divergence distribution before and after reflection is schematically plotted. The McStas implementation results in just lowering the profile, while in reality due to waviness some of the neutron will appear beyond the cutoff (outside the box in the picture) and will not be transported further.



For a conservative estimate of the capture rate we will assume that the neutrons below labeled as unreflected in the McStas simulation all hit the coating in the reflection minimum of the coating.

For m=1 coatings SwissNeutronics deposits a single Ni layer with a thickness of 1200 to 1500 AA on a thin layer of titanium, the latter is needed to prevent delamination. The typical reflectivity curve for a 1500AA nickel layer is depicted in a figure below.



In the reflection minimum the reflection is essentially absent. As in this case the two waves reflected at the two interfaces of a nickel layer nearly exactly extinguish each other outside[4]. The absorption probability of a neutron incident at a coating with normal momentum in the reflection minimum can be thus calculated as

[4] This is similar to what is made use of in production of the anti-reflective optics.

$$P_a = \frac{\Sigma_a(\lambda)}{\Sigma_a(\lambda) + \Sigma_d(\lambda)}\left(1 - e^{-6\,\mathbf{Im}\,q_{\perp,\mathrm{Ni}}\cdot d}\right)$$

Here $q_{\perp}^{\mathrm{Ni}}$ is the momentum normal component which the neutron will have inside the nickel layer if hitting it at the reflection minimum. Σ_a and Σ_d stand for macroscopic cross sections for absorption and diffuse scattering in nickel.

The position of reflection minimum is found from a Brag condition (*d* is thickness of the nickel layer)

$$d \cdot \mathbf{Re}\, q_{\perp,\mathrm{Ni}} = \pi$$

The imaginary part entering the capture probability is found from an imaginary part of neutron optical potential which is related to the product of real and imaginary parts of neutron momentum in the layer:

$$2\,\mathbf{Re}k_{\mathrm{Ni},\perp}\,\mathbf{Im}k_{\mathrm{Ni},\perp} = \frac{2\pi}{\lambda}\left(\Sigma_d(\lambda) + \Sigma_a(\lambda)\right)$$

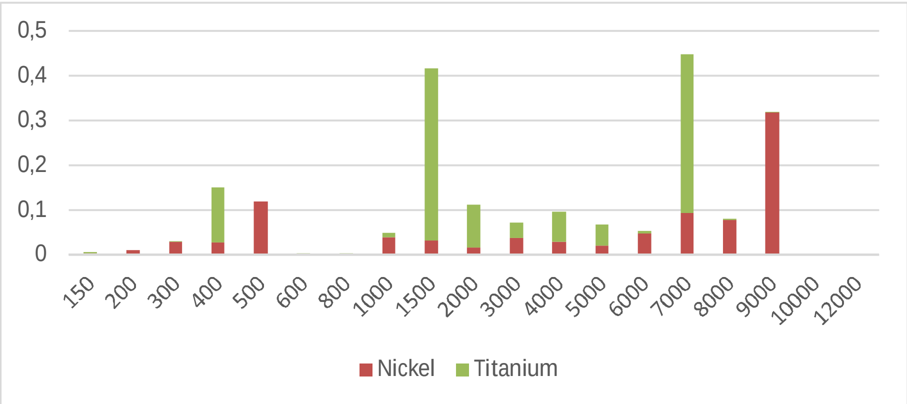
Thus, imaginary part which enters capture probability is expressed as

$$\mathbf{Im}q_{\mathrm{Ni},\perp} = \frac{d}{\lambda}\left(\Sigma_d(\lambda) + \Sigma_a(\lambda)\right)$$

The figure above in addition to a reflectivity curve also shows that parameterization (green line) indeed gives a conservative estimate for a calculated capture probability per incident neutron in the reflection minimum (orange line).

Prompt gamma spectra:

Gamma radiation resulting from neutron capture in the coating and substrate materials has energies up to 11 MeV. The spectra of prompt and decay gammas in nickel, titanium and borosilicate substrate were constructed for 19 energy groups based on inclusive cross sections data downloaded from IAEA web site (PGAA Database files).



A figure illustrates number of photons in the corresponding groups (the number is the upper energy limit in the group) per one captured neutron close to the cutoff of m=5 coating.

McStas implementation of prompt gamma shielding calculations

- It is essential to know the m-value of the coating at reflection point to calculate probabitliy of a capture in the coating materials. A number of McStas components used in construction of the guide systems was customized in order to set a value to a global variable equal to m-value at reflection point:

  - *Guide_custom, Guide_curved_custom (to replace Bender in shielding calculations, has same syntax), Elliptic_guide_gravity_custom, Guide_chanelled_custom*

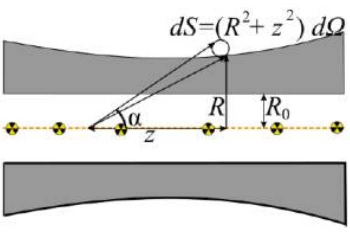
- The McStas component Scatter_Logger by E. Knudsen et has been updated to record m-value at reflection in addition to the characteristics of the intermediate neutron states. The new component was named *Shielding_logger*.

- The McStas component Scatter Log Iterator by E. Knudsen et. al. which normally returns neutron weights corresponding to the reflection loss, has been modified in order to return weights corresponding either to neutron capture in coating materials of the total unreflected weight. The same buffer of saved states is processed for three times by three iterators which are placed in the McStas instrument file one after another. The new components are:

  - *Shielding_log_iteratorNi, Shielding_log_iteratorTi, Shielding_log_iterator_total* (The latter is basically the original scatter log iterator with minimal modifications.)

  - Neutrons capture rate in particular materials along the guides is written to the files by standard Monitor_ND McStas component.

Once the intensity and spectrum of gamma radiation is known, the dose rate for a linear source geometry as seen in the figure,



can be calculated analytically as

$$\dot{H}(R,z) = \frac{1}{4\pi} \int dz\, dE\, K(E)\, I(E,z) B_{\mathrm{dose}}(E, \mu d(z)) \frac{1}{(R^2 + z^2)} e^{-\mu(E) d(z)}$$

$$d(z) = \frac{(R - R_0)\sqrt{R^2 + z^2}}{R}$$

- Table values for the capture γ spectra in Ni, Ti and borosilicate $I(E)$, linear attenuation μ and buildup factors $B(\mu, L)$ of typical shielding materials together with flux to dose conversion factors $K(E)$ are implemented in *Shielding_calculator* component (calculates shielding thickness for given dose outside) and *Dose_calculator* (dose rate outside shielding of fixed thickness). The components simply read the output of the monitors placed within the three iterators (the names of output files have to be specified) and perform the requrested calculations.

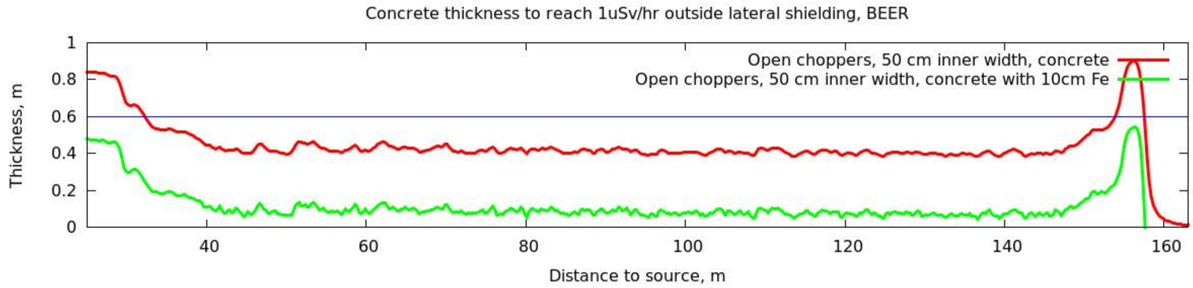
- The "calculator" component implement Flux to dose conversion factors for gamma radiation provided by the ESS and linear attenuation
- The linear attenuation and buildup factors are implemented according to the reference text book on radiation protection by Mashkovich and Kudryavceva. It is possible to use also linear attenuation given by NIST (J. H. Hubbell, S. M. Seltzer) , and different sets of buildup factors, e.g. (Kucuk., 2010). The difference between numbers in different sets is less than few percent.

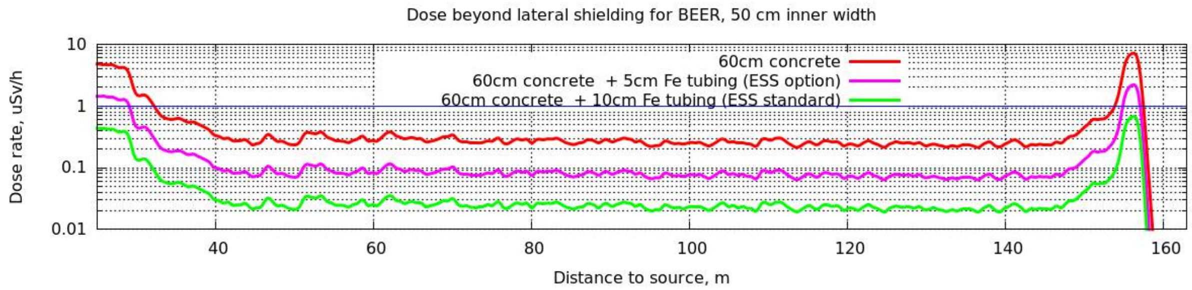
Prompt gamma contribution to dose beyond lateral shielding for ESS insruments

In the plots below results of the calculation for some of the ESS instruments is presented. What is plotted is amount of concrete required to reach 1uSv/hr dose rate outside the lateral shielding with 50 cm inner width, either with (green line) or without (red line) 10 cm extra steel around the guide.

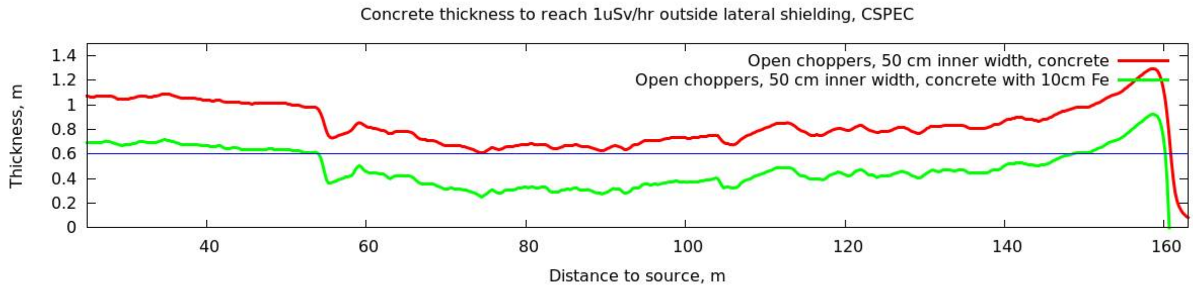
Also the dose rate outside the 60 cm concrete shielding is plotted, either with 10 cm (green line), 5cm (magenta line) or without (red line) steel around the guide. Accelerator power is 5MW.

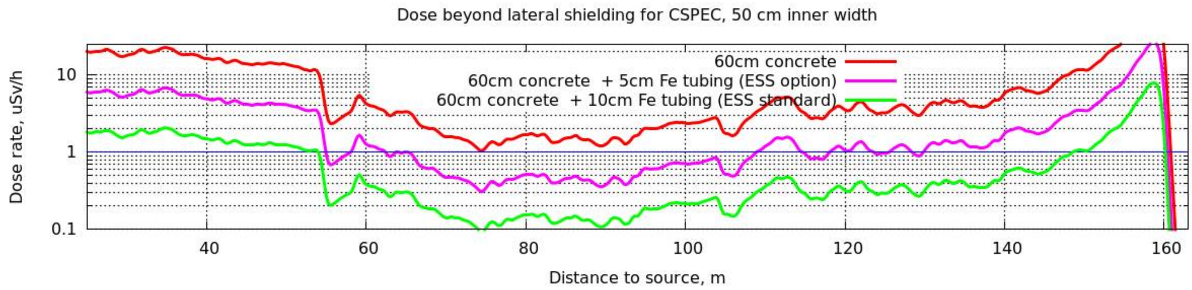
BEER



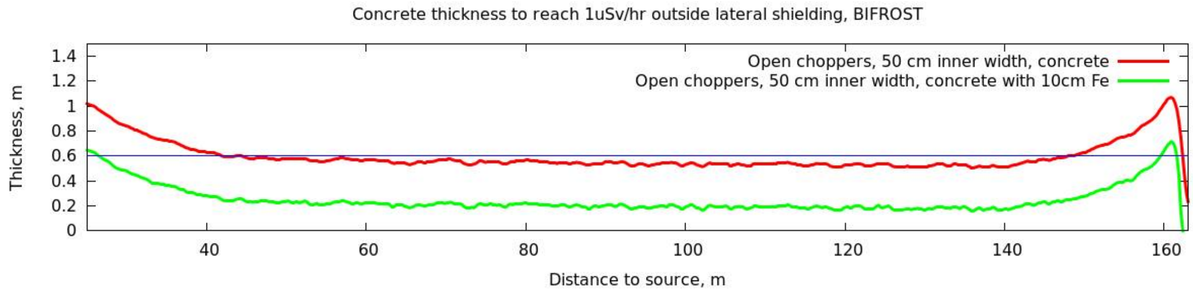


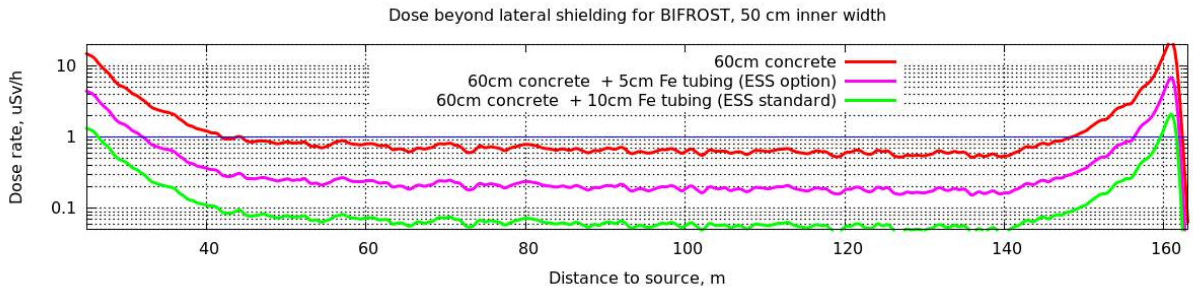
C-SPEC





BIFROST



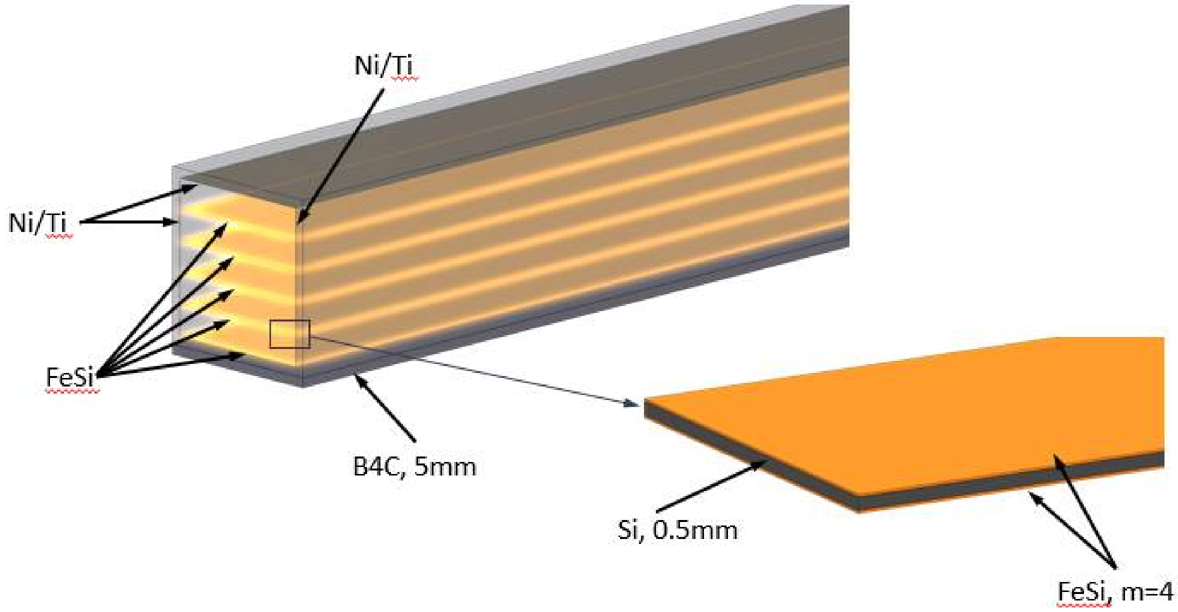


MAGiC

A polarizing guide piece for MAGiC which is placed around 80 m is constructed the following way (Xavier Fabreges):

- Internal cross section 80x80mm^2, length 3m
- Coating Ni/Ti, 3 planes: sides and top, m=1.4; substrate N-BK7, thickness 5mm

- 5 horizontal channels arranged in the vertical direction
- Coating FeSi double side, m=4; substrate Si, thickness 0,5mm
- Sintered B4C layer at the bottom , thickness 5mm, length 3m



The following “worst case” scenario is anticipated for shielding calculations:

1. The reflection off the supermirror takes place at the FeSi coating – Si substrate interface.
2. The incoming neutron is assumed to traverse the whole thickness of the FeSi coating inwards before the reflection.
3. The reflected neutron beam is assumed to traverse the whole thickness of the FeSi coating outwards after the reflection.
4. The transmitted intensity (e.g. wrong polarization or with too high divergence) is assumed to traverse on the average 2 silicon plates and 4 layers of FeSi coating with a possibility for capture in either material.

Correspondingly, if denote p_i incoming neutron weight, p_r reflected weight and p_t=p_i-p_r reflected weight the weight of neutrons captured in Si and Fe reads:

$$p^{Si} \;=\; p_i \frac{\Sigma_a^{Si}\left(\lambda = \frac{2\pi}{k}\right)\cdot d_{Si}}{\sin\theta} + p_r \frac{\Sigma_a^{Si}\left(\lambda = \frac{2\pi}{k}\right)\cdot d_{Si}}{\sin\theta} + p_t[1 - \exp\left(-\frac{\Sigma_a^{Si}\left(\lambda = \frac{2\pi}{k}\right)\cdot 2\cdot D_{Si}}{\sin\theta}\right)]$$

$$p^{\mathrm{Fe}} = p_i \frac{\Sigma_a^{\mathrm{Fe}}\left(\lambda = \frac{2\pi}{k}\right)\cdot d_{\mathrm{Fe}}}{\sin\theta} + p_r \frac{\Sigma_a^{\mathrm{Fe}}\left(\lambda = \frac{2\pi}{k}\right)\cdot d_{\mathrm{Fe}}}{\sin\theta} + p_t \exp\left(-\frac{\Sigma_a^{\mathrm{Si}}\left(\lambda = \frac{2\pi}{k}\right)\cdot D_{\mathrm{Si}}}{\sin\theta}\right)[1 - \exp\left(-\frac{\Sigma_a^{\mathrm{Fe}}\left(\lambda = \frac{2\pi}{k}\right)\cdot 4\cdot d_{\mathrm{Fe}}}{\sin\theta}\right)]$$

Here d_{XX} stands for thickness of the corresponding materials in the coating, D is the thickness of the substrate. $\Sigma_a^{\mathrm{Fe/Si}}$ is the macroscopic absorption cross section of iron/silicon. Diffuse scattering in iron layers and in silicon is neglected (it is very small in silicon, while account of that in iron will just slightly reduce fraction of neutrons reaching silicon substrate). For thermal neutrons:

$$\Sigma_a^{\mathrm{Fe}}(\lambda = 1.8\text{Å}) = 0.2176\ \mathrm{cm}^{-1};\ \Sigma_a^{\mathrm{Si}}(\lambda = 1.8\text{Å}) = 0.00855\ \mathrm{cm}^{-1}$$

For silicon this amounts to absorption mean free path of 116 cm (thermal neutrons), for iron it corresponds to mean free path of 4,59 cm. Total thickness of the corresponding layers in the polarizing m=4 supermirror coating is close to $0.5*10^{-3}$ cm for both silicon and iron. The equations can be rewritten when explicit account of the cross section wavelength dependence is made.

$$p^{\mathrm{Si}} = (p_i + p_r) \frac{\Sigma_a^{\mathrm{Si}}(1.8\text{Å})\cdot d_{\mathrm{Si}} 2\pi}{1.8[\text{Å}]\cdot 0.5\cdot q[\text{Å}^{-1}]} + p_t[1 - \exp\left(-\frac{\Sigma_a^{\mathrm{Si}}(1.8\text{Å})\cdot 2\cdot D_{\mathrm{Si}} 2\pi}{1.8[\text{Å}]\cdot 0.5\cdot q[\text{Å}^{-1}]}\right)]$$

$$p^{\mathrm{Fe}} = (p_i + p_r) \frac{\Sigma_a^{\mathrm{Si}}(1.8\text{Å})\cdot d_{\mathrm{Fe}} 2\pi}{1.8[\text{Å}]\cdot 0.5\cdot q[\text{Å}^{-1}]} + p_t \exp\left(-\frac{\Sigma_a^{\mathrm{Si}}(1.8\text{Å})\cdot 2\cdot D_{\mathrm{Si}} 2\pi}{1.8[\text{Å}]\cdot 0.5\cdot q[\text{Å}^{-1}]}\right)[1 - \exp\left(-\frac{\Sigma_a^{\mathrm{Si}}(1.8\text{Å})\cdot 4\cdot d_{\mathrm{Fe}} 2\pi}{1.8[\text{Å}]\cdot 0.5\cdot q[\text{Å}^{-1}]}\right)]$$

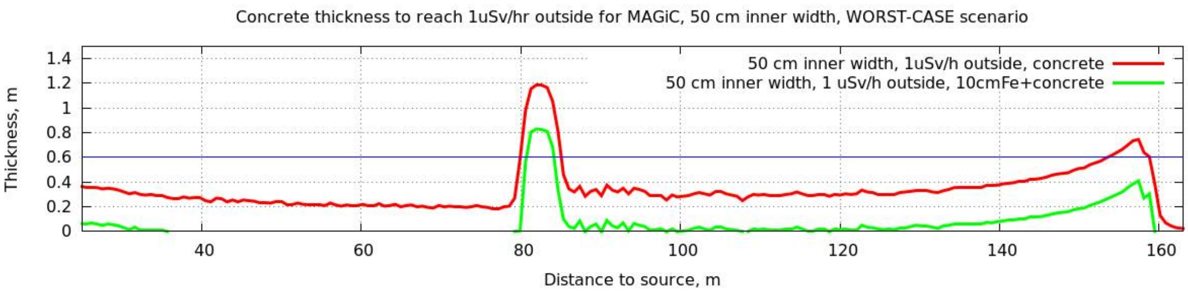
Here q is momentum transfer at reflection expressed in units of AA^{-1}, and the capture cross section is taken for thermal wavelength.

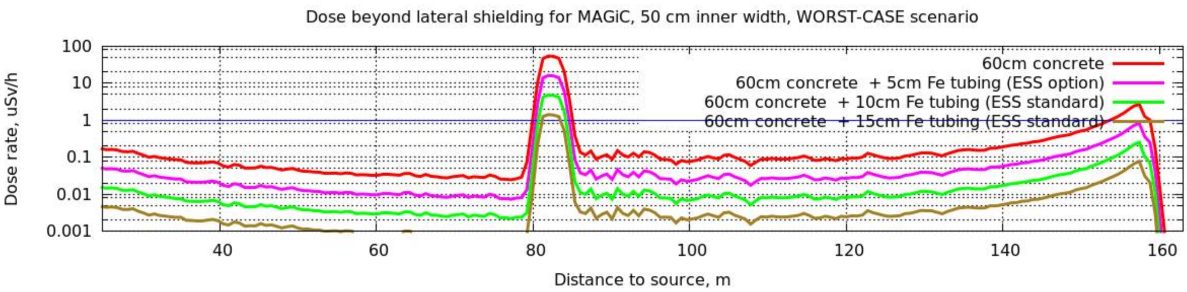
A relaxed scenario for calculation is also possible. From the calculation of neutron absorption in the NiTi coatings performed for Swiss Neutronics layer sequences the neutron capture probability per incident neutron is found to be a linear function of the momentum transfer at reflection. This implies (an easy geometrical calcualtion) that the reflection depth for a neutron is a quadratic function of momentum transfer at reflection. Taking this into accoutn in equations for capture probabilities above this leads to

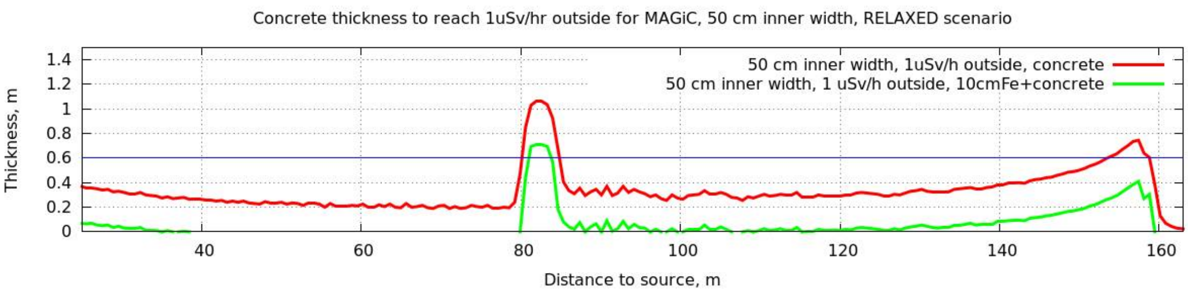
$$p^{\mathrm{Si}} = 2\cdot p_r\cdot\left(\frac{q}{q_{\max}}\right)^2 \frac{\Sigma_a^{\mathrm{Si}}(1.8\text{Å})\cdot d_{\mathrm{Si}} 2\pi}{1.8[\text{Å}]\cdot 0.5\cdot q[\text{Å}^{-1}]} + (p_i - p_r) \frac{\Sigma_a^{\mathrm{Si}}(1.8\text{Å})\cdot d_{\mathrm{Si}} 2\pi}{1.8[\text{Å}]\cdot 0.5\cdot q[\text{Å}^{-1}]} + p_t[1 - \exp\left(-\frac{\Sigma_a^{\mathrm{Si}}(1.8\text{Å})\cdot 2\cdot D_{\mathrm{Si}} 2\pi}{1.8[\text{Å}]\cdot 0.5\cdot q[\text{Å}^{-1}]}\right)]$$

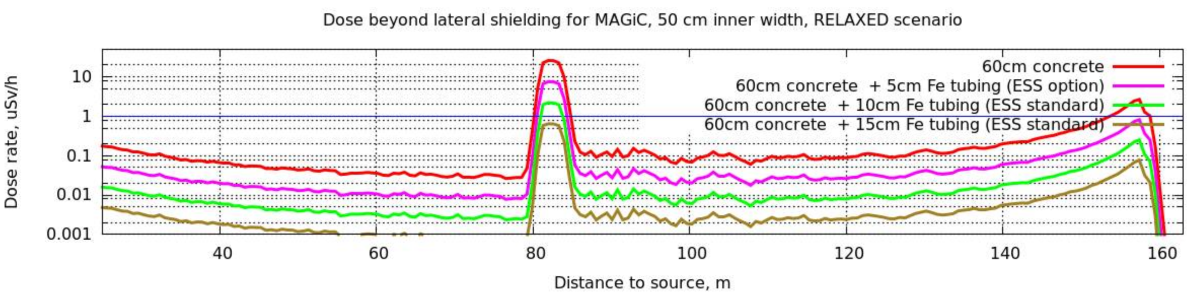
$$
\begin{aligned}
p^{Fe} &= 2 \cdot p_r \cdot \left(\frac{q}{q_{\max}}\right)^2 \frac{\Sigma_a^{Si}(1.8\text{Å}) \cdot d_{Fe} 2\pi}{1.8[\text{Å}] \cdot 0.5 \cdot q[\text{Å}^{-1}]} + (p_i - p_r) \frac{\Sigma_a^{Si}(1.8\text{Å}) \cdot d_{Fe} 2\pi}{1.8[\text{Å}] \cdot 0.5 \cdot q[\text{Å}^{-1}]} \\
&\quad + p_t \exp\left(-\frac{\Sigma_a^{Si}(1.8\text{Å}) \cdot 2 \cdot D_{Si} 2\pi}{1.8[\text{Å}] \cdot 0.5 \cdot q[\text{Å}^{-1}]}\right)\left[1 - \exp\left(-\frac{\Sigma_a^{Si}(1.8\text{Å}) \cdot 4 \cdot d_{Fe} 2\pi}{1.8[\text{Å}] \cdot 0.5 \cdot q[\text{Å}^{-1}]}\right)\right]
\end{aligned}
$$

By q_{max} the coating cutoff is denoted. When the value of momentum transfer at reflection q exceeds the coating cutoff q_{max} the ratio of the two must be replaced by unity. In this case the reflection is assumed to happen at the maximal depth, so the probabilities reduce to the worst-case scenario values.



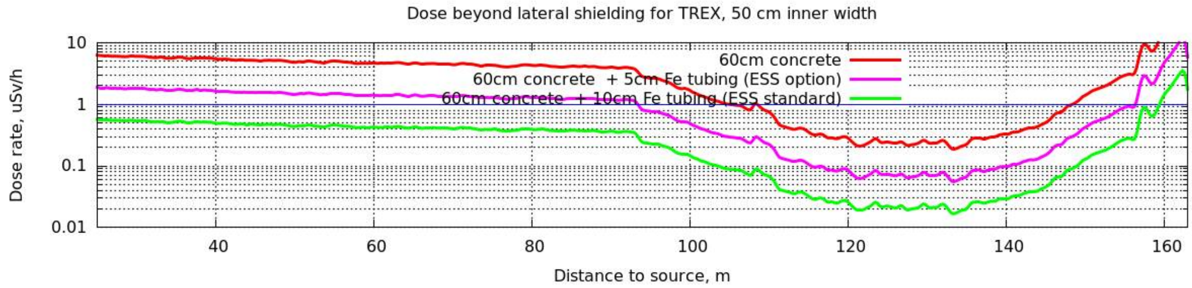


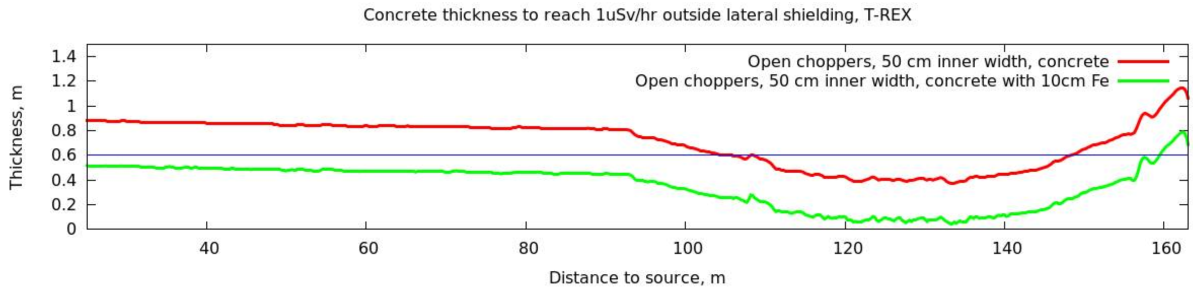




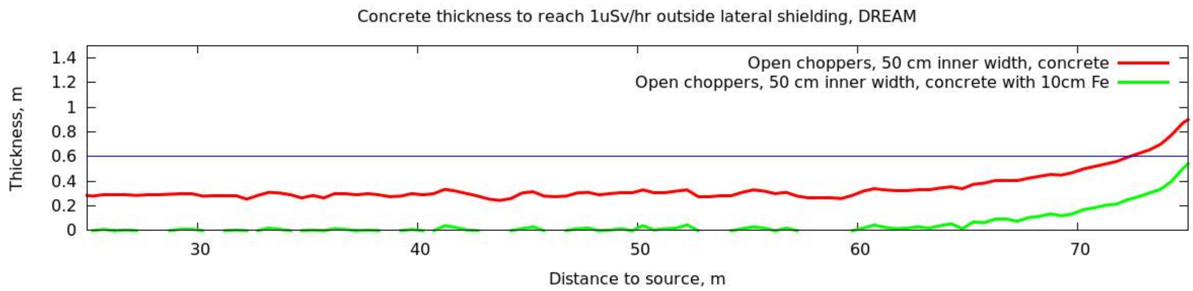
We do calculation for both scenarios. It is apparent that even in the worst case one requres more than 10 cm of iron in addition to 60 cm regular concrete layer. 110 cm of regular concrete or a combination of 15 cm iron and 60 cm concrete correspond to mass integral of around 250 g/cm^2. Such integral may be achieved with 80 cm layer of heavy concrete (density = 3.2 g/cm^3). The worst-case scenario yields 120 cm of regular concrete which has a mass integral equivalent to 86.25 cm of heavy concrete.

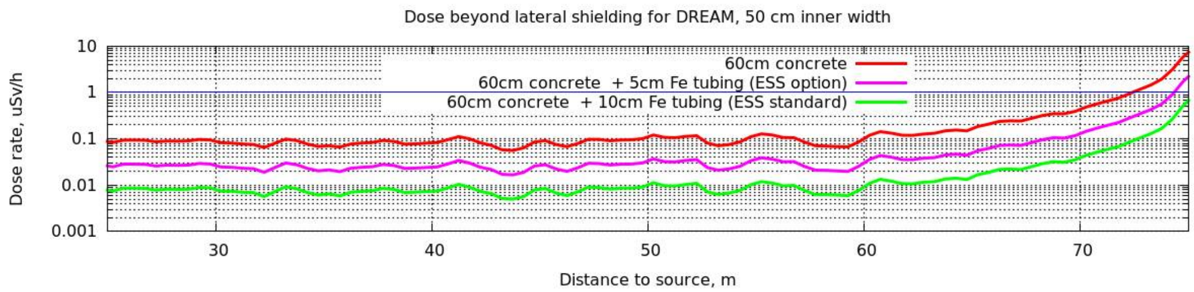
T-REX



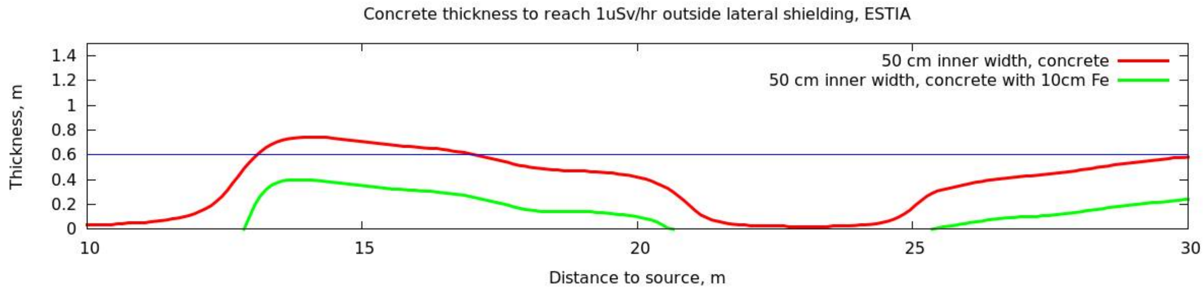


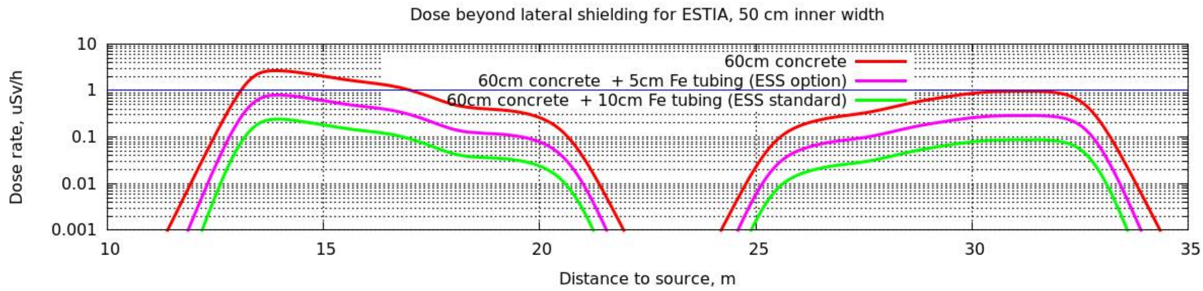
DREAM





ESTIA





Other instruments will be added on a later occasion.

References

J. H. Hubbell, S. M. Seltzer. (n.d.). *Tables of X-Ray Mass Attenuation Coefficients and Mass Energy-Absorption Coefficients from 1 keV to 20 MeV for Elements Z = 1 to 92 and 48 Additional Substances of Dosimetric Interest*.* doi:https://dx.doi.org/10.18434/T4D01F

Kucuk., N. (2010). *Expert Systems with Applications 37*, 3762-3767.

(n.d.). *PGAA Database files.* IAEA. Retrieved from https://www-nds.iaea.org/pgaa/databases.htm

Rodion Kolevatov, Peter Boeni, Christian Schanzer. (n.d.). Neutron absorption in supermirror coatings: effects on shielding. *submitted to NIM A*.