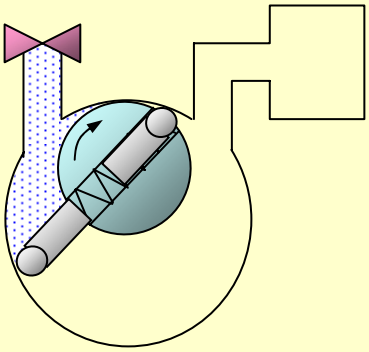
Accelerator technique



$$\nabla \times \overline{B} = \mu_0 \overline{J} + \frac{1}{c^2}\frac{\partial \overline{E}}{\partial t} \qquad \nabla \times \overline{E} = -\frac{\partial \overline{B}}{\partial t}$$

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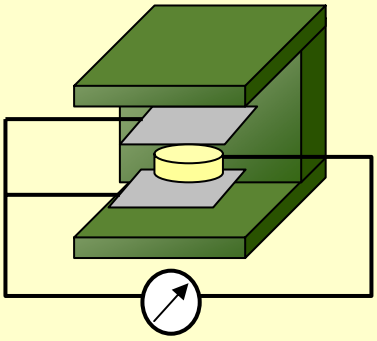

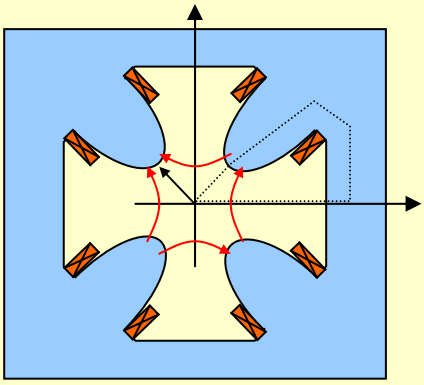

Sverker Werin

2.5th edition

Accelerator technique

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Contents

Foreword

This book is part of the textbook for a course in Accelerator technique given at the department for Accelerator physics at Lund University, Sweden. The total course runs over 5 points (=5 weeks full time study).

The material is intended to give an overview of the techniques and basic physics of accelerators and especially electron accelerators for synchrotron radiation.

1 Introduction

In the world of physics, or chemistry and biology too, there is often a need to create beams of particles which can propagate through space accomplishing a number of tasks:

- Collide with each other or targets to provide means to study the origin of the world around us. (elementary particle physics, nuclear physics)
- To radiate and thus create a beam of light stronger than the sun to let us see new aspects of matter. (synchrotron radiation)
- To induce controlled nuclear reactions (transmutation)
- To hit, heat and destroy cancer tumours.
- Simply be there so we can measure things about them.
- ... and much more

To understand and to build these machines asks for knowledge in accelerator physics and accelerator technique. The first describes the physical phenomena involved in playing with beams of particles, while the latter, which this book is about, treats the different techniques and hardware necessary to really accomplish the accelerator itself.

The accelerator is very seldom just one piece of equipment. There is of course a need to create, handle, accelerate and dispose of the particles in the machines. Depending of the type of particles they are born in ionsources, electron guns, pair production processes ... The particles are handled by magnetic fields in which they bend and turn. By electric fields at different frequencies the particles are accelerated, and some of them end their lifes in beamdumps.

But this is not all! We also have to be able to answer questions like: How many particles are there? Where are they? Are they instable and oscillates? Which energy do they have? To do this and operate an accelerator we need diagnostics. Tools and methods which answers our questions.

To operate much of this equipment there are need for additional apparatuses: klystrons which provide high frequency power, power supplies and other.

Even if we can construct all of these devices, and know about the methods, we will not make the accelerator operate if we do not have a significant lack of something. In the accelerators we have to get rid of all disturbing atmosphere, we need vacuum! This is a technology of its own. Vacuum vessels, pumps, diagnostics, cleaning and of course baking (!).

In this work I want to introduce you to all these different topics. To understand how and why you will also need to know something about the physics behind the machines. Thus there is therefore a quick introduction to accelerator physics, relativity and some electrodynamics.

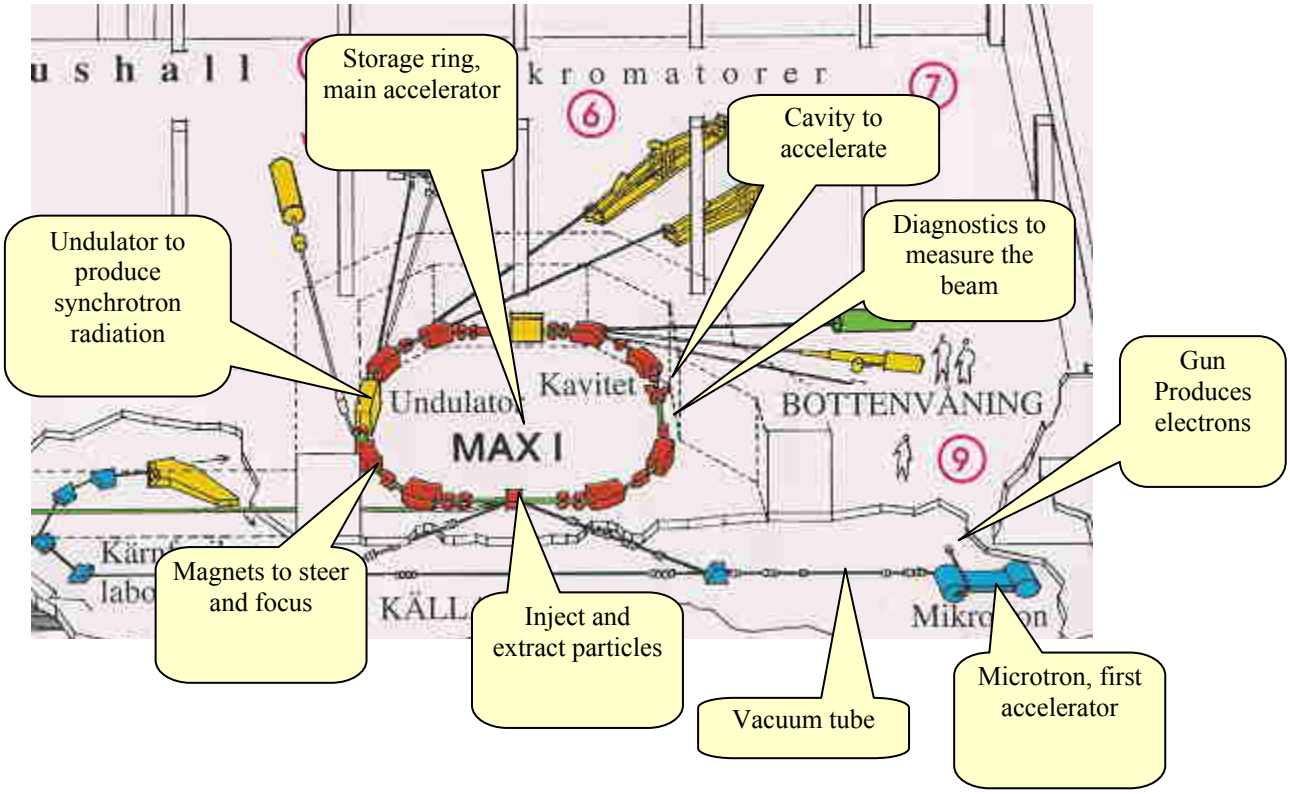


Fig. 1-1. The topics of acceleratorphysics as given at the MAX-laboratory.

2 Physics

In the following chapters the motion and acceleration of charged particles in electric and magnetic fields, the behaviour of these field will be handled. The particles will often be relativistic particles. To form a common base some of the physical relations important for this treatment are given without proof here.

2.1 Basic formulas

The total energy of a particle is given by the sum of the energy of the particle at rest and the kinetic energy due to motion:

$$W = W_o + W_k$$

Eq. 2-1

Where W_o is the energy at rest and W_k the kinetic energy. In the ordinary world around us the energy of an object at rest is so much greater than any kinetic energy we can give it that we can neglect the kinetic energy. We call this the classical world, that is described by classical mechanics. But when we look at small particles, like the elementary particles or particles in the atom nucleus this is not necessarily true anymore. To these particles we can give so much energy that the total energy is dominated by kinetic energy. In this case we talk about relativistic particles or the theory of relativity.

The energy at rest can be written by the help of the mass of the particle:

$$W_o = m_o c^2$$

Eq. 2-2

Where m_o is the mass at rest and c the speed of light (=$3*10^8$ m/s).

We can also write the total energy in a similar form, defining a “new” mass:

$$W = mc^2 = m_o \gamma c^2$$

Eq. 2-3

Where m is the total mass, and γ the "relativistic parameter". If the γ parameter is equal to 1 (one) we have an object at rest and the total energy is only the rest energy. When the velocity increases, the γ increases and so the total energy. The total mass is equivalent to what we normally call mass only when the particle is at rest. The velocities we can give to ordinary objects, like an aeroplane, can not increase the total mass by any significant amount.

An Airbus 330 has a rest mass of $2{,}1*10^{22}$ J. While cruising across the Atlantic the kinetic energy is a mere $6{,}9*10^9$ J

The kinetic energy can thus be written:

$$W_k = W - W_o = m_o c^2 \gamma - m_o c^2 = m_o c^2 (\gamma - 1)$$

Eq. 2-4

The velocity of a particle is often expressed via the β parameter which tells us the fraction of the speed of light. Thus β=0 means no velocity, and β =1 means the speed of light:

$$v = \beta c$$

Eq. 2-5

And we can connect the β with γ via the equations:

$$\beta = \sqrt{1 - \frac{1}{\gamma^2}}, \quad \gamma = \frac{1}{\sqrt{1-\beta^2}}$$

Eq. 2-6

γ starts at the value 1, which means that the particle is at rest (β=0) and the total energy is equal to the rest energy. γ then increases towards infinity. To make β =1, the speed of light, we need to make γ equal to infinity, and thus the total energy equal to infinity. This is of course impossible, but we can get around the problem if the rest mass is equal to zero! (a photon)

2.2 A relativistic particle

We can talk about a "relativistic particle", but when a particle really becomes relativistic is fairly arbitrary. One can invent a number of definitions:

1. The velocity is so close to c (speed of light) that it will not change significantly when we change the energy.
2. The total energy of the particle is larger than the rest energy.
3. The total energy consists of almost only kinetic energy.

In an accelerator we are normally concerned of how particles with different total energies behave. One important issue is whether particles with different energies travel at the same velocity and arrives at the same time to a point down streams. Let us take definition 1 as "our" definition of a relativistic particle.

In figure 1 and 2 the velocity (in reality the β value, which is the velocity divided by c) is plotted for an electron and a proton respectively as a function of total energy. The velocity is zero at the rest energy (0.51 MeV and 938 MeV respectively) and increases first rapidly. When the velocity comes close to the speed of light it does hardly increase any more, or one can say that an enormous increase in kinetic energy is needed to increase the velocity only slightly.
Another important effect is, just as we desired, that at these energies the velocity is *constant*, and does not vary with energy, which is very practical as the time it takes for a particle to pass from A to B is the same regardless of the energy.

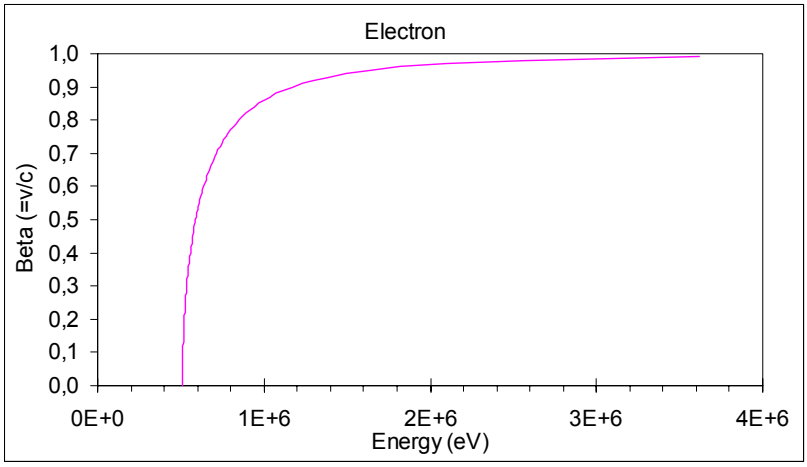


Fig. 2-1. The velocity of an electron at different energies.

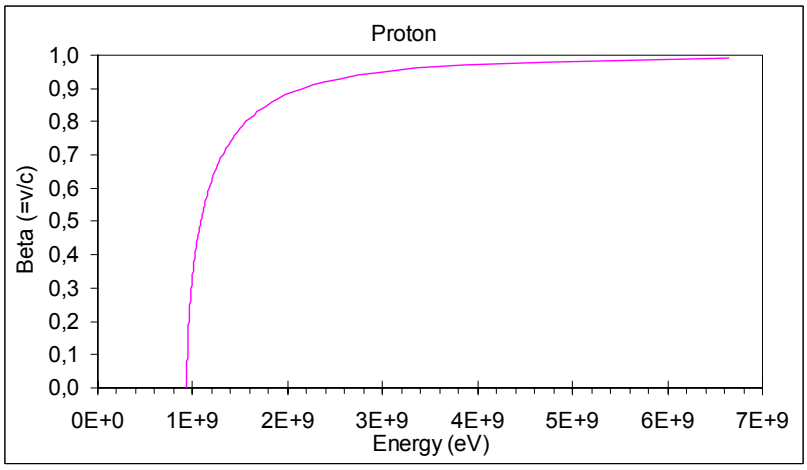


Fig. 2-2. The velocity of a proton as a function of total energy.

In mathematical words we then can say that when β is close to 1 or when γ >>1 the particle is relativistic. But how large should γ be? Assume that γ >= 10 is sufficient. This gives:

$$W = m_o \gamma c^2 = 10 m_o c^2$$

Eq. 2-7

For an electron this means a total energy W_e= 5.1 MeV and for a proton W_p=9.4 GeV to get a relativistic particle. Comparing with the figures above it seems well sufficient. One can also conclude that it is much easier to create a relativistic electron than a relativistic proton.

2.3 Lorentz force equations

Charged particles are influenced by or feel magnetic and electric fields. The forces they experience in these fields are described by the Lorentz equations:

$$\begin{cases} \frac{d}{dt}\left(\gamma m_o \bar{v}\right) = q\left(\bar{E} + \bar{v} \times \bar{B}\right) \\ \frac{d}{dt}\left(\gamma m_o c^2\right) = q\bar{v}\bar{E} \end{cases}$$

Eq. 2-8

Where v is the velocity, q the particle charge, E the electric field and B the magnetic field.
These equations say several things on how to find ways to operate a particle accelerator.

In the second equation the total energy is given by $\gamma m_o c^2$ and the time derivative gives the change in energy of the particle.

- The energy can thus only be influenced by an electric field.

In the first equation the left hand side derivative gives one term related to the change in energy and another term which is the force on the particle.

$$\frac{d}{dt}\left(\gamma m_o \bar{v}\right) = \frac{d\gamma}{dt} m_o \bar{v} + \gamma m_o \frac{d\bar{v}}{dt} = \frac{d\gamma}{dt} m_o \bar{v} + m\bar{a}$$

Eq. 2-9

From the second equation we could see that if there is no electric field, there will be no change in energy and thus the first equation will consist of only the force on the particle. We can further see that

- A particle initially at rest will not be influenced by a magnetic field.
- The magnetic field always acts perpendicular to the velocity of the particle.
- For a relativistic particle (v=c) the electric field has to be very high to compete with a magnetic field. (B = 1 T equals E = 300 MV/m).

2.4 Maxwell's equations

The so called Maxwell's equations tell us about the connections between the electric and magnetic fields in time and space. They are here given completely without any proof. (These equations exist in many different forms)

$$\nabla \times \overline{B} = \mu_0 \overline{J} + \frac{1}{c^2}\frac{\partial \overline{E}}{\partial t} \qquad \nabla \times \overline{E} = -\frac{\partial \overline{B}}{\partial t}$$
$$\nabla \cdot \overline{B} = 0 \qquad \nabla \cdot \overline{E} = \frac{1}{\varepsilon_0}\rho$$

Eq. 2-10

Where B if the magnetic flux density (sloppily called magnetic field), J is the current density, E the electric field strength and ρ the charge density.
Connection of magnetic field strength H to magnetic flux density B is done through:

$$\overline{H} = \frac{1}{\mu_0}\overline{B}$$

Eq. 2-11

2.5 A magnet

Let us now look on a magnet which sits in an accelerator. In an ordinary magnet the magnetic field is constant in time, thus it will not give rise to any electric fields, but remain purely magnetic (E=0, B≠0). This is shown by one of the Maxwell's equations which connects the magnetic fields to the electric fields:

$$\nabla \times \overline{E} + \frac{\partial \overline{B}}{\partial t} = 0$$

Eq. 2-12

In the case where there is no electric field we can see from above that the Lorentz equations become:

$$\begin{cases} \overline{F} = \gamma m_o \dfrac{d}{dt}\left(\overline{v}\right) = q\left(\overline{v} \times \overline{B}\right) \\ \dfrac{d}{dt}(\gamma) = 0 \end{cases}$$

Eq. 2-13

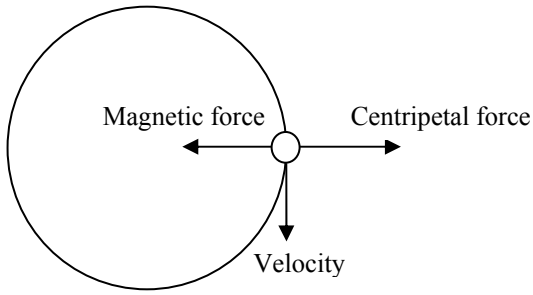


Fig. 2-3. A particle in circular motion.

Just as an example study a particle in circular motion (Fig. 2-3). Classical mechanics give the centripetal force:

$$F = \frac{mv^2}{r}$$

Eq. 2-14

Where r is the radius of the motion. Introducing this into Eq. 2-13 generates (watch out for the x-product, it tells the direction of the magnetic force!):

$$r = \frac{mv}{qB}$$

Eq. 2-15

2.6 Acceleration

To accelerate the particle electrical fields are necessary, while the magnetic ones are useless. This gives the criteria: B=0 and E≠0. But the direction of the electric field is also important. If we want to increase the energy we want acceleration in the same direction as the original velocity: E//v
Introducing this into the Lorentz equations the change in velocity (first equation) and energy (second equation) is given

$$\dot{\gamma} m_o \bar{v} + \gamma m_o \dot{\bar{v}} = q\bar{E}$$
$$\dot{\gamma} m_o c^2 = qvE$$

Eq. 2-16

The change in velocity can be written as

$$\dot{\bar{v}} = \frac{q}{m_o \gamma}\left(E - \frac{\bar{v}\left(\bar{v}\cdot\bar{E}\right)}{c^2}\right)$$

Eq. 2-17

The electric field will both change the energy of the particle and change the amplitude and direction of the velocity.

3 Radiation protection

The reason for all protection against radiation is of course the risk of biological effects. All kind of radiation can affect living organisms in a good or evil way. In this chapter the evil effects will be treated.

In this description most of the values comparing with natural radiation are omitted, as they often are used to give a feeling of diminished risks of radiation. The risks should instead be presented as absolute, relative to the real danger of illness.

3.1 What do we mean by "ionising radiation"?

Radiation may be seen as a flow of particles. If the energy of these particles is sufficiently high to ionise atoms or molecules then the radiation is designated ionising radiation.
The particles could be either charged or neutral and light or heavy:

	Light	**Heavy**
Charged	electrons (β^-) positrons (β^+)	protons (p) helium nuclei (α)
Neutral	photon (x or γ)	neutron (n)

In addition to these we sometimes have to consider rare particles as mesons (π^+, π^-, π^0) and muons (μ^+, μ^-).

The radiation field is defined by:

- the kind of particles.
- energy distribution.
- the spatial distribution of the flux density.

3.2 Different sources of ionising radiation

One can divide the radiation sources in *natural* and *artificial*.

3.2.1 Natural sources

In nature one finds some atomic nuclei that are not stable (radionuclides), but rather transform into some lighter nuclei. In this process they emit either an α-particle or a β^--particle and most frequently also a photon.

One example is the ^{40}K-atom, which amount to 0.01% of all potassium in nature. In the human body we have about 0.1 kg potassium, thus approximately 10 mg of ^{40}K-atoms. In all they emit approximately 3000 β^--particles per second, which amounts to a non-negligible radiation dose. Also in our surroundings, the air and the soil, we find radioactive nuclei. It is predominantly uranium ^{238}U and thorium ^{232}Th, together with their decay product nuclei. It is interesting to note that these radioactive nuclei are relics from the time of birth of the planet earth. It is only because of their long decay times that they are still present.

Another natural radiation source is the cosmic radiation. In free space there is a flux density, consisting mainly of stripped nuclei, of roughly 4 particles/ cm^2/sek. The different nuclei are present at approximately the same abundance as are the atoms in the whole universe. This means roughly 90 % hydrogen, 9 % Helium and all the heavier atoms together 1%. The mean energy of the particles is 10^{10} eV, but energies as high as 10^{20} eV (!) have been observed. However, these primary cosmic particles will never reach us humans (except austronautes travelling in space), since the probability of several cascade interactions in the atmosphere is extremely high. At ground level the main radiation constituent is muons, which interact mainly by ionisation in all materials, as for example the human tissue.

3.2.2 Artificial sources

When it comes to the artificial sources one might first think of radionuclids that are produced and spread by nuclear bombs or by nuclear power plants. ^{90}Sr, ^{137}Cs, ^{131}I, ^{14}C, ^{3}H, ^{85}Kr, ^{239}Pu are examples of such radionuclids. The magnitude of these sources' radiation contribution to the environment is of course subjected to large local variations. Care must be taken to study not only the production processes and decay times, but also the ecological transfer rates.

The use of radiation in the health care, even if averaged per capita, gives by far the largest artificial contribution to human irradiation. One can divide the use of ionising radiation into three groups; x-ray diagnostics, radio-diagnostics and radio-therapy. In radio-diagnostics different radionuclids are injected and traced as they are spread or accumulated in different organs in the body. In x-ray diagnostics one makes, as the name indicates, use of an x-ray source. The radio-therapy is performed either by highly concentrated radionuclid spiecement, such as for example ^{60}Co or by radiation produced by accelerators, for example low-energy electron linear accelerators.

Research facilities that utilise man-made accelerators, devices that accelerate charged particles, are additional artificial sources of radiation. We have already touched upon two kinds of accelerators; the x-ray tube and the linear electron accelerator. No man-made accelerator has yet reached energies equal to the most energetic cosmic particles. Similarly to the use of radiation in health care, research facilities built on accelerators do not expose the public to any radiation.

3.3 The biological system of the human body

The human body is made up of an enormous number of cells. There are several different types of cells: nerve cells, blood cells etc. In each cell there are some main constituents which conduct the work in the cell:

In the cell nucleus there is the DNA-molecule. The DNA is a kind of memory with information about the different processes the cell performs. In the cell nucleus RNA is produced with the DNA as a pattern. The RNA carries the information into the cell to the ribosomes, where the RNA is used as a pattern for the synthesis of protein.

In the cell there are also the mitocondries, which transform energy from sugar and other molecules into more easily available energy.

The radiation damage to the human body is usually directly on these bio molecules, or indirectly via other chemical reactions.

3.3.1 Indirect chemical reactions

About 80% of the cell is made up with water and most of the energy deposited by the radiation will be to water molecules. They then produce free radicals ($\bullet OH$, $\bullet H$ and $\bullet HO_2$) which are very reactive and can change or break up the bindings of bio molecules.

3.3.2 Direct reactions

Direct reactions are normally ionisation of the bio molecule.
When one of the chain in the double helix of the DNA breaks the most likely result is that it repairs itself. In the cases when both chains break the cell is seldom repaired but dies. In some rare cases there will be a lasting error in the DNA which will reproduce when the cell divides. This is called a mutation.

Damage to other molecules in the cell are less critical as there are plenty of them.

When a damage has occurred it can give either genetic or somatic effects. A genetic effect occurs in the reproductive cells and effect only coming generations while somatic effects already the individual exposed and can be either long-term (latent) or short-term (acute).

3.3.2.1 Genetic effects

Studies so far have not been able to show any genetic effects on human beings. The main studies have been made on children born by parents who survived in Hiroshima or Nagasaki. In a group of 50.000 children no statistical rise could be seen. It is important to stress that only easily observable effects have been studied and also only on the *first* generation.

3.3.2.2 Acute somatic effects

Acute somatic effects are caused by the death of a number of cells. Thus the most sensitive types are the ones that often divide themselves which means organs like bone-marrow and intestines. If the bone marrow is damaged the body will soon lack white blood cells and blood plates which weaken the defence against infections and makes coagulation more difficult. If the intestines are damaged the body will loose salt and water after a few days.

These effects are of course coupled to doses (very roughly):
4 Gy on the whole body will cause death by damage on the bone marrow.
2-3 Gy vomiting, diarrhoea and after a few weeks hair loss, bleedings and inflammations.
1 Gy No acute effects.

3.3.2.3 Latent somatic effects

The most well known latent effect is cancer: lung cancer, thyroid gland cancer, leukaemia, breast cancer stomach cancer and skin cancer.
Another latent effect is cataract which will make the eye lens opaque.
It is difficult to reach a picture on the risk after irradiation though studies have been made on populations exposed to radiation as the risks have to be treated statistically.

3.3.2.4 Effects on the foetus

These effects can not be seen as genetic as the damage is directly to the foetus. In an early stage of development the organism is much more sensitive to radiation as it is growing rapidly. The affects are among others: mental disturbances and inhibition of growth.
Thus acceptable limits of irradiation during pregnancy are probably so low that many radiological works are not possible.

3.4 Dosimetry

In radiation protection work the main duty - apart from predicting radiation levels - is to perform measurements from which one can conclude if the present radiation field pose any health risk; and in that case take action so that radiation exposure is brought down to tolerable levels.
In order to quantify such a risk, the most natural quantity to measure seems to be the energy absorbed from the radiation, by the human body. Since not always the whole body is irradiated homogenously, it is more convenient to define a local quantity, *absorbed dose, D*, which tells us *the energy deposited per mass unit*, in a very small volume of tissue, or more generally, in any material.

$$D = \frac{d\overline{E}}{dm}$$

Eq. 3-1

where $\overline{E}$ is the expectation value of the deposited energy in the volume element and m is its mass. The SI-unit [J/kg] is most often referred to as [Gy] (Gray). Often one is also interested in the rate at which rate the energy is deposited, that is the *absorbed dose rate*:

$$\dot{D} = \frac{d}{dt} D$$

Eq. 3-2

It has turned out that the biological effects due to radiation do not scale lineary with the absorbed dose. At a the microscopic level, different types of radiation will deposit energy more or less densly. In different specific cases a *relative biological effectiveness, RBE,* of different kinds of radiation has been measured. As a standard radiation one use 200 kV xrays, for which *RBE*=1. As expected, the *RBE* grows with increasing dE/dx. By multiplying the absorbed dose and the *RBE* one would get a better measure of the biological effect from a certain radiation field. However, the *RBE* is defined for a number of different specific cases, and we want to use some quantity that can be used more generally. One has therefore defined a *quality factor, QF*, by which the absorbed dose should be multiplied to give the *dose equivalent*, *H*.

$$H = QF \cdot D$$

Eq. 3-3

The value of the dose equivalent achieved, may then be used to compare the risks posed by different radiation fields The SI-unit for dose equivalent is [J/kg] or [Sv] (Sievert).

Table 3-1. Dose limits for persons working with ionising radiation

	Period of time	**Quantity**	**Limits**
Workers in general	Annual	Effective dose	50 mSv
		Dose equivalent to the lens of the eye	150 mSv
		Dose equivalent to the skin	500 mSv
		Dose equivalent to hands, forearms, feet and ankles	500 mSv
	In addition, for five consecutive years	Effective dose	100 mSv
Students and trainees, aged 16-18 years	Annual	Effective dose	6 mSv
		Dose equivalent to the lens of the eye	50 mSv
		Dose equivalent to the skin	150 mSv
		Dose equivalent to hands, forearms, feet and ankles	150 mSv

The concept of equivalent dose may be used both for separate organs and the whole body, if homogeneously irradiated. In case the body is not irradiated homogeneously one has to use yet another quantity, *the effective dose*, in order to quantify the risks. Different organs in the body have different sensitivity to radiation and thus weighting factors can be defined (examples: bone marrow - 0.2, Liver - 0.05, skin - 0.01) if not the whole body is exposed. The value for the whole body irradiation should be multiplied by the weigthing factors to get the risk if "only" one organ is irradiated.

To conlude, it is important to stress that radiation dosimetry may be performed along two different paths.

- One way is to measure the actual radiation field, and fom these data derive the absobed dose and the equivalent dose. However, this is difficult and when it comes to dose rates in bodies that affects the radiation field significantly, it will become even more complicated.

- The other way is to measure the absorbed dose rate by help of a detector that *resembles* biological material, a tissue-equivalent detector. Without any knowledge of what kind of field is present, the quality factor is not known and, hence, the equivalent dose cannot be determined.

3.5 Basic Interactions

In this part the basic interactions involved in stopping the electrons will be described. This means not only interactions 'electron ⇔ matter' but also interactions from all of the secondary particles that could possibly be produced during the stopping process. Some of the secondary particles will interact and give rise to new secondaries, which will again interact etc. In this way a cascade of particles will be produced. By studying the basic interactions we can conclude how this cascade or "shower" develops in different materials, and from this knowledge it is possible to make statements on the shielding design.

A pleasant fact concerning the problem, is that our primary particle (electron) is a lepton, and leptons do not interact strongly, i.e. they do not feel the nuclear forces. Instead they interact electromagneticly, and the secondary particles will be pre-dominantly electrons, positrons and photons (At MAX-lab we may neglect muons due to the low cross-section for muon pair production below 1.5 GeV). There would have been no exceptions if it was not for the fact that nuclei are charged and can be excited electromagneticly. An excited nucleus can get rid of its extra energy by emitting one or several nucleons. These are hadrons, which feel the strong force, and may interact with other nuclei. Nevertheless, electromagnetic processes are dominant.

3.5.1 Electromagnetic processes

3.5.1.1 Collision losses

A charged particle will interact electromagnetically and ionise or excite atoms while transferring energy.
The energy loss per unit length also called stopping power is given by

$$-\left(\frac{dE}{dx}\right)_{coll} = 4\pi NZ \frac{e^4}{(4\pi\varepsilon_0)^2} \frac{z^2}{m_e v^2} B$$

Eq. 3-4

where E is the kinetic energy of the particle, N the atomic density of the material, Z the atomic number. Z is the charge number of the particle , v the velocity and B a slowly varying function.

- The stopping power is larger at lower velocities.
- The stopping power is independent of the mass of the particle.
- Highly charged particles are more efficiently stopped.
- A high NZ of the stopping material means a high stopping power.

At low energies the formula above is not correct.

Examples on penetration ranges:

	Energy (MeV)	**In air**	**In water**
α-particle	5	4 cm	50 um
proton	5	30 cm	0.3 mm
proton	100	80 m	8 cm
electron	5	20 m	3 cm
electron	100	200 m	30 cm

3.5.1.2 Radiation losses

The incoming particle interacts with the electrons of the stopping material. Thus the incoming particle is deflected, accelerated, and emission of electromagnetic radiation will occur.
The stopping power is given by:

$$-\left(\frac{dE}{dx}\right)_{rad} = \frac{16}{411} NZ^2 \frac{e^4}{(4\pi\varepsilon_0)^2} \frac{z^4}{(Mc^2)^2} E \ln\left(\frac{233M}{Z^{1/3} m_e}\right)$$

Eq. 3-5

The energy loss due to bremsstrahlung has the following characteristics:
a) More radiative losses for higher energy particles
b) The radiation loss i dependent of the mass of the particle
c) The efficiency of the stopping material is proportional to NZ^2.

The photons can interact by: pair production, compton scattering or photoelectric effect. The attenuation of the photons follows an exponential decay:

$$I(x) = I_0 e^{-\mu x}$$

Eq. 3-6

where u is the linear atttenuation coefficient taking different values for the different processes.

3.5.2 Hadronic processes

Only hadronic processes initiated by an electron beam, will be studied. We have for this case already stated that the electromagnetic processes are dominant. Hence, the radiation field will mainly be composed of electrons, positrons and photons.

If a nucleus becomes excited to an excess energy above the nucleon binding energy, it may relaese a nucleon. It is almost always a photon, and not an electron or positron, that excites the nucleus, because the latter processes have much smaller cross-sections. The threshold energy is typically in the range 4-6 MeV for heavy materials, and 10-19 MeV for lighter materials. The cross-section for this so-called "giant photonuclear resonance" has quite a broad maximum around 20 MeV but falls off rapidely above 30 MeV. At much higher energies, around 200-300 MeV, another nuclear mechanisms allows for nucleon production with a cross-section of almost the same order as the giant resonance. It is called the "photo-pion resonance".

With a proper radiation shield the electromagnetic shower will be developed so far that most photon energies outside the shield are below the giant resonance threshold. This implies that only (γ , n)-reactions are of concern, since (γ , p) or (γ , α) processes will lead to a fast energy transfer by ionisation within the sheilding material.
The neutrons produced show a spectrum of energies with a mean energy at a few MeV depending on target nucleus. Most important though, is that the emission is *isotropic*.

3.5.2.1 Neutron stopping processes

The neutrons can only interact strongly, that is interact with nuclei. Interactions will predominantly be through *elastic scattering* and *neutron capture*.
The neutron capture process will remove the neutron. However this happens mostly at low neutron energies, less than 1 keV. It is therefore of high importance to slow down the produced neutrons, and this is achieved by a shielding material that effectively removes energy from the fast neutrons.

Light nuclei are effective to slow down the neutrons, but heavy nuclei merely act as fixed scatterers. Materials like water, paraffin and concrete (consisting mostly of O, H and Si atoms) are therefore used to protect agianst neutrons.
The neutron is not a stable particle. It decays into three particles, a proton an electron and a neutrino. However, the neutron mean lifetime is about 15 minutes so in practice the decay will rarely take place. Instead a neutron capture process will be dominant. Hydrogen itself is quite an

effective capturer, but an even better capturer is Boron. Concrete may be borated to give an effective shield against neutrons.
Even though the number of neutrons produced is not at all as high as the number of electromagnetic particles, they could in radiation protection work still constitute main radiation field outside the shield

3.6 Radiation shielding

To design a radiation shield one first must adopt a principle of what radiation levels are acceptable. Then one should look at what kind of radiation that has to be considered and also what are the power levels in the accelerator. Finally one takes in most cases a combination of lead and concrete to provide the shielding necessary.

Questions that are such as:
- The peak power of the accelerator might be huge, but should one look on an average?
- Which operation mode produces most radiation? (injection, collision, extraction…)
- Which currents will be used?
- Should one consider "worst cases" like total beam loss in one spot?
- How many hours will the machine operate?

At MAX-lab one has chosen to have radiation levels outside the concrete shielding which are below the natural background of 5 mSv/year. One then consider three different types of radiation:
- The electromagnetic component (gamma and electrons/positrons)
- The intermediate and low-energy neutrons (E_n<10 MeV)
- The high-energy neutrons (E_n>10 MeV).

The complete accelerator system is enclosed in concrete of varying thickness as certain parts have higher radiation levels due to mainly the injection process. Some parts are then enclosed in lead very close to the accelerator.

3.6.1 **Example:** Equivalent dose from the electromagnetic component.

As an example let's look at a part of the new injector for MAX-lab. At a certain critical point in the machine we have a flux of $2.5*10^{10}$ electrons/s at 0.25 GeV in a thick iron magnet. The iron is roughly 20 cm.

By assuming that the initial shower development in the iron is similar to the one in a copper target we can take some empirical data (from DESY, Hamburg) saying that after 20 cm of copper a flux of $5\cdot10^{9}$ electrons/s at 6.3 GeV, gives the dose 300 mSv.

On this magnet there will be a layer of 16 cm of lead and 90 cm of concrete wall.
The electromagnetic component of the dose rate immediately outside this shield will then be given by this scaling formula:

$$H_\gamma = \frac{\Phi_e}{\Phi_0}\frac{E_e}{E_0}H_{\gamma 0}e^{-l_{lead}\cdot\lambda_{lead1}}e^{-l_{concrete}\cdot\lambda_{concrete}}\cdot 3600\text{mSv/h}$$

Eq. 3-7

where Φ is the electron flux, E the electron energy, $H_{\gamma 0}$ the dose, l the material thickness and λ the attenuation constant.

The attenuation constants are given by:
$\lambda_{lead} = 0.7\ \text{cm}^{-1}$ $\lambda_{concrete} = 0.054\ \text{cm}^{-1}$
and thus the dose rate in this case by:

$$H_\gamma = \frac{2.5\cdot10^{10}}{5\cdot10^{9}}\frac{0.25}{6.3}300e^{-16\cdot0.7}e^{-90\cdot0.054}\cdot 3600\text{mSv/h} = 23\mu\text{Sv/h}$$

Eq. 3-8

This is then the dose rate from the electromagnetic component and to this one has to add the contributions from other components, such as slow and fast neutrons.

3.7 Detectors

To detect the radiation levels of different kinds of radiation a number of detectors are available.

Gas chamber detectors use the effect that photons and electron/positrons ionise a gas in a tube. The produced ion pair is collected by a voltage between an anode and cathode and the current is then proportional to the amount of radiation. If a higher voltage is applied a "total discharge" builds up for each ion pair produced making detection easier, but the proportionality is lost. This device is called a Geiger-Müller counter, or GM-tube.
The sensitivity for very high energy photons is limited though in the gas chambers.

Thermo Luminiscence detectors use the fact that the radiation can excite electrons in the detector material to states where no spontaneous de-excitation route exists. By later, after the exposure to radiation, heat the material the electrons can return to their groundstate while emitting a photon. By counting the photons the amount of radiation can be detected.

Neutron detector. To detect neutrons an gas chamber, ionisation chamber, is enclosed in a hydrogen rich material which moderates, slow down, the neutrons which then can be detected.

4 Accelerators

A particle accelerator is a device that can increase the kinetic energy of a particle. Remembering the Lorentz force equation of chapter 2 we can se that the one uses electric fields in different forms to accelerate particles. To change the energy it is a necessary condition that the charge of the particles is different than zero (0). If we would like obtain a high energy neutral particle, we would need to accelerate a charged particle, which is then used to generate the neutral particle.

To fulfil the needs to handle many different particles in different energy ranges a number of accelerators have been constructed. They vary in field characteristics:

Electric accelerating field: uniform / low frequency / high frequency
Magnetic guiding field: zero / homogenous / varying in space and/or time

A few "families" of accelerators can be defined depending of these characteristics (Table 4-1).

Table 4-1. The particle accelerator zoo.

		B-field	Radius	Frequency
Linear machines	accelerate particles in a straight line • LINAC • electron gun • electrostatic generators	B = 0	r = ∞	df/dt = 0
Circular spiralling machines for heavier particles	Both relativistic and non relativistic particles • cyklotrons	(dB/dt = 0)	dr/dt ≠ 0	
Circular spiralling electron machines	Relativistic particles • microtrons		dr/dt ≠ 0	df/dt = 0
Circular machines	• electron synkrotron • proton synkrotron • storage ring	dB/dt ≠ 0	dr/dt = 0	df/dt = 0

4.1 Linear accelerators

4.1.1 Electrostatic accelerators and the electron gun

The most simple type of accelerator uses a DC electric field with no magnetic field. By applying the voltage between two points in space an electric field is created and a charged particle experiences a force and is accelerated.

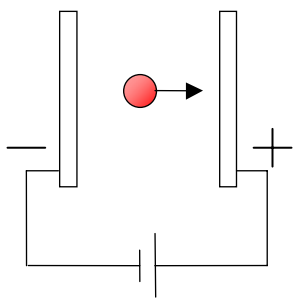


Fig. 4-1. An electron (negatively charged) is accelerated

An electron placed between two plates with different polarity will be attracted to the positively charged plate Fig. 4-1.
Most well known are may be the van der Graaff accelerators first built in the 1930ies. The main part of the accelerator is the van der Graaff generator which produces very high voltages between the electrodes. A conveyor belt carries positive charge up to a collector and the high-voltage

electrode. Refined technical methods have given machines called: Pelletron, Laddertron, Tandem accelerator etc. The voltage produced in these accelerators mounts to several MV.

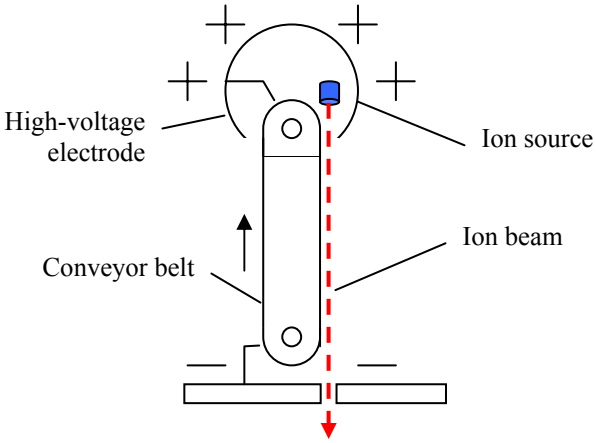


Fig. 4-2. The van der Graaff generator.

The electron gun also uses a steady electric field for acceleration, but it is mainly a device to produce electrons.

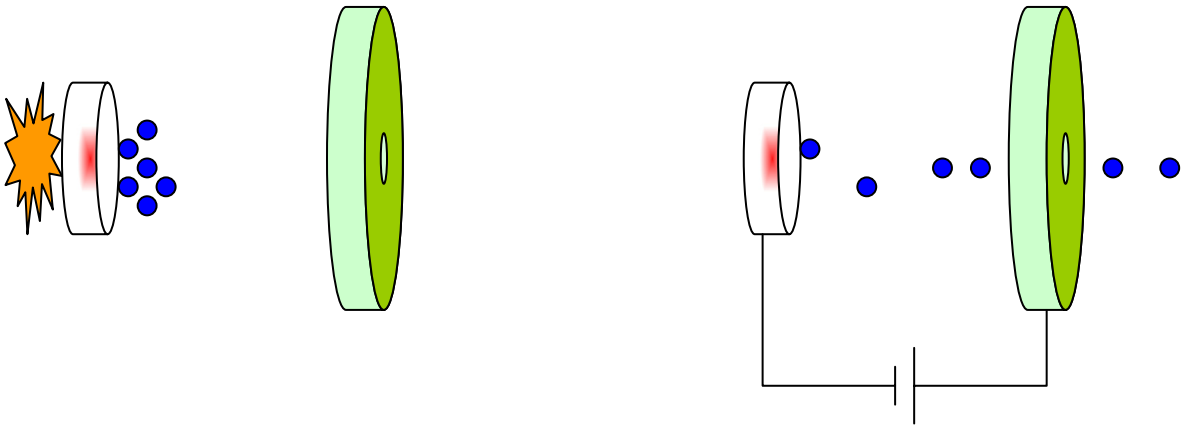


Fig. 4-3. The electron gun. a) The cathode is heated up and electrons emitted. b) The electrons are accelerated towards the anode.

In the electron gun (Fig. 4-3) the cathode is heated to a temperature when electrons are emitted. A voltage is put between and cathode and the anode and the electrons are accelerated towards the anode. A small hole in the anode "fools" the electrons and a beam of electrons is achieved. The energy of the beam is given by the voltage. A voltage of 1000 V gives an energy of 1000 eV for the particle. In practise voltages up to many kV can be applied before sparking occurs between the cathode and the anode. To reach higher energies several accelerating gaps and/or rapidly alternating fields can be used. The alternating fields are switched very rapidly and a spark does not have time to build-up.

There are several techniques to achieve the emission of electrons from the cathode. A growing method is to use photoemission where the cathode is irradiated by a strong laser pulse and electrons emitted. This method gives several advantages compared to ordinary heating:

- The timing of the emission can be precisely defined.
- The area of emission can be very localised.
- The amount of electrons (charge) emitted can be very high.

4.1.2 LINAC

LINAC is an acronym for LINear ACcelerator and simply means an accelerator in which particles follow a straight line.

As mentioned above it is necessary to divide the acceleration process into steps and/or operate the voltage (E-field) in an alternating fashion to reach high energies. The frequencies necessary are high and one thus talks about radio frequency accelerators.

One problem directly arises here: timing. As the particles are accelerated they, before being relativistic, gain velocity. A time varying field thus has to be controlled to achieve a proper phase between the field and the particles.

By a method proposed by Ising and Wideröe, Wideröe constructed in the 20ies a linear accelerator. It consists of tubes which are charged by an alternating source. Each tube carries the same potential, but with alternating sign, and the acceleration occurs in the gaps between the tubes. Every second gap thus gives an acceleration and every other a deceleration. The length of the tubes is increasing and tuned in such a way that the time it takes for a particle to pass a tube is constant despite the increasing velocity.

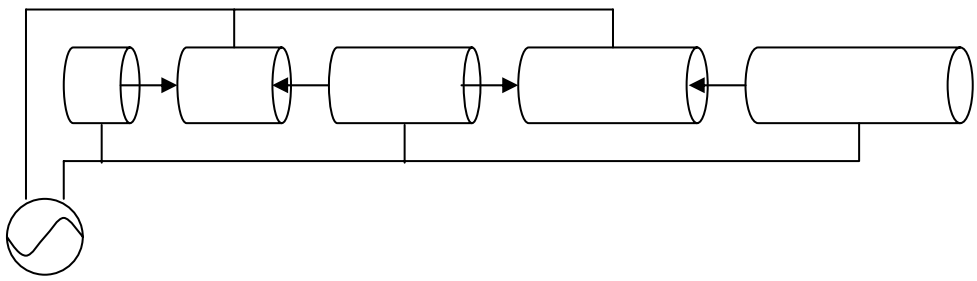


Fig. 4-4. The Wideröe accelerator.

As the field changes direction it is just a small moment when acceleration occurs. The particles that pass the gap just at that occasion are properly accelerated, all others are lost. Wideröe managed to accelerate heavy ions up to 50 keV.

Instead of powering each tube or electrode separately an electromagnetic wave can be made to either propagate or oscillate within a structure. Alvarez mor or less enclosed the Wideröe structure in a long tube (Fig. 4-5). A wave enetering the structure at one end will travel down through the tube and high electric fields will bild up at the gaps.

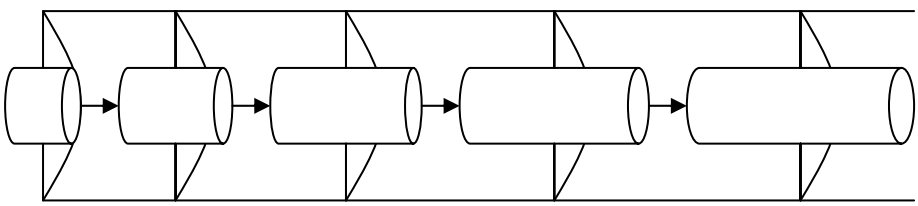


Fig. 4-5. Alvarez structure.

The Alvarez structure can handle fields with much higher frequencies and voltages than the Wideröe structure and the field in the gaps points in the same direction in all gaps. The Alvarez structure is still used to accelerate ions.

The waves in such a LINACs can be either standing waves or travelling waves (Fig. 4-6). In a standing wave the field strength at one points oscillates between a positive and a negative value. In between these points there are node points, which always has zero field strength. A travelling wave moves along the structure with a certain phase velocity. Thus a point of maximum field will move along the structure and all points will at some moment have maximum field strength.

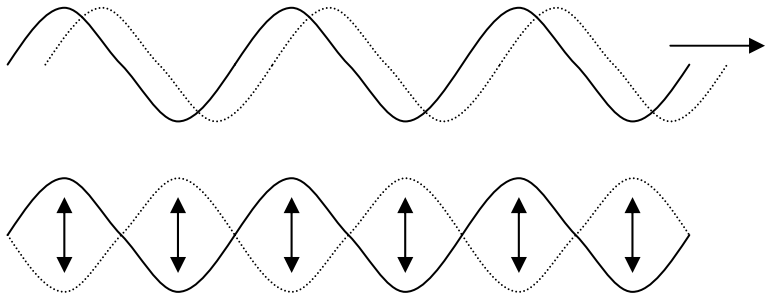


Fig. 4-6. A travelling wave and a standing wave.

A travelling wave LINAC is in principle a tube in which a number of discs are introduced at specific intervals. These discs, the amount, the hole size and the distances, define the velocity with which the wave propagates. It should be tuned to follow the particles along the accelerator. In an electron LINAC this becomes easy as the electrons are relativistic above 5 MeV, and the wave propagation velocity should be constant.

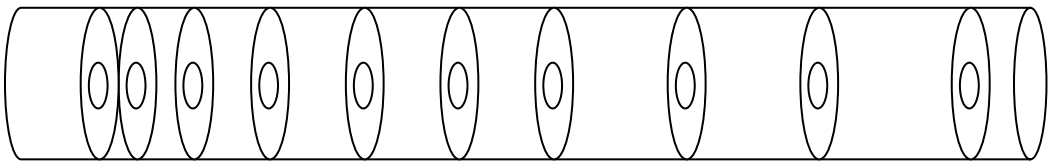


Fig. 4-7. A travelling wave LINAC.

A standing wave LINAC consists of a number of cavities in which the field oscillates.

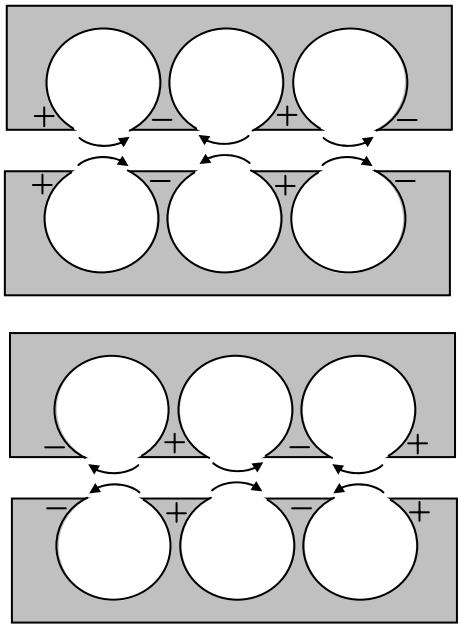


Fig. 4-8. A standing wave linac. The fields at half a period time difference.

The power into a LINAC stucture, or most high power RF systems, is supplied by a klystron (see chapter on Guns and klystrons) which is an RF amplifier.

The particles in a LINAC experiences what is called *autophasing*. This means that they automatically finds a suitable phase relative the wave in the LINAC.

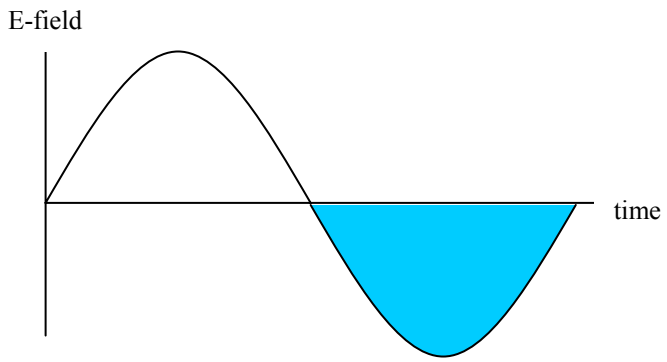


Fig. 4-9. The accelerating phase of the electric field (for negative charges).

As an example an electron present at a certain point during the striped time will gain energy, while the other electrons will loose energy and be lost (Fig. 4-9).

But electrons arriving at different times during the "gain time" will gain different amount of energy. If we define a reference field as the field which gives the electron exactly the right amount of energy to stay in phase with the accelerating field, all other electrons will receive more or less energy (Fig. 4-10).

The electrons 1 and 1' will gain slightly less energy than necessary and thus arrive a little later in next period. The electrons 2 and 2' will gain slightly more energy and arrive a little earlier.

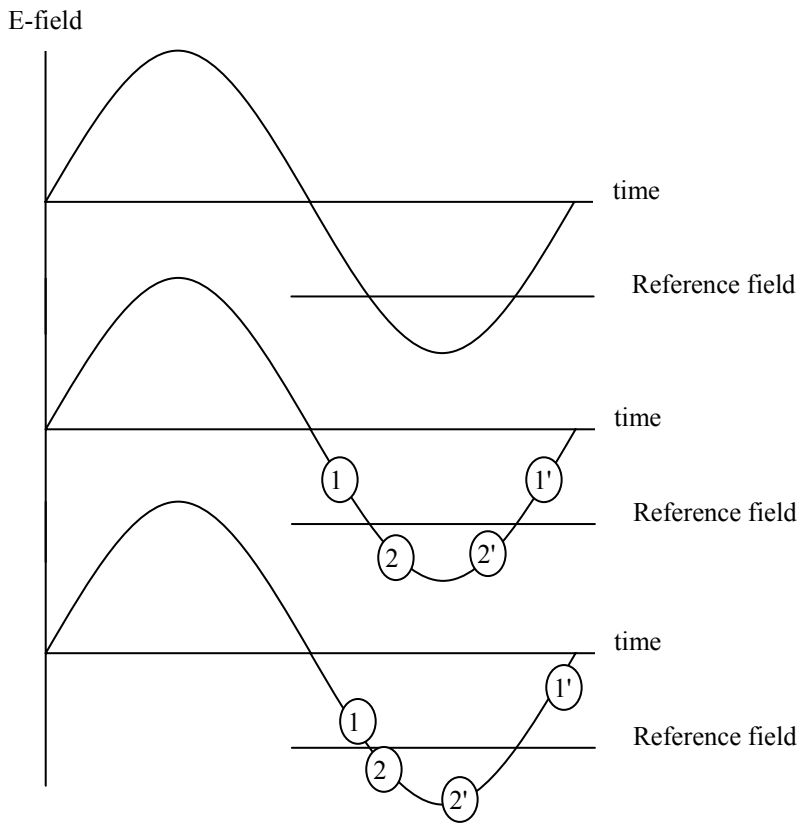


Fig. 4-10. Autophasing in a LINAC.

The electrons with primes (') are moving away from the reference field and will be lost. The electrons without primes are moving closer to the reference field and will oscillate around this stable point. They are in a way "automatically" finding the right phase.

4.2 Circular accelerators

In a circular accelerator the basic condition is given by the circular motion in a magnetic field.

$$r = \frac{mv}{qB}$$

Eq. 4-1

Which can be rewritten into

$$v = \frac{rqB}{m}$$

Eq. 4-2

Thus the time for one revolution is given by

$$T = \frac{2\pi r}{v} = \frac{2\pi m}{qB}$$

Eq. 4-3

Here several machines exist which use either: time constant magnetic field B, constant radius r or constant time T. Below follows a choice of some typical machines.

4.2.1 The (classical) Cyclotron

The cyclotron is a machine with a constant magnetic field where the radius increases as the particle gains energy. The time for one revolution must remain constant though, which is the main limitation of this device.

A constant magnetic field (a vertical field in Fig. 4-11) gives a motion in the horizontal plane between the two electrode planes. The electrodes are powered such that a horizontal electric field is present between the two semicircles. The frequency of this field is typically 10 MHz. The particle is crated in a source in the middle of the cyclotron and accelerated in increasing circles. In each passage of the gap between the electrodes it gains energy.

Between two passages of the electrode gaps the field has to change direction. As the frequency with which the electric field changes is constant the time between two passages always has to be the same. According to Eq. 4-3 the time is given by:

$$T = \frac{2\pi r}{v} = \frac{2\pi m}{qB} = \frac{2\pi m_o \gamma}{qB}$$

Eq. 4-4

When the velocity is low γ stays close to one and the time is more or less constant. Assume that a γ <= 1.01 is ok, then the final energy of the cyclotron is $W=1.01W_k$. For an electron this would mean a "useless" 5 keV but for a proton near 10 MeV.

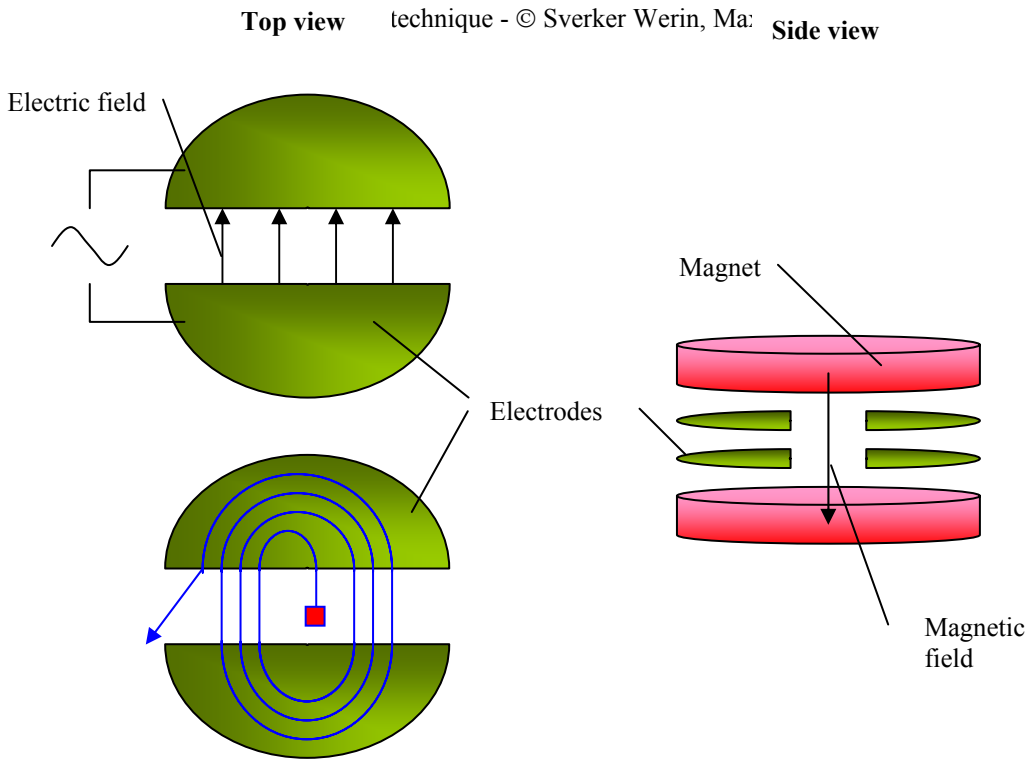


Fig. 4-11. The Cyclotron.

4.2.2 The Synchrocyclotron

In order to override the problem of dephasing after a number of turns one can changes the frequency of the electric field. From Eq. 4-4 one sees that when γ increase the time, T, has to do the same, this means that the frequency, $f = 1/T$, has to decrease. Much higher energies can be achieved, but a disadvantage is that particles can only start to accelerate when the frequency is at maximum. The beam will be pulsed with a much lower average intensity than for an ordinary cyklotron.

4.2.3 The Isochronous cyklotron

From Eq. 4-4 one can see another possibility and that is to change the magnetic field. If the magnetic field becomes stronger for higher energies = larger radia a constant time, T, could be achieved (Fig. 4-12).

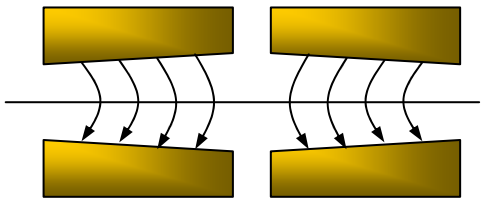


Fig. 4-12. The magnetic field for an "improved" cyklotron.

Unfortunately this does not work. A particle which is slightly off the median plane of the device will not experience a perfectly vertical field. The force, which is always perpendicular to the velocity and the magnetic field, will move the particle further away from the median plane and the particle will thus be "defocused" and lost after a while.

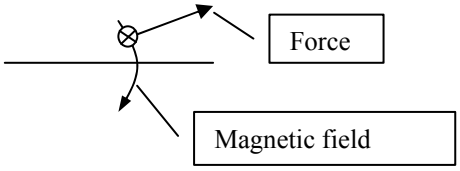


Fig. 4-13. Focusing from the field in Fig. 4-12.

This problem can be solved though by introducing a magnet which varies in both r and θ. As the particle passes the edge with a small angle the field extending out from the edge will give a vertical force. This force will focus the particle back towards the medium plane and counteract the defocusing force introduced by the magnetic field increasing with the radius. The particle will be stable and the machine is called *Isochronous cyklotron* or isosynchrotron (Fig. 4-14).

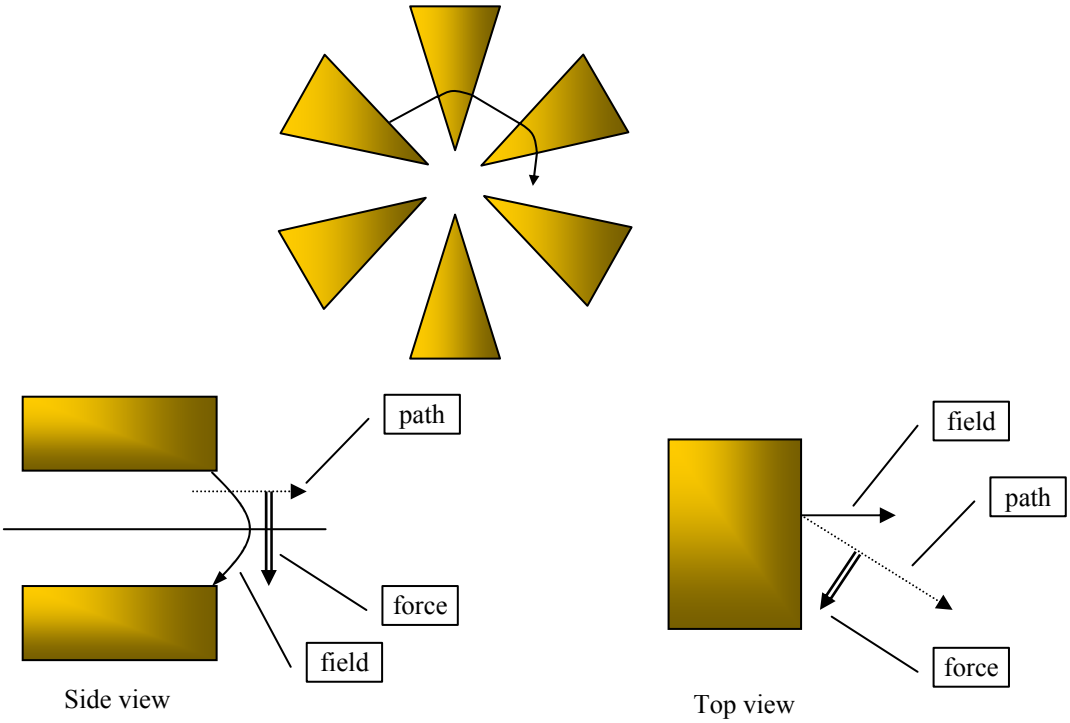


Fig. 4-14. The Isosynchrotron and edge focussing in the machine.

4.2.4 The Mikrotron

The different cyclotrons discussed above are all suitable for "heavy" particles. For electrons (and positrons) sligthly different machines are used. The acceleration is normally made by a high frequency (500 MHz and 3 GHz are typical frequencies) electric field which is "stored" in a cavity. In the microtron such a cavity is placed inside a large magnet. The time it takes for the electron to go one turn in the microtron corresponds to an integer number of periods of the HF-field. Each turn will be an integer number of periods longer than the previous. The timing is here easy to keep as the velocity is constant (relativistic particles). The revolution time is given by

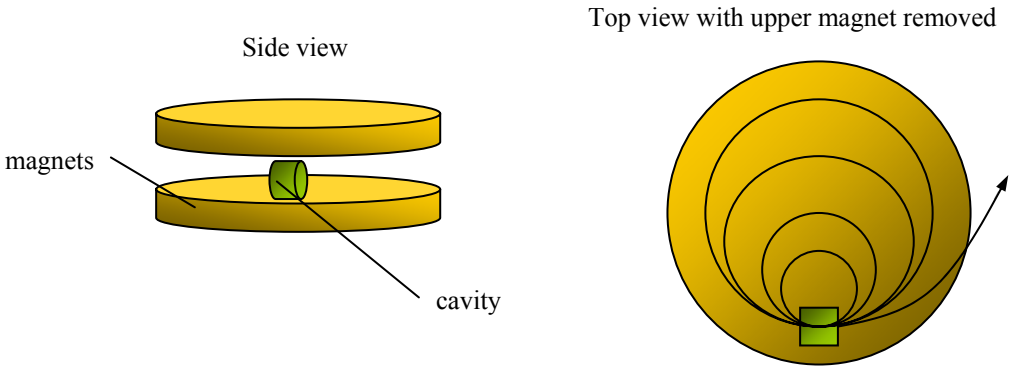


Fig. 4-15. The microtron.

$$T = \frac{2\pi r}{v} = \frac{2\pi m}{qB} = \frac{2\pi W}{qBc^2} = \frac{2\pi}{qBc^2}\left(W_o + W_k\right)$$

Eq. 4-4

The increase in time for one revolution is

$$\Delta T = \frac{2\pi}{qBc^2}\Delta W$$

Eq. 4-5

Which whould be an integer number of periods in the HF-field.

$$n\lambda = n\frac{c}{f} = nc\Delta T = nc\frac{2\pi}{qBc^2}$$

Eq. 4-6

A slightly different machine is the racetrack mikrotron in which the main magnet is split into two halfs and the accelerating cavity is enlarged into a LINAC. The principle is the same but larger energies can be given to the electron in each turn. A racetrack mikrotron can accelerate electrons up to a few 100 MeV.

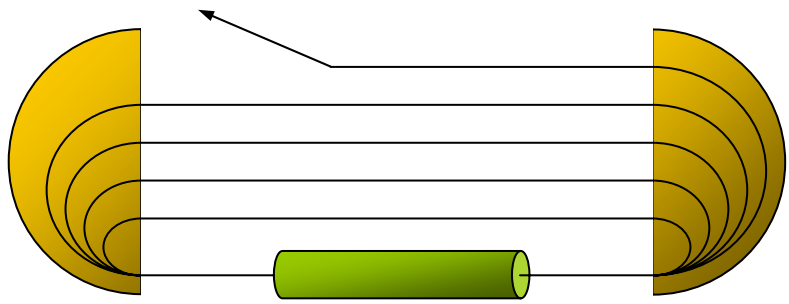


Fig. 4-16. The Race track microtron

4.2.5 The Synkrotron

A synkrotron is an accelerator in which the radius,r, and the frequency, f, are kept constant while the magnetic field increases with time.

$$\left.\begin{aligned}\frac{dr}{dt} &= 0\\ \frac{dB}{dt} &\neq 0\end{aligned}\right\} r = \frac{mv}{qB} = \frac{1}{qB}m_o\gamma c\sqrt{1-\frac{1}{\gamma^2}} = \frac{1}{qBc}\sqrt{W^2 - W_o^2}$$

Eq. 4-7

Thus a change in energy has to be followed by achange in magnetic field and vice versa. (One here only count parts in which the particles turn, and not the straight sections).

For an electron synkrotron

$$T = \frac{2\pi\langle R\rangle}{c} = const$$

Eq. 4-8

Where <R> is the radius of a "perfect" particle. The orbit in this kind of machine is kept on exactly the same place all the time. The energy is increased by increasing the magnetic field of the machine, and the particles has to follow with this energy.

In Fig. 4-17 the top graph shows the electrons passing the E-field at stable conditions. Then the magnetic field is increased, and the electrons take a slightly shorter path around the machine and arrives a little bit to early (middle graph). They get an energy change proportional to the E-field times their negative charge. Thus the middle electron will gain energy and close in on the stable path, while the other two will loose energy and fall further behind and later be lost from the machine.

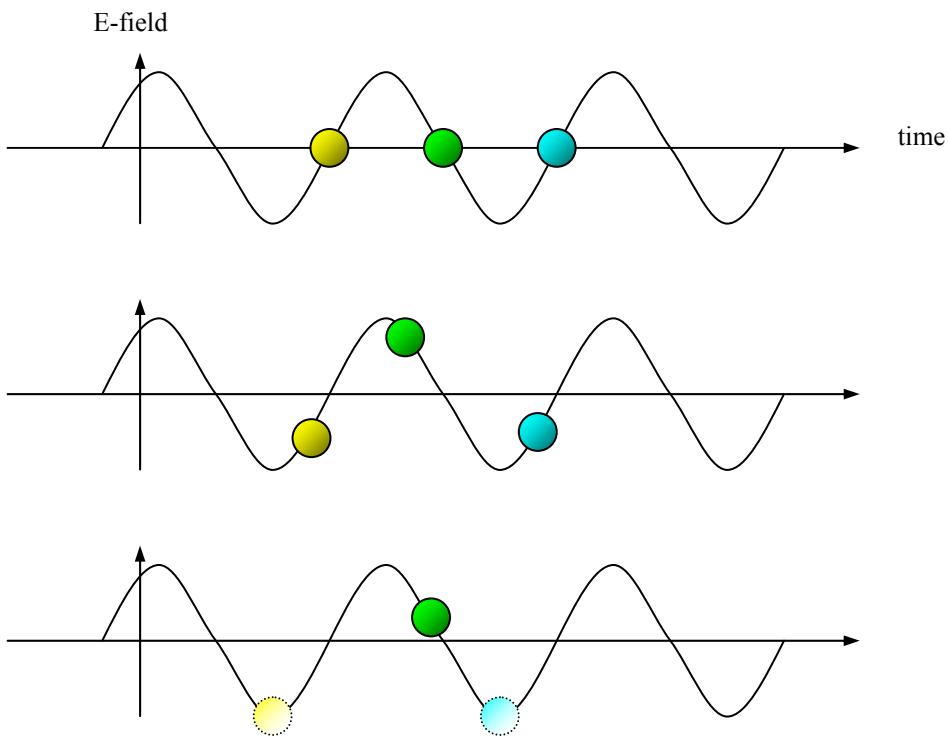


Fig. 4-17. Electrons passing the electric field at different times.

A storage ring (Fig. 4-18), which consists of a number of separated magnets, is using the synchrotron concept.

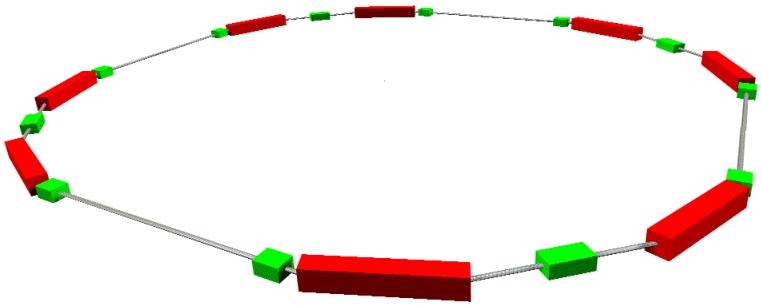


Fig. 4-18. A storage ring. (One of the proposed lattices for a new MAX-lab booster)

4.2.6 The Betatron

A very special accelerator is the Betatron which does not possess any electrodes or cavities. Instead it uses a very basic principle of the magnetic field.
One of Maxwells equations:

$$\nabla \times \overline{E} = \frac{\partial \overline{B}}{\partial t}$$

Eq. 4-9

gives that a time varying magnetic field (B) produces an electric field (E).

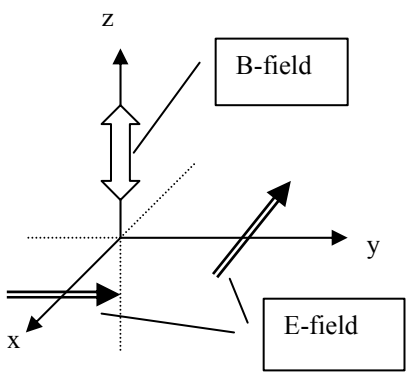


Fig. 4-19. The fields in a Betatron.

By introducing a time varying vertical magnetic field $\overline{B}(t) = B(t)\hat{z}$ and inserting it into the equation above

$$\frac{\partial}{\partial x} E_y - \frac{\partial}{\partial y} E_x = \dot{B}_z$$

Eq. 4-10

The magnetic field B will keep the particles in a circular orbit, and as the magnetic field is changing it will also produce an electric field, E, which accelerates the particles. The energy gain per pass is small but the particle will make a large number of turns as the frequency of the magnetic field is low (often 50 Hz). The Betatron can produce a particle beam of several 100 MeV.

5 Magnets

Before really getting into the topics of magnets we have already used some magnets to discuss the idea behind accelerators. Most of this could be understood by the knowledge of any ordinary magnet, like a horseshoe magnet from high school.

We will now move into a domain where we try to find magnets which can handle the beam of particles we have created in one of the basic accelerators. We need to transport the beams in straight lines and bend it through corners or around a ring. We need to focus the beam, as it otherwise will blow up by the repulsive forces of common charge. There are different ways to do this which we shall study.

The magnets here are different due to the different number of pole tips they have. We talk about di(two)pole magnets, quadru(four)pole magnets etc. First we will look on these two special magnets, and then continue on to general expressions for any combination of magnet, with any number of pole tips.

Most of these ideas in practise use electrical magnets, but we will briefly mention the existence of permanent magnets.

5.1 Why magnetic fields?

The Lorentz force equation tells us that:

$$\overline{F} = q\left(\overline{E} + \overline{v} \times \overline{B}\right)$$

Eq. 5-1

An electrical magnet quite easily produces 1 T of magnetic flux density. As the velocities we are using normally are close to the speed of light, a competitive electric field would have to be 3 10^8 V/m. Such a field is not trivial to produce and to give the detailed shape we need.

5.2 Accelerator magnets

Important parameters of a magnet are

The permeability:	$\mu = \mu_r/\mu_0$	Vs/Am
Magnetic flux:	Φ	Wb = Vs
The magnetic flux density:	B	T = Vs/m^2
The magnetic fields strength:	H	A/m

The permeability is a material constant which for vacuum is $\mu_0 = 4\pi*10^{-7}$. The magnetic flux density is related to the magnetic field strength through: B=μH. The magnetic flux is given from

$$\Phi = \int BdA$$

as an integration over a surface.
When we in everyday language talk about magnets and the strength of a magnet we most often refer to the magnetic flux density. (magnetic field, field, magnet strength ....)

Another useful expression is "Ampere circular integral" which says that if there is a current lead the integral of the magnetic field strength around the lead is equal to the total current:

$$\oint Hds = NI$$

where I is the current and N the number of current leads.

5.3 The Dipole

With these expressions at hand we can start out by looking at a magnet which we will call dipole magnet. The name comes from the fact that there are two poles (magnetic ends) on the magnet. (We will here assume that the magnet has a core of iron). The magnetic flux density is here constant in the gap between the iron poles.

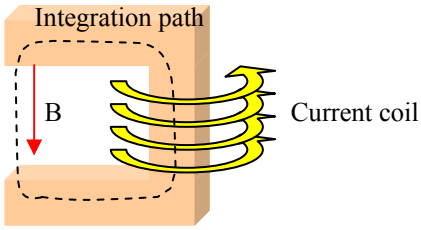


Fig. 5-1. A dipole magnet.

By integrating the magnetic field strength along the dashed line (across the gap and around the iron) we can get the current in the coil, or vice versa.

The gap has a height of h and the length of the iron is l.

$$\oint H ds = hH_{gap} + lH_{iron} = nI$$

Eq. 5-2

The flux density at the two sides of the iron-air interface is constant which gives us that

$$H_{gap}\frac{\mu_{air}}{\mu_0} = H_{iron}\frac{\mu_{iron}}{\mu_0}$$

Eq. 5-3

As the permeability, μ_{iron}, is much larger than that for air ($\mu_{air} = 1$) the field strength for the iron is much less. Thus the integral above can be approximated to:

$$\oint H ds = hH_{gap} = h\frac{B}{\mu_0} = nI \Rightarrow B = \frac{nI\mu_0}{h}$$

Eq. 5-4

It has to be remembered though that this is an approximation. In a real magnet there are end fields and the iron can saturate etc. which give other effects.

5.4 The quadrupole

In a quadrupole magnet there are four poles (magnet ends) each with current coils around them. The magnetic field has a much more complex layout.

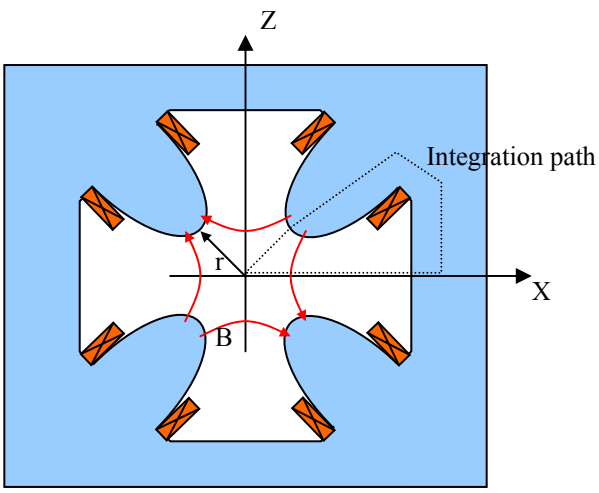


Fig. 5-2. A quadrupole magnet.

The magnetic flux density in a quadrupole is given by

$$\overline{B}_z = -gx\hat{z}$$
$$\overline{B}_x = -gz\hat{x}$$

Eq. 5-5

Thus there is no flux density in the very centre of the magnet, what we call on axis. If we look at a point in the direction of a pole tip, x=z and we can write

$$\overline{B}_r = \sqrt{B_x^2 + B_z^2}\,\hat{r} = g\sqrt{x^2 + z^2}\,\hat{r} = gr\hat{r}$$

Eq. 5-6

Also for this magnet we can set up the circular integral of the magnetic field strength. There are three regions.
1) from the centre to the pole tip.
2) through the iron.
3) from the iron along the axis to the centre.

$$nI = \oint \overline{H}d\overline{s} = \int_0^R \overline{H}d\overline{r} + \int_{iron} \overline{H}d\overline{s} + \int_{axis} \overline{H}d\overline{s}$$

Eq. 5-7

where R is the distance from the centre to the pole tip. As decided for the dipole, we can neglect the contribution from the path through the iron (2). As H is parallel to B which is perpendicular to s, the integral along the axis is 0. Thus only the integral from the centre to the pole tip remains.

$$nI = \int_0^R \frac{\overline{B}_r}{\mu_0} d\overline{r} = \int_0^R \frac{g\overline{r}}{\mu_0} d\overline{r} = \frac{gR^2}{2\mu_0} \Rightarrow g = \frac{2\mu_0 nI}{R^2}$$

Eq. 5-8

Now we know the strength of the magnet with the given magnetic fields, but we do no actually know what the poles look like.

As there are no current sources on the pole tip surface we can write

$$\nabla \times \overline{B} = 0 \Rightarrow \overline{B} = -\nabla V$$

Eq. 5-9

which means that there exists a scalar potential for the flux density on the surface, which is given by

$$\overline{B} = -\left(\frac{\partial V}{\partial x}, \frac{\partial V}{\partial y}, \frac{\partial V}{\partial z}\right)$$
$$\overline{B} = -(gz, 0, gx)$$

Eq. 5-10

thus the potential is given by:

$$V = gxz$$

Eq. 5-11

which describes a hyperbolic surface.
A hawk eye then sees that pole tips drawn above are not the ones of hyperbolic surfaces. Yes and no. They are close to the pole tip but not a distance away, and the reason is that the field in the centre region is defined to a very high precision by the area around the pole tips. If one would like to have good fields far away from the centre, one should have to really make large pole tips fulfilling the potential above.

5.5 The sextupole

To make the picture a little bit more complete I will here sketch also a magnet with six pole tips, the sextupole magnet. The method of deriving the parameters of this magnet is very similar to the quadrupole, but more cumbersome, and will thus just be sketched.

The magnetic flux density from this kind of device is given by

$$\begin{cases} B_x = g'xz \\ B_z = \frac{1}{2} g'\left(x^2 - z^2\right) \end{cases}$$

Eq. 5-12

Regarding the magnetic flux density along the radius from the centre point to a pole tip generates

$$\overline{B}_r = \sqrt{\overline{B}_x^2 + \overline{B}_z^2} = \sqrt{g'^2 x^2 z^2 + \frac{g'^2}{4}\left(x^4 - 2x^2 z^2 + z^4\right)}\hat{r} = \frac{g'}{2}\sqrt{\left(x^2 + z^2\right)^2}\,\hat{r} = \frac{g'}{2} r^2 \hat{r}$$

Eq. 5-13

The next step is to set up the integral over the path from the centre point to the pole tip. The rest of the path gives no contribution as the discussion for the quadrupole.

$$nI = \oint \overline{H} d\overline{s} = \int_0^R \frac{\overline{B}_r}{\mu_0} d\overline{r} = \int_0^R \frac{g' r^2}{2\mu_0} dr = \frac{g' R^3}{6\mu_0} \Rightarrow g' = \frac{6\mu_0 nI}{R^3}$$

Eq. 5-14

We thus have an expression for the strength of a sextupole. The actual shape of the pole surface can in principle be extracted from the same method with the scalar potential generating the surface as for the quadrupole, but the solution is much more complicated.

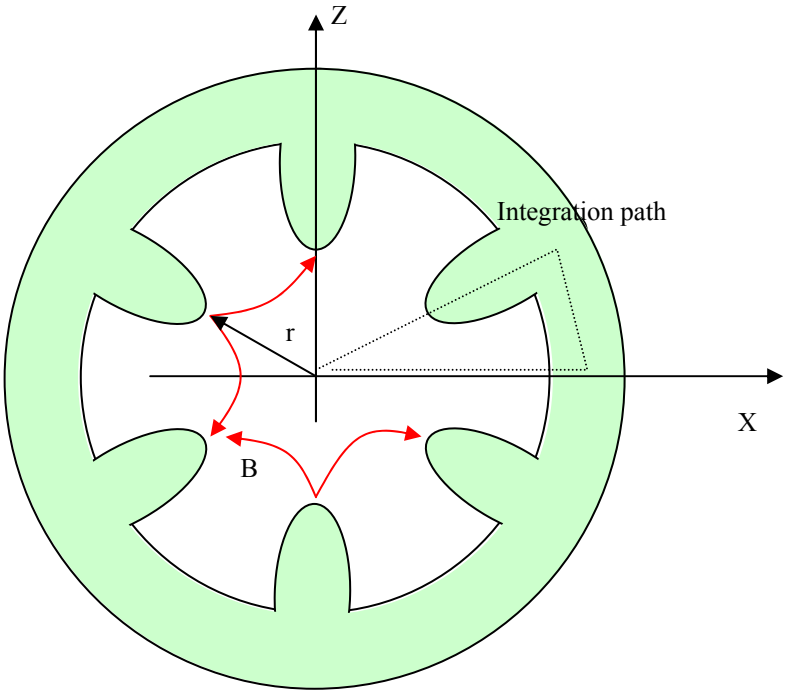


Fig. 5-3. A sextupole magnet.

5.6 A general magnet

Having looked at three examples of magnets it is now time to look at a general magnet. It will be a magnet which combines any magnet pole tip configuration. (think for example of a dipole with some sextupole character included?!).

The Maxwells equations for a current line free region say:

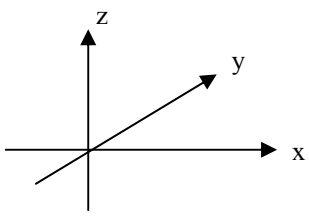
$$\nabla \bullet \overline{B} = 0 \Rightarrow \overline{B} = \nabla \times \overline{A}$$
$$\nabla \times \overline{B} = 0 \Rightarrow \overline{B} = -\nabla V$$

Eq. 5-15

Where A is a vector potential and V a scalar potential.
From this follows that

$$\overline{B} = \nabla \times \overline{A} = -\nabla V$$

Eq. 5-16



We can assume that the magnetic flux density, B, is uniform along the y-axis. From this it is possible to deduce that

$$\overline{A} \perp (x,z)\, plane \Rightarrow \overline{A} = A_y \hat{y}$$

Rewriting Eq. 5-16 then gives

$$B = \left(\frac{\partial A_y}{\partial y}, -\frac{\partial A_y}{\partial x}, 0 \right) = -\left(\frac{\partial}{\partial x} V, \frac{\partial}{\partial y} V, \frac{\partial}{\partial z} V \right)$$

Eq. 5-17

Before we continue we have to take help of some mathematical tools.

$$\begin{aligned} &\textit{If } \widetilde{A} \textit{ is an analytical function it can be written} \\ &\widetilde{A}(\zeta) = \sum_{n=0}^{\infty} (\lambda_n + i\mu_n)\zeta^n \end{aligned}$$

and

$$\begin{aligned} &\textit{The Cauchy – Riemann condition says that if} \\ &\frac{\partial A}{\partial x} = \frac{\partial V}{\partial z} \textit{ and } \frac{\partial A}{\partial z} = -\frac{\partial V}{\partial x} \\ &\textit{then} \\ &\widetilde{A}(\zeta) = \widetilde{A}(x+iy) = A(x,z) + iV(x,z) \\ &\textit{is an analytical function} \end{aligned}$$

This construction of $\widetilde{A}$ is an analytical function as the criteria is exactly what is given in Eq. 5-17.

The potential can then be written as

$$\begin{aligned} V(x,z) &= \mathrm{Im}\left(\widetilde{A}(\zeta)\right) = \mathrm{Im}\sum_{n=0}^{\infty} (\lambda_n + i\mu_n)\ (x+iz)^n = \\ &= \mathrm{Im}\left\{(\lambda_0 + i\mu_0) + (\lambda_1 + i\mu_1)(x+iz) + (\lambda_2 + i\mu_2)(x+iz)^2 + \ldots\right\} = \\ &= \mathrm{Im}\left\{\mu_0 + \lambda_1 z + \mu_1 x + 2\lambda_2 xz + \mu_2\left(x^2 + z^2\right) + \ldots\right\} \end{aligned}$$

Eq. 5-18

We can now continue and write the magnetic flux density as

$$\begin{aligned} \overline{B}_x &= -\frac{\partial V}{\partial x}\hat{x} = -(\mu_1 + 2\lambda_2 z + 2\mu_2 x + \ldots) \\ \overline{B}_z &= -\frac{\partial V}{\partial z}\hat{z} = -(\lambda_1 + 2\lambda_2 x + 2\mu_2 z + \ldots) \end{aligned}$$

Eq. 5-19

To make things resemble things we have done before, lets make two definitions:

$$\begin{aligned} a_n &= \frac{n\mu_n}{B_{main}} r_0^{n-1} \\ b_n &= -\frac{n\lambda_n}{B_{main}} r_0^{n-1} \end{aligned}$$

Eq. 5-20

Introducing the definitions of Eq. 5-20 into Eq. 5-19 generates

$$\overline{B}_x = -B_{main}\left(a_1 - \frac{b_2}{r_0}z + \frac{a_2}{r_0}x + \ldots\right)$$
$$\overline{B}_z = B_{main}\left(b_1 - \frac{a_2}{r_0}z + \frac{b_2}{r_0}x + \ldots\right)$$

Eq. 5-21

With these expressions at hand let us look at the case when n=1

$$\overline{B}_x = -B_{main}a_1$$
$$\overline{B}_z = B_{main}b_1$$

Eq. 5-22

If we now call all the components carrying b as "normal components" and the ones carrying a as "skew component" (inclined, rotated), we can see than the normal component of order n=1 is exactly a dipole magnet with a vertical magnetic field.

Encouraged by that knowledge, lets continue to n=2

$$\overline{B}_x = -B_{main}\left(-\frac{b_2}{r_0}z + \frac{a_2}{r_0}x\right) \Rightarrow (normal) \Rightarrow B_{main}\frac{b_2}{r_0}z = -gz$$
$$\overline{B}_z = B_{main}\left(\frac{a_2}{r_0}z + \frac{b_2}{r_0}x\right) \Rightarrow (normal) \Rightarrow B_{main}\frac{b_2}{r_0}x = -gx$$

Eq. 5-23

We are just at the quadrupole definition of Eq. 5-5. In reality there are also the magnets using the "skew components". This would mean a dipole bending vertically or a quadrupole rotated 45°.

5.7 Permanent magnets

So far we have not discussed the materials of the magnets as they were electrical magnets, and we simply assumed an iron core and a coil around. Another way of producing magnetic fields is by using permanent magnets. These are materials that can be "magnetised" and then keep a magnetic flux density B called remanence afterwards.

A number of different materials can be used such as:

- Metallic alloys mainly of iron
- Ferrites which are ceramic compounds with iron.
- Rare earth-cobalt magnets, most often Samarium-cobalt (Sm_xCo_y) compounds.
- Neodymoium-iron-boron compositions like $Nd_2Fe_{14}B$.

The operation of permanent magnets has to follow the "hysteresis curve", Fig. 5-4. The material is magnetised by exposing it to a magnetic field. When removing the field the flux density does not fall to zero, but a certain amount remains. This is called remanence, flux density at zero magnetic flux. It can in practise not be used as the magnet will be exposed to a magnetic flux when inserted into a magnetic circuit (Fig. 5-5), thus there will be a working point at a slightly lower flux density.

The actual behaviour of a permanent magnet can be simulated in almost any of all finite element codes available.

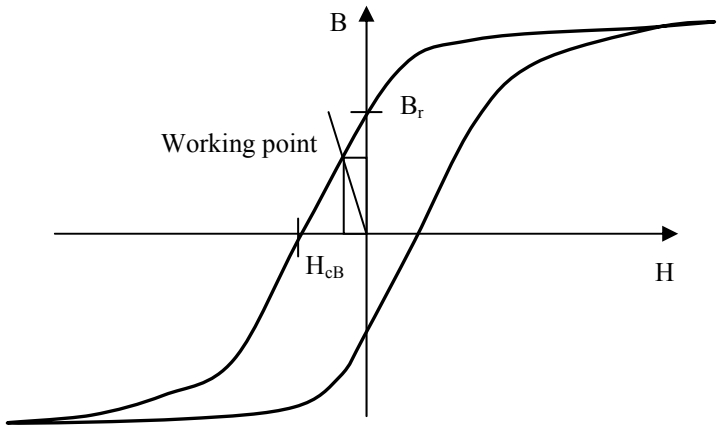


Fig. 5-4. Hysteresis curve for a permanent magnet with working point.

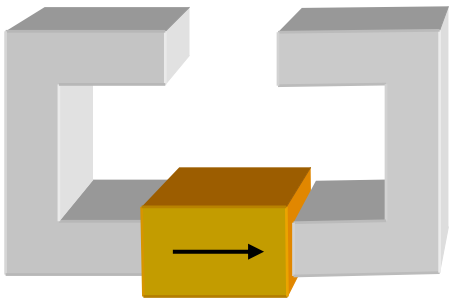


Fig. 5-5. Permanent magnet circuit (example).

6 Beamdynamics

A particle, or a beam of particles, that moves in an accelerator is steered by a number of magnets. The path of the particle will turn and bend around the machine and the whole beam of particles will be focused and defocused. We can find "ideal" particles that travel in the very centre of the path in the accelerator, and we can find the other particles which move around and oscillate close to the path of the "ideal" particle.

These movements and the oscillations in the accelerators are called *beam dynamics*. The layout of magnets around the accelerator is called *lattice*, and in this chapter we will first look on how a single particle travelling through the lattice of the accelerator behaves. When we think we have a clue on that, we will continue to look on a beam of particles and see how the whole beam behaves.

Before getting into the real task lets quickly look on a particle in circular motion, and an example on how a particle in circular motion can be focused (weak focusing).

Beam dynamics also includes the behaviour of particles and beams while experiencing accelerating fields and thus forces along the path of the particles. Some details of this will be discussed at different places in this book, but a fuller picture is far beyond the scope of this work.

6.1 Circular motion

As a less complicated example on how to study a charged particle moving in a magnetic field we shall look on a particle moving in a circular orbit. The particle will, as in classical mechanics, experience the centripetal force:

$$F_{cp} = \frac{mv^2}{r}$$

Eq. 6-1

A vertical magnetic field, as in Fig. 6-1, will act on the particle to make it move in a circular path. (There is a handy rule on how to find the direction of the force. Left hand rule: thumb – velocity, index finger – magnetic flux density, long finger – force. The left hand is valid for negatively charged particles, like an electron. The right hand is correct for positively charged particles). This force has to be equal to the centripetal force, otherwise the particle will start to spiral outwards or inwards. The different directions are important and the plane of motion will be perpendicular to the direction of the magnetic flux density.

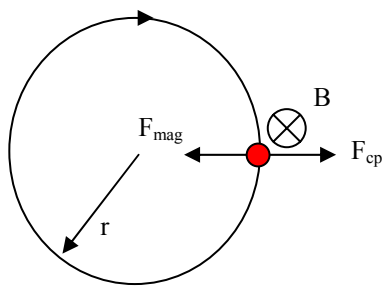


Fig. 6-1. Circular motion

The magnetic force is given by the Lorentz equation, which for a case with no electric field is:

$$F_{mag} = qvB$$

Eq. 6-2

with the magnetic flux density, B.

$$B = \frac{mv}{rq}$$

Eq. 6-3

Thus we have to adjust the magnetic flux density (B) such that a particle with a certain energy and charge move with just the radius we want.

> An electron with an energy of 500 MeV has a total mass of $\gamma m_0 \approx 1000 * 9{,}1*10^{-31}$ Kg, a velocity $v \approx c = 3{,}0*10^{8}$ m/s and a charge of $q = -1{,}6*10^{-19}$ As. If we desire a motion with a radius of 3 m we need a magnetic flux density B = 0,57 T. (using Eq. 6-4).

6.2 Weak focusing

In the section above we have in principle designed an accelerator lattice in which electrons travel around an orbit with a given radius. A problem soon arises and that is that all particles are not on the same path from the beginning.

If the circular path painted above is the ideal path we want our particles to follow we have to take special care to particles coming to far out ($r_{particle} > \rho$) or in ($r_{particle} < \rho$) (where ρ is the radius of the ideal path). Our desire would be that a particle coming to far out should feel a stronger magnetic force than the centripetal force and v.v.

$$F_{mag} \begin{array}{ll} < F_{cp} & if\ r < \rho \\ > F_{cp} & if\ r > \rho \end{array}$$

Eq. 6-4

ρ being the radius of the ideal circular path.

If this is accomplished the particles will pushed back to the ideal path, they will be "focused".

What should the magnetic field look like?

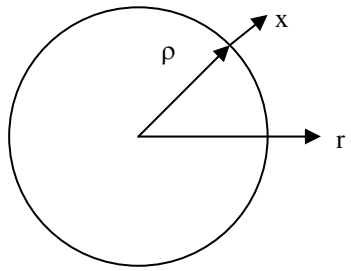


Fig. 6-2. Deviation from circular motion

Lets look on a small deviation from the ideal circular orbit, a particle moving with a radius

$$r_{particle} = \rho + x = \rho\left(1 + \frac{x}{\rho}\right)$$

Eq. 6-5

which gives a centripetal force

$$F_{cp} = \frac{mv^2}{r} = \frac{mv^2}{\rho\left(1+\frac{x}{\rho}\right)} \approx \frac{mv^2}{\rho}\left(1 - \frac{x}{\rho}\right)$$

Eq. 6-6

To fulfil the condition above to achieve focusing around the ideal path the magnetic field has to vary with the radius, r. Lets assume that the magnetic field has an amplitude given by

$$B = B(r) = B_0(1 - n\frac{x}{\rho})$$

Eq. 6-7

where n is a field index.
Inserting this into the criteria above gives:

$$qvB_0\left(1 - n\frac{x}{\rho}\right)\begin{cases} < [for\ x < 0] \\ > [for\ x > 0] \end{cases} \frac{mv2}{\rho}\left(1 - \frac{x}{\rho}\right) \Rightarrow n < 1$$

Eq. 6-8

By applying a magnetic field with a field index n<1 we can achieve focusing. Unfortunately this is only in the plane of the circular path. In the other plane, lets call it the vertical plane, there should not be any activitity, but if there is we need a focusing force even there.

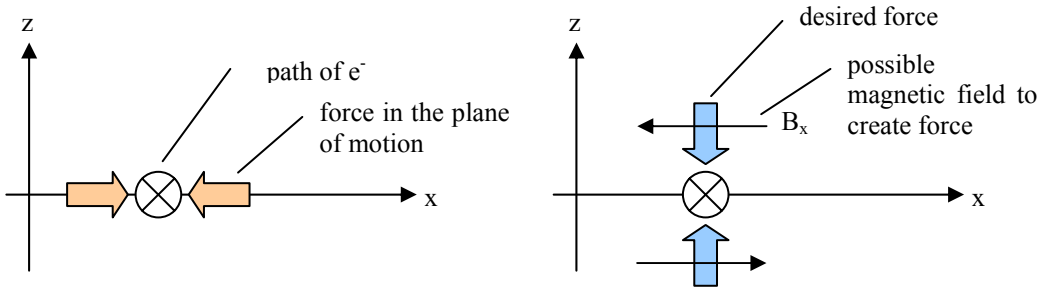


Fig. 6-3. The necessary focusing forces, and the magnetic fields to give vertical focusing forces.

Such a restoring field should look like:

$$B_x = -B_{ox}z$$

Eq. 6-9

($B_x < 0$ for $z > 0$ and v.v.)

To get hand on the connection between the vertical field and the horisontal field we use one of the Maxwell equations:

$$\nabla \times \overline{B} = \frac{4\pi}{c}\overline{J}$$

Eq. 6-10

where J is the current density, which in the actual region is 0 (no electric coils or current lines).
This gives:

$$\overline{J} = 0 \Rightarrow \nabla \times \overline{B} = 0 \Rightarrow \frac{\partial B_z}{\partial r} = \frac{\partial B_r}{\partial z}$$

Eq. 6-11

From which follows that (using the $B_z = B(r)$ above)

$$\frac{\partial B_z}{\partial r} = \frac{\partial B_z}{\partial x} = -B_0\frac{n}{\rho}$$

Eq. 6-12

and

$$B_x = B_x(z) = -B_0 \frac{n}{\rho} z$$

Eq. 6-13

The criteria for focusing above gives:

$$-\frac{B_0}{\rho} n < 0 \Rightarrow n > 0$$

Eq. 6-14

We now have to demands on the field index n, which combined give:
$0 < n < 1$

This method is called weak focusing, and unfortunately the focusing forces are just weak. To get around this problem the concept of strong focusing has been developed. Here one accepts that there is not focusing in both directions simultaneously, but instead it varies along the accelerator. While B=B(r) in weak focusing, B= B(r,θ) in strong focusing.

6.3 The coordinate system and equations of motion

To make a more general approach we need to write the equations of motions of the particles in a form suitable for the kind of problem we are studying. The forces on the particle in a magnetic system can be written both by the Lorentz equation and an ordinary force equation:

$$\overline{F} = q\overline{v} \times \overline{B}$$
$$\overline{F} = m\overline{a} = m\ddot{\overline{R}}$$

Eq. 6-15

Where R is describing the position of the particle.
A first step is to find a practical reference system for the future calculations. It is convenient to assume that all motions are circular motions. The radius can be infinity though, which then generates a straight path. We define an "ideal" path in a circle, with a radius ρ_0, and then look at small deviations from this "ideal" path (Fig. 6-4).

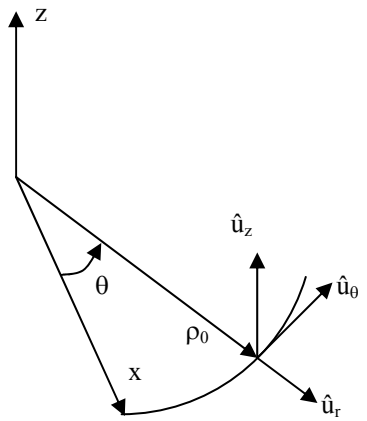


Fig. 6-4. Coordinate system for particle motion.

The actual position of a particle is given by

$$\overline{R} = r\hat{u}_r + z\hat{u}_z$$

Eq. 6-16

The local coordinate system, painted by the û vectors, follows an ideal particle which always travels on the ideal path around the accelerator. This system changes as the angle θ changes:

$$\begin{cases} \Delta\hat{u}_r = \Delta\theta\,\hat{u}_\theta \\ \Delta\hat{u}_\theta = -\Delta\theta\,\hat{u}_r \\ \Delta\hat{u}_z = 0 \end{cases}$$

Eq. 6-17

This can be used to deduce the change over time of the local coordinate system:

$$\begin{cases} \dfrac{d\hat{u}_r}{dt} = \dfrac{d\hat{u}_r}{d\theta}\dfrac{d\theta}{dt} = \dot{\theta}\,\hat{u}_\theta \\ \dfrac{d\hat{u}_\theta}{dt} = \dfrac{d\hat{u}_\theta}{d\theta}\dfrac{d\theta}{dt} = -\dot{\theta}\,\hat{u}_r \\ \dfrac{d\hat{u}_z}{dt} = \dfrac{d\hat{u}_z}{d\theta}\dfrac{d\theta}{dt} = 0 \end{cases}$$

Eq. 6-18

With this knowledge we can write the velocity of any particle as the time derivative of the particle postion, Eq. 6-16 :

$$\frac{dR}{dt} = \dot{r}\hat{u}_r + r\dot{\hat{u}}_r + \dot{z}\hat{u}_z + z\dot{\hat{u}}_z =$$
$$= \dot{r}\hat{u}_r + r\dot{\theta}\,\hat{u}_\theta + \dot{z}\hat{u}_z$$

Eq. 6-19

In a similar way we can construct the acceleration of any particle:

$$\frac{d^2R}{dt^2} = \ddot{r}\hat{u}_r + \dot{r}\dot{\hat{u}}_r + \dot{r}\dot{\theta}\hat{u}_\theta + r\ddot{\theta}\hat{u}_\theta + r\dot{\theta}\dot{\hat{u}}_\theta + \ddot{z}\hat{u}_z + \dot{z}\dot{\hat{u}}_z =$$
$$= \hat{u}_r\left(\ddot{r} - r\dot{\theta}^2\right) + \hat{u}_\theta\left(2\dot{r}\dot{\theta} + r\ddot{\theta}\right) + \ddot{z}\hat{u}_z$$

Eq. 6-20

We now have the acceleration of the particle (Eq. 6-20). The same notation can be introduced in the Lorentz equations in Eq. 6-15 which yields:

$$\Rightarrow -ev \times B = -e\begin{vmatrix} 1 & 2 & 3 \\ v_r & v_\theta & v_z \\ B_r & B_\theta & B_z \end{vmatrix} = -e\left[v_\theta B_z - v_z B_\theta, v_z B_r - v_r B_z, v_r B_\theta - v_\theta B_r\right] =$$
$$= e\left[r\dot{\theta}B_z - \dot{z}B_\theta, \dot{z}B_r - \dot{r}B_z, \dot{r}B_\theta - r\dot{\theta}B_r\right]$$

Eq. 6-21

Assuming that the magnetic field along the path of the electron is zero, B_θ=0, yields for the equations of motion in different directions (from Eq. 6-15):

$$\begin{cases} \hat{u}_r: & m\left(\ddot{r} - r\dot{\theta}^2\right) = -er\dot{\theta}B_z \\ \hat{u}_\theta: & m\left(2\dot{r}\dot{\theta} + r\ddot{\theta}\right) = -e\left(\dot{z}B_r - \dot{r}B_z\right) \\ \hat{u}_z: & m\left(\ddot{z}\right) = -e\left(-r\dot{\theta}B_r\right) \end{cases}$$

Eq. 6-22

The two equations for the deviations perpendicular (r and z) to the original direction (θ) are the ”general equations of motion”. With these at hand we can in principle extract the motion in an arbitrary magnetic field.

6.4 A di- and quadrupole magnet

Our next task is to write the equations of motion in a real magnetic field. A magnetic field could of course be of arbitrary shape, but let us here concentrate on the fields from a dipole and a quadrupole magnet. In the previous chapter we found the fields from a dipole, quadrupole and sextupole magnet etc. Let us look at a magnet composed of both a dipole and a quadrupole magnetic field. Such a field is given by:

$$B_z = B_0 - gx$$
$$B_r = B_x = -gz$$

Eq. 6-23

The radius of the path of the particle is given by $r=\rho_0+x$, where ρ_0 is constant and x defines a small deviation.

Until now we have looked on all behaviour as change over time. Lets now change from time, t, to the distance along the path, s, where s=vt. This coordinate change can be written by:

$$\begin{aligned} &a) \quad \dot{r} = \dot{x} = \frac{dx}{dt} = \frac{dx}{ds}\frac{ds}{dt} = x'v \\ &b) \quad \ddot{r} = \ddot{x} = v^2 x'' \\ &c) \quad \ddot{z} = v^2 z'' \end{aligned}$$

Eq. 6-24

By introducing Eq. 6-24 into the ”general equations of motion” Eq. 6-22 and using the magnetic fields from Eq. 6-23 plus that $r\dot{\theta} \approx v$ we get:

$$\begin{cases} m\left(v^2 x'' - \frac{v^2}{r}\right) = -ev(B_0 - gx) \\ mv^2 z'' = -evgz \end{cases} \Rightarrow \begin{cases} x'' = \frac{1}{r} - \frac{eB_0}{mv} + \frac{eg}{mv}x \\ z'' = -\frac{eg}{mv}z \end{cases}$$

Eq. 6-25

This form of the equations of motion does not make us very happy, so lets rewrite it somewhat. If we use that:

$$\frac{1}{r} = \langle r = \rho + x \rangle = \frac{1}{\rho + x} = serie... \approx \frac{1}{\rho}\left(1 - \frac{x}{\rho}\right) as\ \rho >> x$$

and

$$mv = p = p_0 + \Delta p = p_0\left(1 + \frac{\Delta p}{p_0}\right)$$

where

$$p_0 = m_0 v_0 = e\rho_0 B_0$$

We also define the parameter

$$k = \frac{eg}{mv}$$

The equations of motion of Eq. 6-25 can now be rewritten into (using that $\Delta p/p_0$ is small):

$$\begin{cases} x'' - \left(k - \frac{1}{\rho^2}\right)x = \frac{1}{\rho}\frac{\Delta p}{p_0} \\ z'' + kz = 0 \end{cases}$$

Eq. 6-26

From the different definitions of the magnetic field above, we can see that for a *dipole magnet* k=0 and for a *quadrupole magnet* $\rho = \infty$. From this we find the equations of motion, or the path equations, for either a dipole:

$$\begin{cases} x'' - \frac{1}{\rho^2} x = \frac{1}{\rho} \frac{\Delta p}{p_0} \\ z'' = 0 \end{cases}$$

Eq. 6-27

or a quadrupole:

$$\begin{cases} x'' - kx = 0 \\ z'' + kz = 0 \end{cases}$$

Eq. 6-28

For the dipole we can see that the effect is limited to one plane, while the other is undisturbed. The term including $\Delta p/p_0$, an energy deviation, is only effective for a dipole. This we will later call dispersion.

The quadrupole on the other hand is more symmetric between the two planes, except a different sign on the focusing term.

6.5 Solve the path equation

In this subchapter we will search for a solution of the path equations. All the equations above (Eq. 6-27 and Eq. 6-28) can be written in a general form, which is similar for all cases:

$$x'' - K(s)x = \frac{1}{\rho} \frac{\Delta p}{p_0}$$

Eq. 6-29

Of which we can write the solution as consisting of two parts: the homogenous solution and the partial solution:

$$X(s) = X_{Hom}(s) + X_{Part}(s)$$

$$\begin{cases} X''_{Hom} + K(s)X_{Hom} = 0 \\ X''_{Part} + K(s)X_{Part} = \frac{1}{\rho} \frac{\Delta p}{p_0} \end{cases}$$

Eq. 6-30

The homogenous solution can be written as

$$X(s) = C(s)x_0 + S(s)x'$$

Eq. 6-31

where C(s) is a ”cosine-like” function and S(s) a “sine-like” function.

If we define a function D(s) as

$$D(s) = \frac{X_{Part}(s)}{\frac{\Delta p}{p_0}}$$

Eq. 6-32

The partial equation can be written as

$$D'' - K(s)D = \frac{1}{\rho}$$

Eq. 6-33

The total solution is now given by

$$\Rightarrow \begin{cases} X(s) = C(s)x_0 + S(s)x_0' + D(s)\dfrac{\Delta p}{p_0} \\ X'(s) = C'(s)x_0 + S'(s)x_0' + D'(s)\dfrac{\Delta p}{p_0} \end{cases}$$

Eq. 6-34

where x_0 and x'_0 are the initial values and D the partial solution, given by:

$$D(s) = S(s)\int_{s_0}^{s} \frac{1}{\rho(t)} C(t)dt - C(s)\int_{s_0}^{s} \frac{1}{\rho(t)} S(t)dt$$

Eq. 6-35

From equation Eq. 6-34 is is possible to see that one can write the solution on another form, called matrix form, which looks like

$$\overline{X}(s) = \begin{pmatrix} x(s) \\ x'(s) \\ \dfrac{\Delta p}{p_0} \end{pmatrix} = \begin{pmatrix} C(s) & S(s) & D(s) \\ C'(s) & S'(s) & D'(s) \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x(s_0) \\ x'(s_0) \\ \dfrac{\Delta p}{p_0} \end{pmatrix} = \underline{M}\,\overline{X}(s_0)$$

Eq. 6-36

Where M is called the transformation matrix as it ”transforms” the input values at s_0 to the output values at s. If we can construct this matrix for all different elements of our accelerator, we can also so how a particle with specific input data behaves at different points around the accelerator. But still we don’t know the functions C, S and D.

6.6 Matrix formulation

In this sections we will try to put some more substance into the transfer matrix formulation given above. Find actual expressions for different elements.

First lets just make a note that a system consisting of different elements, will have one matrix for each element. It is from these matrices possible to construct a matrix for the whole system by multiplying the matrices together in the order that they appear in the machine. One then multiplies the matrices "backwards" to get the proper total system.

$$\underline{M}_{total} = \underline{M}_n \cdots \underline{M}_3 \underline{M}_2 \underline{M}_1$$

Eq. 6-37

6.6.1 Drift section

The first simple thing too look at is a drift section. This is a part of the accelerator which has no magnetic elements at all. Sounds kind of strange, but it might straighten out some question marks on the construction of the transfer matrix.

We can without any fancy method find that the coordinates after a drift section of length *l* are:

$$\begin{cases} x(l) = x_0 + lx_0' & x'(l) = x_0' \\ z(l) = z_0 + lz_0' & z'(l) = z_0' \end{cases}$$

Eq. 6-38

The angle (or direction) of the particle path, *x'*, does not change in a drift section as there are no objects to change it. The position of the particle, $^{x(l)}$, after the distance l is given by the starting position, x_0, plus the angle times the distance, $l\,x_0'$.

Writing Eq. 6-38 into a transfer matrix becomes:

$$\underline{M}_x = \begin{pmatrix} 1 & l & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \underline{M}_z$$

Eq. 6-39

6.6.2 A di- and quadrupole magnet

We previously looked at an example of a combined function magnet, and wrote the equations of motion for that magnet, but we never explicitly solved the equation. It is now time to work a little further on the solution and write the result on the transfer matrix form. The equation of motion looks in principle the same for the two cases and thus a common approach can be used.

If we start out with the homogenous equation a general form is

$$X'' + KX = 0$$

where K takes different values depending of the magnet type and the plane of motion we are studying.

We here assume the solutions according to the cosine like and sine like functions:

$$\begin{matrix} C(s) = A\cos(as) \\ S(s) = B\sin(bs) \end{matrix} \; \textit{with the conditions at } s = s_0 \begin{cases} C = 1 & S = 0 \\ C' = 0 & S' = 1 \end{cases}$$

Eq. 6-40

If we here assume that K is constant with s, which is true within each element or magnet, we get:

$$a = b = \sqrt{K}$$
$$A = 1$$
$$B = \frac{1}{\sqrt{|K|}}$$

Eq. 6-41

Defining a $\varphi(s) = s\sqrt{K}$ we can write the transfer matrix as

$$\underline{M} = \begin{pmatrix} \cos(\varphi) & \frac{1}{\sqrt{|K|}}\sin(\varphi) \\ -\sqrt{|K|}\sin(\varphi) & \cos(\varphi) \end{pmatrix} \qquad K > 0$$

Eq. 6-42

$$\underline{M} = \begin{pmatrix} \cosh(\varphi) & \frac{1}{\sqrt{|K|}}\sinh(\varphi) \\ \sqrt{|K|}\sinh(\varphi) & \cosh(\varphi) \end{pmatrix} \qquad K < 0$$

Eq. 6-43

For cases where we have to take an energy error into account, we also need to get our hands on the partial solution, which now can be achieved through Eq. 6-35:

$$D(s) = S\int_0^s \frac{C}{\rho}dt - C\int_0^s \frac{S}{\rho}dt$$

Eq. 6-44

producing

$$\begin{pmatrix} D \\ D' \end{pmatrix} = \begin{pmatrix} \frac{1}{\rho K}(1-\cos(\varphi)) \\ \frac{1}{\rho\sqrt{K}}\sin(\varphi) \end{pmatrix} \qquad K > 0 \quad and\ v.v.$$

Eq. 6-45

We can see from Eq. 6-80 that contribution to the dispersion only occur when ρ(s)<∞, that means in the dipole magnets. Dispersion exists at other points in the machine, but it can not change more than in the dipoles. In a machine without dipole magnets, a straight machine, there will be no difference in transverse position for particles with different energies, which is what one would expect.

6.6.3 Quadrupole

In the clean ecample of a quadrupole magnet where $\rho = \infty$ and K=k we get the transfer matrix

$$\underline{M}_{quad} = \begin{pmatrix} \cos(\varphi) & \frac{1}{\sqrt{k}}\sin(\varphi) & 0 \\ -\sqrt{k}\sin(\varphi) & \cos(\varphi) & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad k > 0, \quad \varphi = l\sqrt{k}$$

Eq. 6-46

and similarly for k<0 which is the other plane of motion.

6.6.3.1 Thin lens approximation

For the case of a quadrupole magnet it is often convenient to use a matrix which is independent of the length. We call this element a "thin lens" as it has zero length, and just the optical properties of the actual magnet. Mathematically this is achieved by letting l→0 while keeping kl constant.

Making this operation on the matrix in Eq. 6-46 yields

$$\underline{M} = \begin{pmatrix} 1 & 0 & 0 \\ -lk & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Eq. 6-47

Such a matrix is just what one gets in ordinary optics if looking at an optical lense with focal length $f = \frac{1}{lk} >> l$

The matrix for the two planes is thus given by

$$\begin{pmatrix} 1 & 0 & 0 \\ -\frac{1}{f} & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad and \quad \begin{pmatrix} 1 & 0 & 0 \\ \frac{1}{f} & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Eq. 6-48

The problem is of course that this element has not the correct length of the magnet. Happily there is an easy way out!
Let us assume that this "thin lens" is surrounded by two drift sections with half of the ordinary magnet length. Thus a total expression would be something like:

$$\begin{pmatrix} 1 & \frac{l}{2} & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ \frac{1}{f} & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & \frac{l}{2} & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Eq. 6-49

The actual magnet is given by multiplying the matrices together.

6.6.4 Dipole

The solution for the dipole magnet is now just as simple to write. But watch out, there is something nasty coming below!
A dipole has a bending radius in the bending plane but no focusing in any plane:

$$K = \frac{1}{\rho^2} \quad k = 0$$

This generates the transfer matrices:

$$\underline{M}_{bending\ plane} = \begin{pmatrix} \cos(\varphi) & \rho\sin(\varphi) & \rho(1-\cos(\varphi)) \\ -\frac{1}{\rho}\sin(\varphi) & \cos(\varphi) & \sin(\varphi) \\ 0 & 0 & 1 \end{pmatrix}$$

$$\underline{M}_{non\ bending\ plane} = \begin{pmatrix} 1 & l & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Eq. 6-50

The matrix for the non bending plane is just the same as a drift matrix. In the bending plane we find some extra terms now, which comes just from the influence of energy deviations. Remember here that we are always looking at coordinates (position or energy) relative to the ideal or perfect particle, which is assumed to pass just on the ideal path. An energy error is therefore a particle with slightly more or less energy than this ideal particle.

Example:
Assume the bending angle to be 30^0 and the bending radius 1 m. This gives a transfer matrix in the bending plane

$$\begin{pmatrix} 0{,}866 & 0{,}5 & 0{,}134 \\ -0{,}5 & 0{,}866 & 0{,}5 \\ 0 & 0 & 1 \end{pmatrix}$$

A perfect particle enters the magnet with $\begin{matrix} x_0 = 0 \\ x_0' = 0 \\ \frac{\Delta p_0}{p} = 0 \end{matrix}$ and thus exits with $\begin{matrix} x_1 = 0 \\ x_1' = 0 \\ \frac{\Delta p_1}{p} = 0 \end{matrix}$

It is still a perfect particle.

Another particle is a little off axis when entering $\begin{matrix} x_0 = 0{,}01 \\ x_0' = 0 \\ \frac{\Delta p_0}{p} = 0 \end{matrix}$ and thus exits with

$$x_1 = 0{,}00866$$
$$x_1' = -0{,}05$$
$$\frac{\Delta p_1}{p} = 0$$

This particle is closer to the perfect path (focused) but has received a small angle.

A third particle has a little excess energy $\begin{matrix} x_0 = 0 \\ x_0' = 0 \\ \frac{\Delta p_0}{p} = 0{,}05 \end{matrix}$ and exits with $\begin{matrix} x_1 = 0{,}134 \\ x_1' = 0{,}5 \\ \frac{\Delta p_1}{p} = 0{,}05 \end{matrix}$

It is now slightly off axis and has also an angle.

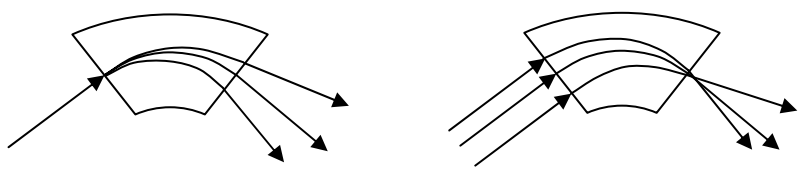


Fig. 6-5. A sector bending magnet. Particles with different energies follow different paths (left) and particles with different position in the bending plane get focused (right).

Now to the nasty thing about this. These matrices are only valid for dipole magnets which are **sector magnets**. In a sector magnet the path of the particle into the magnet is perpendicular to the magnet edge, and the same going out.
Unfortunately this is not always a convenient way to produce a dipole magnet, as they are often made up from thin slices, laminates. By such a production method they become rectangular and the entrance into the magnet and the exit are not perpendicular to the magnet surface any more.

6.6.4.1 Rectangular magnet

We can regard a rectangular magnet as a sector magnet where one has added and removed some magnet material at the entrance and exit.

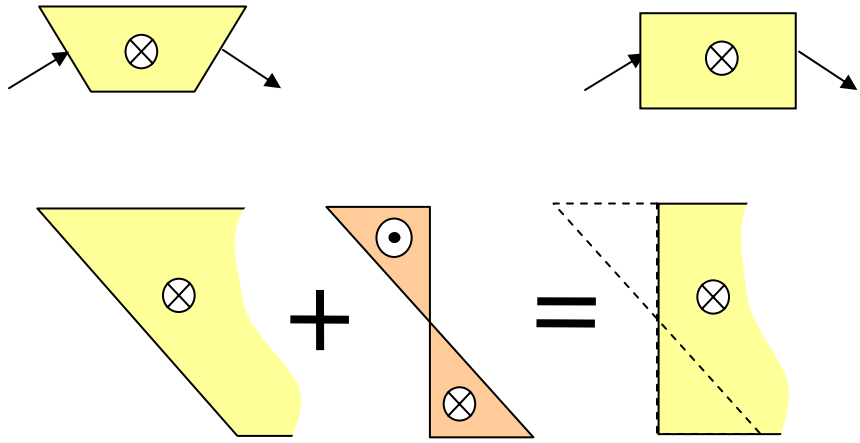


Fig. 6-6. The difference between a sector magnet (tope left) and a rectangular magnet (top right). Add a wedge to the sector magnet edge and the sum will be a rectangular magnet.

The difference can be seen as a small wedge with a magnetic field that adds to the existing field and subtracts from the existing field some strength. Lets look at this small wedge in more detail.

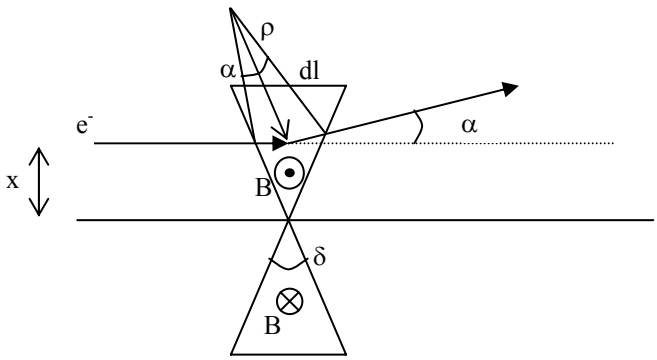


Fig. 6-7. Focusing of the edge wedge.

The bending radius ρ is related through ρα=dl, where dl=x tan(δ). The angle α=x tan(δ)/ρ. This is the action of a lense with the focal length f= x/α (f>>x) which also gives 1/f=tan(δ)/ρ. We know from above that such a thin lense has a transfer matrix given by (omitting the third column and row):

$$\begin{pmatrix} 1 & 0 \\ \frac{\tan(\delta)}{\rho} & 1 \end{pmatrix}$$

Eq. 6-51

The total transfer matrix for a rectangular magnet is thus given by the transfer matrix of a sector magnet corrected with one such thin lense at edge end.

$$\begin{pmatrix} 1 & 0 \\ \frac{\tan(\delta)}{\rho} & 1 \end{pmatrix} \begin{pmatrix} \cos(\varphi) & \rho\sin(\varphi) \\ -\frac{1}{\rho}\sin(\varphi) & \cos(\varphi) \end{pmatrix} \begin{pmatrix} 1 & 0 \\ \frac{\tan(\delta)}{\rho} & 1 \end{pmatrix}$$

Eq. 6-52

This effect is called edge focusing.
Still we have to gain knowledge what happens in the non bending direction. We have identified the magnetic field of the wedge by a vertical magnetic field with a strength $B_y = xB_{wedge}$.
Maxwells equations tells us that in a source free region $\nabla \times B = 0$ which gives that

$$\frac{\delta B_y}{\delta x} = \frac{\delta B_x}{\delta y} \quad \Rightarrow \quad B_{wedge} = \frac{\delta B_x}{\delta y} \quad \Rightarrow \quad B_x = yB_{wedge}$$

Eq. 6-53

What does this magnetic field give? Lets try to solve the directions and forces.

In Fig. 6-8 the magnetic field in the x-direction is drawn. A particle being off axis in the non-bending plane is focused by the wedge. (The example is for a negatively charged particle, like an electron). The result is a focusing lens which we can write as a thin lens. Thus the transfer matrix in that direction is given by (with one lens entering the magnet and one while exiting):

$$\begin{pmatrix} 1 & 0 \\ -\frac{\tan(\delta)}{\rho} & 1 \end{pmatrix} \begin{pmatrix} 1 & l \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -\frac{\tan(\delta)}{\rho} & 1 \end{pmatrix}$$

Eq. 6-54

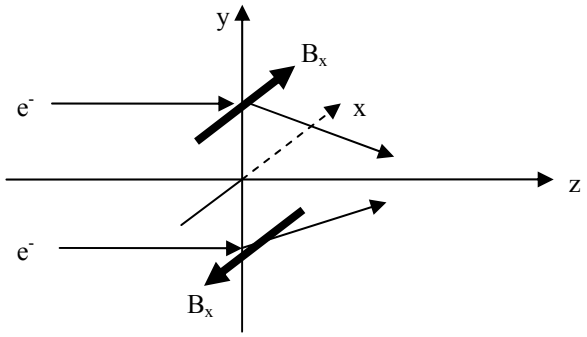


Fig. 6-8. Focusing by the "wedge" in the rectangular magnet.

6.7 Second order

We have up until now, and will in the future, only looked on behaviours that are described by first order parameters. In the above description of the edge effects in a bending magnet one could include effects of the field components in second or higher order, B^n. Most other effects could also be calculated for higher orders. If one does so the transfer matrices become more complicated with many new columns and rows describing the dependence of factors such as: x^2, x'^2, xx', xy, x dp/p etc.

6.8 Betatron oscillation

We have now in some detailed looked on how a single particle behaves while passing the different magnetic elements in an accelerator. We have not studied sextupoles and higher order magnets, but the principle is the same.

From this “single particle” picture will will now try to paint a picture on how an ensemble of particles, a beam, behaves. To do this we will look in “phase space”, which here is drawn by the two coordinates x and x’ (position and angle). In phase space we will define an ellipse with a certain area inside which a certain fraction of all particles are found. By following the extension of this ellipse we can follow the extent of the particle beam in the machine.

If we take the homogenous part of the path equation, Eq. 6-29:

$$x'' - K(s)x = 0$$

Eq. 6-55

and look at a circular machine, where K(s) is periodic: K(s+L)=K(s). The magnets that the particle passes are after one turn (or distance L) the same again. Under this condition we have the so called Hill's equation to which the solution can be written:

$$\begin{cases} x(s) = \sqrt{\varepsilon}\sqrt{\beta}\cos(\Phi(s) - \Phi_o) \\ x'(s) = \sqrt{\varepsilon}\left[\dfrac{\beta'}{2\sqrt{\beta}}\cos(\Phi(s) - \Phi_o) - \sqrt{\beta}\Phi'\sin(\Phi(s) - \Phi_o)\right] \\ x''(s) = \ldots \end{cases}$$

Eq. 6-56

where ε and ϕ_0 are constants and β and ϕ functions of s. (This is a special case for an harmonic oscillator and the proof is not given here).
By inserting Eq. 6-56 into Eq. 6-55 we can extract that

$$\phi' = \frac{1}{\beta}$$

Eq. 6-57

It is also customary to define the following two parameters:

$$\alpha = -\frac{\beta'}{2} \qquad \gamma = \frac{1+\alpha^2}{\beta}$$

Eq. 6-58

The β, γ and also α are called the Twiss parameters, but please watch out as β, γ and also α often also are used for other phenomena!
This put together will give us a way to write the position and angle of any particle around a circular machine.

$$\begin{pmatrix} x(s) \\ x'(s) \end{pmatrix} = \begin{pmatrix} \sqrt{\varepsilon}\sqrt{\beta(s)}\cos(\phi(s) - \phi_0) \\ -\dfrac{\sqrt{\varepsilon}}{\sqrt{\beta(s)}}(\sin(\phi(s) - \phi_0) + \alpha\cos(\phi(s) - \phi_0)) \end{pmatrix}$$

Eq. 6-59

The position of the particle is an oscillation given by the cosine term. The amplitude of the oscillation is given by $\sqrt{\varepsilon\beta(s)}$. The term ϕ(s) tells the phase of the oscillation around the accelerator. Eq. 6-59 is one way to express an ellipse (see Fig. 6-9) in the x – x' space by choosing all values on ϕ_0 from 0 to 2π. β(s) and ϕ(s) are given by the lattice (magnets etc.) and ε and ϕ_0 defines the position on the ellipse for each particle at a certain s.

We can also see that the overall size in some way is defined by the ε constant, which is called the emittance.

β, which varies around the machine, scales this size, emittance, by a certain value for every position, every s. This scaling of the size is called the beta function. If β is large we have a large size (x) with a small divergence (x') and vice versa.

The ϕ function tells us where on the ellipse we are, and the α the inclination or rotation of the ellipse. A particle with a smaller ε than another particle will never reach a larger maximum amplitude.

The area of an ellipse is given by the product of the two half axises and π. To make life a little easier lets choose a point s where β'(s)=0 and thus α=0. The two half axises are drawn by two "orthogonal particles". Lets choose these ones such as for the first $\phi_0 = \phi(s)$ which gives $\cos(\phi(s)-\phi_0)=1$, thus the next one has $\phi_0 = \phi(s)+\pi/2$. The area is then given by

$$Area = \pi\sqrt{\varepsilon}\sqrt{\beta(s)}\frac{\sqrt{\varepsilon}}{\sqrt{\beta(s)}} = \pi\varepsilon$$

Eq. 6-60

which is independent of s, and thus does not vary around the machine. This is an important fact and is called *Liouvilles theorem*.
The consequence is that a particle with a smaller ε will always draw a smaller ellipse than a particle with larger e. If we pick out the particle with the largest ε, we know that all other particles draw **smaller** ellipses.

6.8.1 Phase ellipse

So far we have talked some about the phase space and about the phase ellipse, but what does is actually say about the beam in our machine?
The phase ellipse is drawn around the particles in the beam. One might choose to paint it around the particles within one standard deviation, or the one within the full width half maximum (FWHM) or any other definition, but the most common is one standard deviation and thus the size of one standard deviation of the beam is:

$$\sigma_x = \sqrt{\varepsilon\beta(s)}$$

In the same manner we can write the divergence of the particle beam as:

$$\sigma'_x = \sqrt{\frac{\varepsilon}{\beta(s)}}$$

The result is quite obvious and we can see that a large beta values gives a large and parallel, non divergent beam. If we create a focus for the particle beam, we reduce the betafunction and thus achieves a small beam, but at the same time the divergence increases. This is just what we can expect if we think in terms of an optical beam.
[Another effect which comes into play in a focus is the repulsive forces between charges, space charge. But we will not explore that area here].

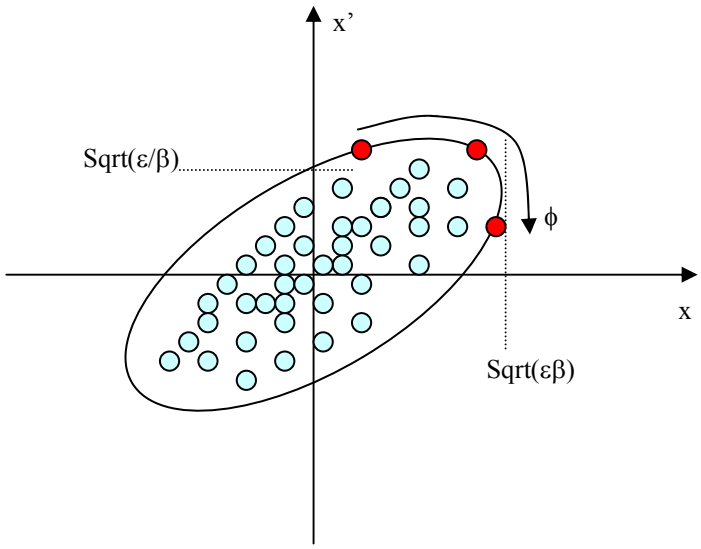


Fig. 6-9. The phase ellipse is built of all particles in the beam.

The tilt of the phase ellips is given by the a-parameter. At a symmetry point the ellips is symmetrical to the two axises, but it then “rotates” or “change form” on the way to the next symmetry point in the clockwise direction. In a focus the beam is narrow and high.

Individual particles are spread around the ellips with different phase values.
As ε is independent of s the area of the ellips is a constant.
Now, what do the different factors explain?

β The **beta function**. It gives the amplitude of the motion, *y(s),* around the machine. It also defines the divergence, *y'(s),* of the motion as proportional to 1/β.
ε The **emittance**. It gives the size of the ellipse around the machine. The amplitude of the motion scales with this factor.
α gives the "tilt" of the ellipse.

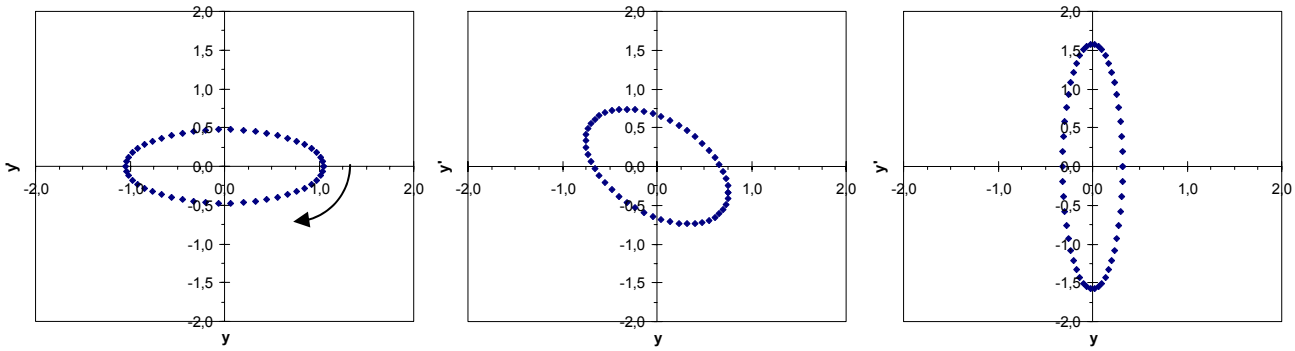


Fig. 6-10. The phase ellipse.
a) β= 2.2, α=0, ε=0.5; b)β=1.14, α=0.5, ε=0.5; c) β=0.2, α=0, ε=0.5

What happens when the particle moves down the machine? From s_0 to s_1.

1. The phase increases by $\phi(s_1) - \phi_0$, and thus the "position" on the ellipse moves along its edge. (see the arrow in Fig. 6-10a.)
2. The β changes from $\beta(s_0)$ to $\beta(s_1)$ and the α from $\alpha(s_0)$ to $\alpha(s_1)$, thus the ellipse will change form, as the length of the half axis changes with β and 1/β, and the tilt of the ellipse with α.

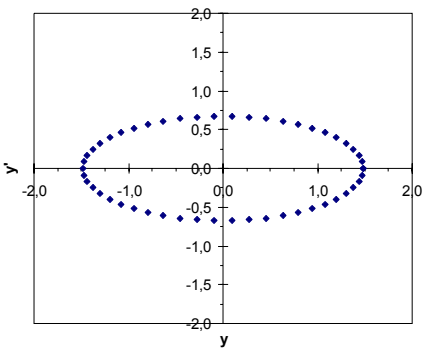


Fig. 6-11. The phase ellipse, β=2.2, α=0, ε=1.0

If one by some means change the e the size of the ellipse will change size.. (compare Fig. 6-10 a and Fig. 6-11).

6.8.2 Transformation of the Twiss parameters

We are now vaguely more happy, as we only have another mathematical way of drawing the behaviour of the particles. If we could get some knowledge of how the α, β and γ are transformed

around the machine, we would have the same tools available that we have for following a single particle around.

As a first step to find such a transformation, preferably on the matrix form, we shall rewrite the expression for the emittance.
An ellipse can generally be expressed by:

$$Ay^2 + Byy' + Cy'^2 = D$$

Eq. 6-61

We already know that:

$$\begin{cases} y = \sqrt{\varepsilon\beta}\cos \\ y' = -\sqrt{\dfrac{\varepsilon}{\beta}}(\sin + \alpha\cos) \end{cases}$$

Eq. 6-62

We are free to choose D = ε, and by identifying the terms we get:

$$A = \gamma \qquad B = 2\alpha \qquad C = \beta$$

Eq. 6-63

and

$$\gamma y^2 + 2\alpha yy' + \beta y'^2 = \varepsilon$$

Eq. 6-64

As the emittance, e, is conserved around the machine, we know that

$$\gamma y^2 + 2\alpha yy' + \beta y'^2 = \varepsilon = \gamma_0 y_0^2 + 2\alpha_0 y_0 y_0' + \beta_0 y_0'^2$$

Eq. 6-65

By our matrix formulation from above we can write:

$$\overline{y} = \underline{M}\,\overline{y_0} = \begin{pmatrix} C & S \\ C' & S' \end{pmatrix}\begin{pmatrix} y_0 \\ y_0' \end{pmatrix} \rightarrow \overline{y_0} = \underline{M}^{-1}\overline{y} = \begin{pmatrix} S' & -S \\ -C' & C \end{pmatrix}\begin{pmatrix} y \\ y' \end{pmatrix}$$

Eq. 6-66

Inserting Eq. 6-66 into Eq. 6-65 and identifying the terms give

$$\begin{pmatrix} \beta(s) \\ \alpha(s) \\ \gamma(s) \end{pmatrix} = \begin{pmatrix} C^2 & -2SC & S^2 \\ -CC' & SC' + S'C & -SS' \\ C'^2 & -2S'C' & S'^2 \end{pmatrix}\begin{pmatrix} \beta_0 \\ \alpha_0 \\ \gamma_0 \end{pmatrix}$$

Eq. 6-67

We now have the tools to follow an ensemble, beam, of particles around the machine. If we at the starting point fit an ellipse described by β_0, α_0 and γ_0 around the particles, we can follow this ellipse around the machine. The matrix transforming the ellipse parameters is slightly more complicated than the single-particle transfer matrix, but it is only constructed out of the same elements.

6.8.3 Transfer matrix

To extract more knowledge out of the mathematics from above, we should also express the transfer matrices in terms of the Twiss parameters (α, β and γ).
Any path of a particle can be expressed by:

$$\begin{cases} y(s) = a\sqrt{\beta}\cos\phi + b\sqrt{\beta}\sin\phi \\ y'(s) = \ldots \end{cases}$$

Eq. 6-68

The initial conditions for any path are
S=0 and ϕ=0
Which generates:

$$\begin{cases} y_0 = a\sqrt{\beta_0} \\ y'_0 = -\dfrac{a}{\sqrt{\beta_0}}\alpha_0 + \dfrac{b}{\sqrt{\beta_0}} \end{cases}$$

Eq. 6-69

Using Eq. 6-69 in Eq. 6-68 gives

$$\begin{pmatrix} y(s) \\ y'(s) \end{pmatrix} = \begin{pmatrix} \sqrt{\dfrac{\beta(s)}{\beta_0}}(\cos\Delta\phi + \alpha_0\sin\Delta\phi) & \sqrt{\beta_0\beta(s)}\sin\Delta\phi \\ -\dfrac{1}{\sqrt{\beta_0\beta(s)}}\left[(\alpha(s)-\alpha_0)\cos\Delta\phi + (1+\alpha(s)\alpha_0)\sin\Delta\phi\right] & \sqrt{\dfrac{\beta_0}{\beta(s)}}(\cos\Delta\phi - \alpha(s)\sin\Delta\phi) \end{pmatrix} \begin{pmatrix} y_0 \\ y'_0 \end{pmatrix}$$

Eq. 6-70

We have earlier said that we are looking on a periodic machine. This means that it consists of one or several periods which are identical (identical magnets in identical order). Thus we can say that after one period when $s=s_0+L$ we have $\beta=\beta 0$ and $\alpha=\alpha 0$.
It is also nice to start in a "symmetry point". From such a point it does not matter which way we go: backwards or forwards. In such a point the following is true:

$$\beta(s_0+ds) = \beta(s_0-ds) \quad \rightarrow \quad \beta'(s_0) = 0 \quad \rightarrow \quad \alpha(s_0) = 0$$

If we start in one such symmetry point and continue around the machine to another. This might mean one turn in a round machine, or to a place where the machine starts to repeat itself. At this point the transfer matrix takes the form:

$$\begin{pmatrix} \cos\mu & \beta_0\sin\mu \\ -\dfrac{1}{\beta_0}\sin\mu & \cos\mu \end{pmatrix}$$

Eq. 6-71

where $\mu = \phi(s)-\phi_0$, which is change in phase angle from one symmetry point to the next one. This phase angle can be written as $\mu=2\pi Q$, where Q denotes something called the betatron tune, the number of oscillations the particles perform from one symmetry point to the next. If there are n such symmetry points around the machine the total tune will be nQ.

Another nice thing about looking at the transfer matrix from one symmetry point to another is that we can extract the beta function value at that point. By choosing the two matrix elements (1,2) and (2,1) we get:

$$\frac{S}{C'} = \frac{\beta(s_{sym})\sin\mu}{-\dfrac{1}{\beta(s_{sym})}\sin\mu} = -\beta^2 \quad \Rightarrow \quad \beta(s_{sym}) = \sqrt{-\frac{S}{C'}}$$

Eq. 6-72

We have thus actually defined the beta function around the machine, as we from this value have the possibility to transform it around the machine by the matrix formulations above. We still have to remember that this is not the actual size of the beam, and the size is given by the beta function value times the emittance!

We also know α in the starting point (=0) and can thus calculate it at any position around the machine. The γ parameter was a construction out of α and β and is thus also known. What we do not know is the emittance, and we can not describe the beam totally without it. Unfortunately the emittance is NOT uniquely defined in a circular machine by these methods. It is necessary to know more about the beam, such as energy loss, damping mechanisms, acceleration etc. to find a value on the emittance. This is beyond the scope of this book though, so until then we must accept that for a beam with a certain emittance we can define the beam in quite some detail around a machine.

If we do not have a round machine, which repeats itself, we would not be able to find a unique solution for the beta function and the α-parameter, and thus the beam would remain fairly unknown. Here we have to rely on information of what our beam looks like while entering the accelerator.
This can be the case in a so called transport line from a particle source up to an experimental station or the round accelerator. The transport line is only passed once in one direction, and thus not repeating itself in any sense, and a stable mathematical solution does not exist.

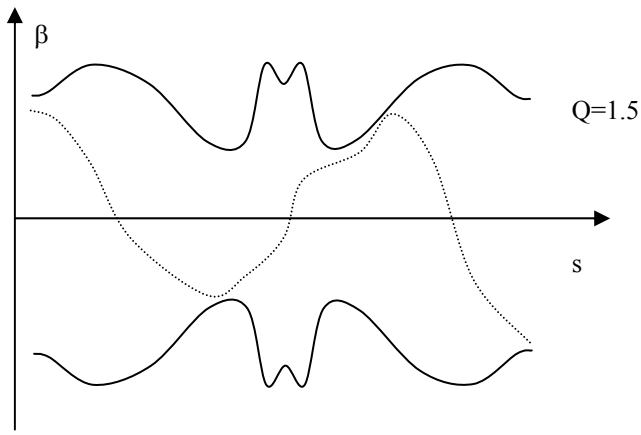


Fig. 6-12. Betatron function and the path of an individual particle.

6.9 Momentum compaction

In some cases one is interested in the difference in path length for particles with energies other than the perfect energy when they travel around the machine. This relation is described by the momentum compaction.

$$\frac{\Delta l}{l} = \alpha_m \frac{\Delta P}{P}$$

Eq. 6-73

Where l is the length of the machine, α_m the momentum compaction and P the momentum of the particle.

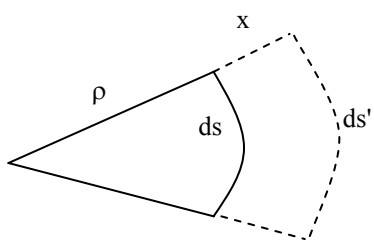


Fig. 6-13. A longer path for a particle with a higher energy.

From Fig. 6-13

$$ds' = \frac{ds}{\rho}(\rho + x) = ds\left(1 + \frac{x}{\rho}\right)$$

Eq. 6-74

and

$$x(s) = D(s)\frac{\Delta P}{P_0}$$

Eq. 6-75

If we look at a whole turn of the machine we can write the change in circumference as

$$\Delta C = \int_0^{s_c} \frac{x(s)}{\rho(s)} ds = \int_0^{s_c} \frac{D(s)}{\rho(s)} \frac{\Delta P}{P_0} ds$$

Eq. 6-76

from this we can write the momentum compaction as

$$\alpha_m = \frac{\Delta C}{C}\frac{P_0}{\Delta P} = \frac{1}{C}\int_0^{s_c} \frac{D(s)}{\rho(s)} ds$$

Eq. 6-77

6.10 Motion with momentum deviation and closed orbit distortion

We earlier solved the Hills equation and then quite happily omitted one term we had carried along before:

$$X'' - K(s)X = \frac{1}{\rho}\frac{\Delta P}{P_0} = F(s)$$

Eq. 6-78

By including the last term of Eq. 6-78 we have the partial equation as given in [xxx]. We did also write the actual solution to the partial equation, but we could not solve it as we did not know what the functions that built the solution looked like.
The partial solution to this equation is given by:

$$X_{part}(s) = S(s)\int_{s_0}^{s} F(t)C(t)dt - C(s)\int_{s_0}^{s} F(t)S(t)dt =$$
$$\frac{\sqrt{\beta(s)}}{2\sin(\pi Q)}\int_{s}^{s+L} \sqrt{\beta(t)}F(t)\cos[\phi(t) - \phi(s) - \pi Q]dt$$

Eq. 6-79

This we can now use for two things. The first is to find the solution for a particle with a energy deviation (from the perfect energy) and the second thing is to study errors which gives deviations from the perfect orbit (closed orbit) for all particles.

We earlier wrote a term called the dispersion as

$$D(s) = \frac{X_{part}(s)}{\frac{\Delta P}{P_0}}$$

and the position of the particle became:

$$x(s) = C(s)x_0 + S(s)x_0' + D(s)\frac{\Delta P}{P_0} =$$
$$C(s)x_0 + S(s)x_0' + \frac{\sqrt{\beta(s)}}{2\sin(\pi Q)}\int_s^{s+L}\sqrt{\beta(t)}\frac{1}{\rho(s)}\cos[\phi(t)-\phi(s)-\pi Q]dt\frac{\Delta P}{P_0}$$

Eq. 6-80

Let us assume that we have a nice round accelerator where the particles move on the "perfect" closed orbit. At one point $s=s_{kick}$ we introduce an error, or a "kick" of some kind to the beam. From Eq. 6-79 we have the partial solution and the integration becomes simpler as the F(s) only exists in one point $s=s_{kick}$:

$$X_{part}(s) = \frac{\sqrt{\beta(s)}}{2\sin(\pi Q)}\int_s^{s_1}\sqrt{\beta(t)}F(t)\cos[\phi(t)-\phi(s)-\pi Q]dt =$$
$$= \frac{\sqrt{\beta(s)}}{2\sin(\pi Q)}\sqrt{\beta(s_{kick})}F(s_{kick})\cos[\phi(s_{kick})-\phi(s)-\pi Q]L$$

Eq. 6-81

Where F(s) represents the "kick strength" and L the length of the kick.
F as given in Eq. 6-78 can also be written as a change in radius of the beam path

$$F(s) = \Delta\frac{1}{\rho} = \frac{q\Delta B}{p_0}$$

Eq. 6-82

Due to an error in a bending magnet or a small additional bending magnet.

From this we can see a few things:

- The form of the path due to the kick has an amplitude proportional to the beta function, and there are Q (betatron tune) oscillations over one turn (due to the $\phi(s)$ in the cosine function).
- If Q is an integer the amplitude of the path will go to infinity! This can be interpreted as if we have an error in the machine the particle should not come back with the same phase every turn, because it will then get another kick adding to the first. If the phase is not the same there will be an oscillation but no catastrophe.

In reality the machine is very sensitive to integer Q numbers as there are always small faults in a machine. The machine will also be sensitive to half integers and one third integers, but not as sensitive.

We can draw a tune diagram which shows which Q, betatron tunes, that are allowed and where to find them etc. (Fig. 6-14). It is a schematic way to see if we are close to resonances, and if wecan move to a new operating point without crossing any resonances.

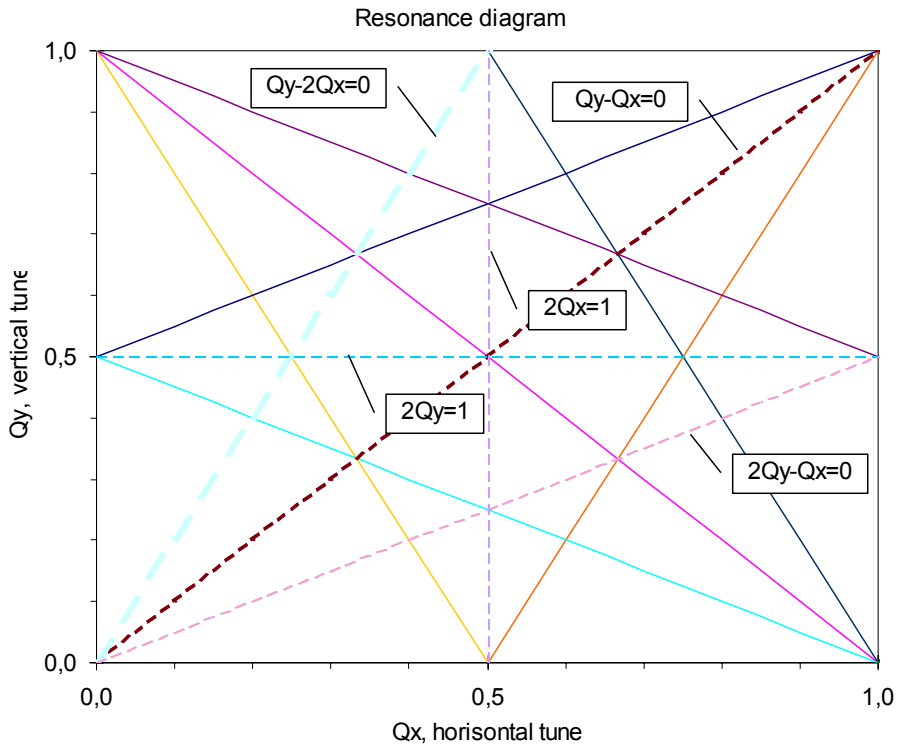


Fig. 6-14. Tune diagram (to 2nd order)

7 Particle sources and first acceleration

(prev. Guns, klystrons och cavities)

After the odyssey around the different basic accelerator types a while ago we have spent a long time on the magnets and beamdynamics. It is now time to return a bit and look on the actual components giving acceleration, or more correct an increase in energy. In this context we will also treat the creation of particles, and then mainly electrons.

As most of the devices performing the acceleration needs high power high frequency fields, we must have a look on the generators of these powers, the klystrons, and how the power is transported from the klystrons to the accelerator.

All accelerators begin with a particles source. This source is different for each species of particles, but in most cases the initial source is an electron source. The electrons are then used either directly or in a process to "create" the desired particle. Often there is a need for a rapid acceleration of the particles, especially electrons, directly after the creation. In many cases the quality of these very first steps are giving the overall performance of the accelerator system. This is especially true for single pass Free Electron Lasers. Special techniques and effects thus have to be regarded, such as space charge effects and coherent synchrotron radiation.

7.1 Cathodes

To "create", or rather extract, electrons one uses processes in which electrons are made to leave a solid surface. Two main methods are at hand, either by thermal emission or photo emission. A combination of the two is also possible.

The first method uses in principle a piece of metal which one heats to high temperatures, say 1000-2000^0 C. A "cloud" of electrons is thus produced just outside of the surface of the metal. The metal one uses can be an ordinary metal but due to better efficiency one often choose special compounds like BaO or LaB6. From the LaB6 one extract very high currents, the material can also withstand quite bad vacuum which is convenient. The drawback is that it need somewhat higher temperatures, close to 1800^0 C. The BaO type cathodes are commercially available and operates at lower temperatures, 1200^0 C. They need a better vacuum and are thus more rapidly deteriorated while inserted into the gun. The current possible to extract is also lower than for the LaB6 type.

In photoemission a high energy laser pulse illuminates the cathode. A figure of merit is here the quantum effincy (QE) of the material. It tells us how many electrons are created by each photon. (typical numbers are in the range of QE=10^{-4} – 10^{-5}). The basic criteria for the method to work is that the photon energy is at least the work function for the material in question.
The cathodes are most simply made out of metals. Copper (Cu) has en work function of 4.7 eV asking for a photon wavelength of <266 nm giving a QE around $1*10^{-4}$. Higher QE can be achieved by treating the Cu surface properly (polishing, cleaning etc) and by introducing the laser beam at an angle to the surface. To reach large improvements though other materials are necessary. Higher QE and lower photon energies can be achieved by semi conductor materials. This makes the laser driving the device significantly less complicated. The price to be paid is instead that the cathode materials are expremely sensitive. They more or less have to be produced inside of the accelerator and never exposed the anything else than ultra high vacuum. They normally also have a limited lifetime. From hours to maybe a few months.

The desire to use photo cathodes is the urge to improve the electron beam performance. A large amount of charge should be produced within a very limited time interval (a few ps) from a very small surface. A laser can typically produce such pulses, while a thermionic cathode emits electrons all the time.

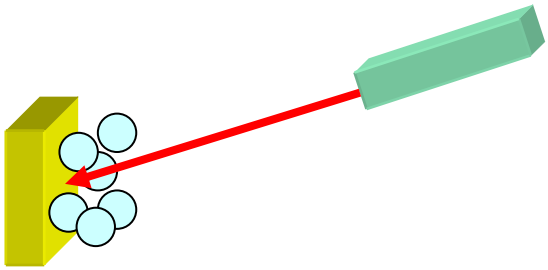


Fig. 7-1. A laser producing photo electrons on a cathode surface.

7.2 Electron guns

Many different species of guns have been developed to fill the demands from different applications. The basic guns consists of a thermionic (heated) cathode placed in a high voltage gap.

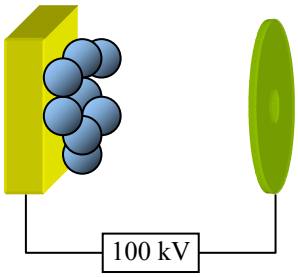


Fig. 7-2. Principle of a thermionic electron gun.

This is the gun used in traditionial TV-sets (until the birth of the flat screens). Guns are divided into continous emitting devices (CW) or pulsed devices. For accelerators one is often interested in the emittance of the beam, the current, the charge in a pulse (if pulsed), the repetition rate and the pulse length.

7.2.1 Traditional guns, Diode and Triode guns

It is neither possible nor desirable to operate other than small guns completely DC, as the power needed is rather high. The heating is a slow process though, and this is not a way to create a pulsed operation. Instead one applies the high voltage between the cathode and the anode in pulses, typically a few microseconds long. This is not trivial either, but by proper construction of the electronics it can be achieved.

Another fruitful way to go is to use a triode gun. In this gun a grid is introduced between the cathode and the anode, close to the cathode. By changing the voltage of the grid the gun can be switched on and off. As the grid is much closer to the cathode, the voltage difference needed is much lower and much more rapid switching can be achieved. The main accelerating voltage is still pulsed, but one can now choose to let just a short pulse of electrons by, when the main voltage really has reached its maximum.

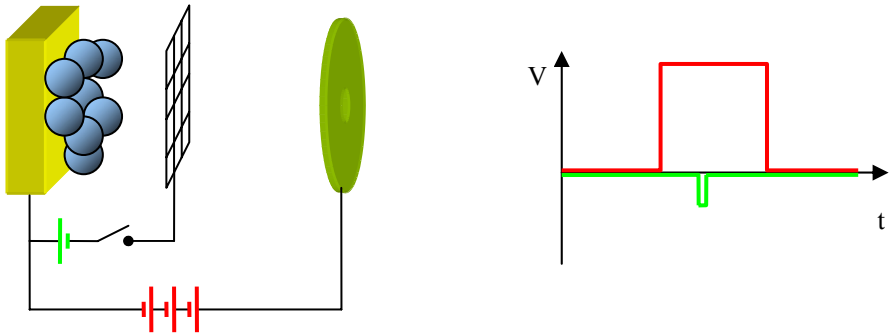


Fig. 7-3. The triode gun. A grid is pulsed with a short lower voltage pulse to gate the acceleration of electrons.

7.2.1.1 Focusing in the DC gun

The beam leaving the cathode is accelerated towards the anode by the electric field created by the potential difference. But this is not all! The electrons also feel the repulsive forces of their neighbouring electrons in the beam and tend to be deflected out from the beam core, defocused.

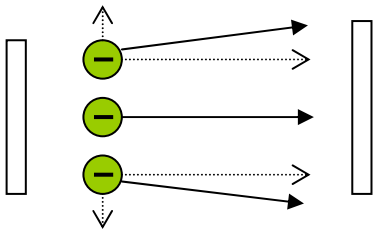


Fig. 7-4. Defocusing inside an electron bunch due to space charge.

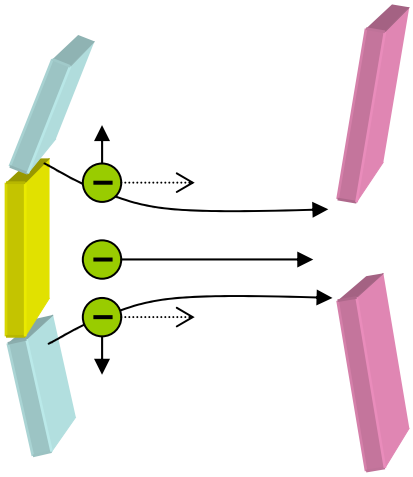


Fig. 7-5. Focusing electrodes around the cathode of an electron gun.

The defocusing forces can be counteracted by placing electrodes outside the cathode with a proper shape.
In the planar case, as in fig xx, the angle of the electrodes can be calculated to 67.5^0 for the case that one wants to extract as much current as possible. One can construct other geometries for the gun and the most convenient is thus a cylindrical geometry.

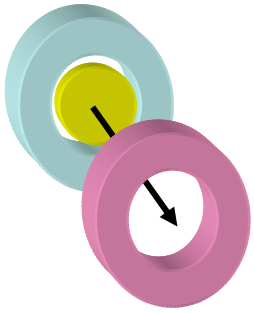


Fig. 7-6. Cylindric configuration of an electron gun.

7.2.2 RF-gun

In an ordinary gun the electrons are accelerated up to say 100 KeV in the gun and then travels for a while until the next accelerating element is reached. This gives plenty of time for space charge (see below) to destroy all nice features of the beam.

In an RF-gun the cathode is placed inside an RF-cavity (like the first cell of a linac structure). The accelerating electric fields at the surface can become very large, and it is easy place additional cells directly after the first one.
[The first cell should only be a half cell as one wants high fields on the surface of the cathode. For a full cell the fields become zero on the walls in the middle].

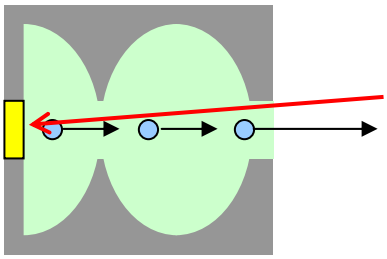


Fig. 7-7. Electrons moving inside the cavity structure of an RF-gun.

If the cavities are operating at 3GHz (a common choice) the total length of the first half cell and the whole cell is only 7-8 cm and the resulting energy from the first half cell is typically 500 KeV and the total energy gain in the order of 2500 KeV.
In this structure an electron emitted from the cathode at the perfect moment is accelerated in the first half cell. It takes some time for it to pass into the second cell and when it does so the fields change direction and acceleration can occur in the second cell as well. (The fields in the cells are oscillating with opposite direction in the two cells). An electron starting off at a different time will not experience any accelerating field in the first cell, and thus not get anywhere (not even out from the cathode surface).
There is though no sky completely without clouds. An electron starting out just slightly late in the accelerating phase comes to the end of the first cell just when the field has changed direction. It has enough energy to get on to the next cell but is not able to pass before the fields has switched again. It is then accelerated backwards (!) and can hit the cathode coming back. This is called backbombardment and has two effects. First it can/will destroy the cathode, secondly it will heat the cathode and thus more electrons are generated and the process might “run-off”.

There are several cures. One is to shape the geometry to minimize the effects. One can also reduce the current or one can gate the emission by a laser pulse.

7.2.3 Super conducting gun

In some applications one is interested in running with a continuous beam (CW-operation) or nearly continuous. This can not be achieved in a normal cavity as the resistance, and thus losses, in the

copper structure are far too and would result in enourmous powers. One solution at hand is to operate superconducting devices where the losses are far smaller and can allow CW operation. This is already a well operating technology for cavities in storage rings and is available for linacs. The sources are still a little bit more troublesome especially as one have to introduce a "heating" on the cathode surface by a laser and to achieve emittance compensation magnetic fields are used. The magnetic fields can completely remove the superconducting ability of the material and cause a quench (break down) of the system.
In Fig. 7-8 there is a drawing of a gun aimed for the proposed BESSY FEL (Berlin, Germany).

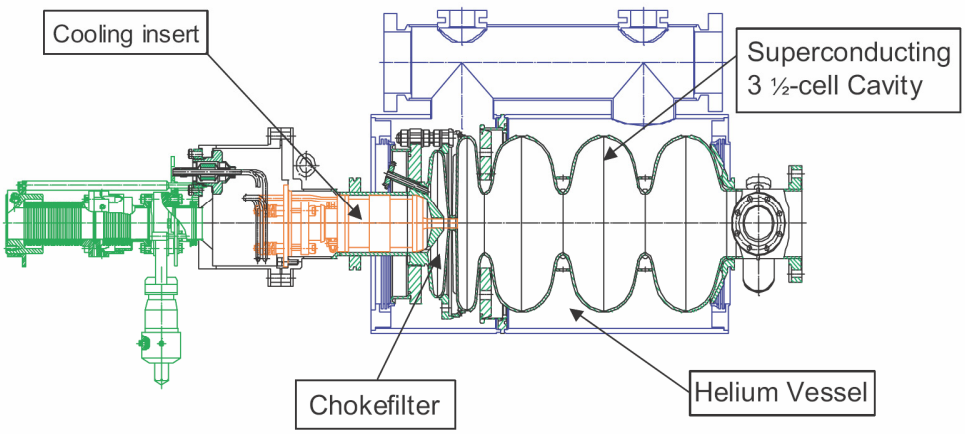


Fig. 7-8. A superconducting gun design.
(BESSY FEL, Technical Design Report, Berlin 2004)

7.2.4 Low Emittance Guns (LEG)

The photo cathode RF guns have difficulties in reaching emittances below 1 mm mRad (normalised) at any significant charges. To overcome this "limit" new concepts are developed. One such concept is using field emitters. The electrons are emitted from one, or several, very thin tips. The emission is gated either in a triode configuration or by triggering the emission with laser light. Due to the extremely small dimensions the emittance of the beams will also be small.

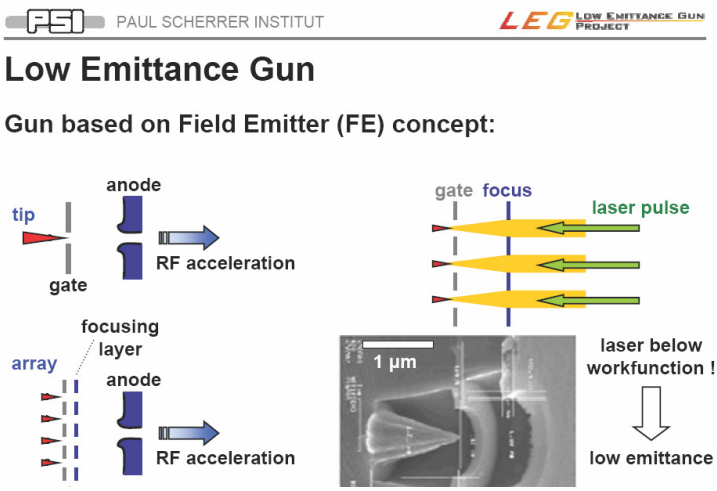


Fig. 7-9. The Low Emittance Gun layout at PSI (by René Bakker, PSI)

Another way to approach the problem is not to restrict the emission of the partictles but rather produce an abundance of particles and then "scrape off" the ones not needed and thus a small emittance will remain. A long high charge electgron bunch is produced in a triode configuration. This is then compressed and accelerated to the desired performance.

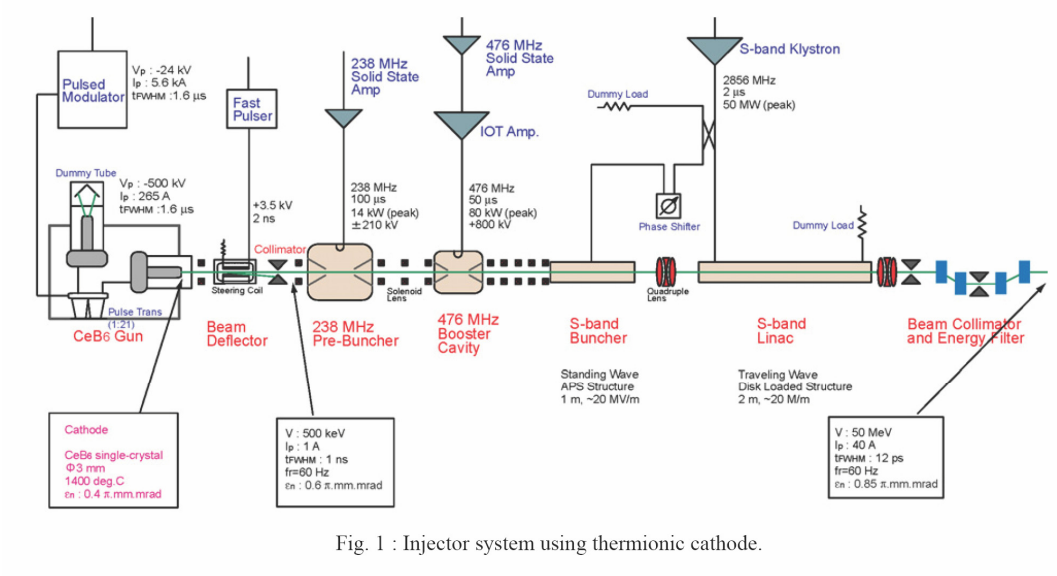


Fig. 1 : Injector system using thermionic cathode.

Fig. 7-10. Layout of the thermionic gun for the SCSS X-FEL (conceptual design report, Riken Japan 2005)

7.3 Laser acceleration

A completely different method for accelerating particles to high energies extremely quickly is laser acceleration. This might be a solution for complete accelerators in the future but the closest application is to form a complete first accelerator up beyond the energies where space charge effects occur. The accelerating fields are very much higher than anything achievable in an RF-gun and over just a few millimetres the particles can reach beyond 100 MeV.

One way of doing laser acceleration is to focus a laser with a very short bunch into a gas jet. The laser beam will make a plasma of the gas and electrons will be pushed out from the path of the laser beam. Just behind the laser bunch there will be very few electrons left while the atoms, now ionised, remain as they are much heavier and do not move as easily. The electrons will be attracted by the positive ion area and accelerated. At the same time the laser pulse propagates and thus the electrons are dragged along with the plasma bubble behind the laser pulse.

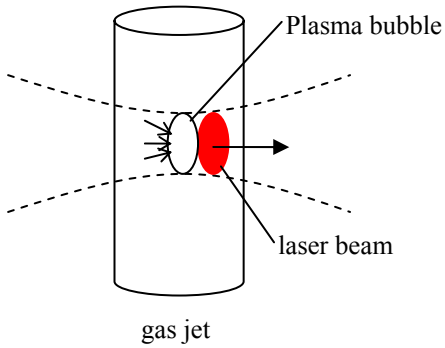


Fig. 7-11. Laser acceleration.

7.4 Effects in the electron beam

7.4.1 Space charge, emittance growth and compensation

Space charge effects originates from the fact that particles with the same charge repel eachother and in many accelerators there is a desire to make the particle density as large as possible. When the density increases the repulsive forces increase and thus the bunch or beam of particles can

disrupt. Fortunately a beam is only sensitive to space charge forces at lower energies. Above 100-200 MeV almost no effect can be seen.

The space charge forces will, if the charge density is high enough, make the bunch diverge and thus destroy the emittance of the beam. As the emittance is a key parameter for Free Electron Lasers there is an absolute need to preserve it at low values. In fact there is a possibility to reduce the emittance of the beam just after the electron gun.

When a high current bunch exits the gun it suffers from space charge and the electrons tend to move away from the beam axis.

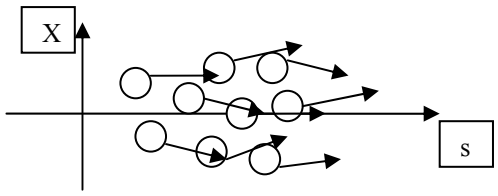


Fig. 7-12. A low charge beam propagating

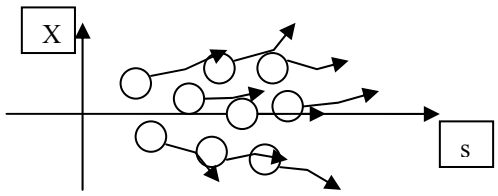


Fig. 7-13 The movement of a similar beam but with high charge and under influence of space charge forces. The beam is growing with increased angular errors.

Or drawn in a phase space diagram

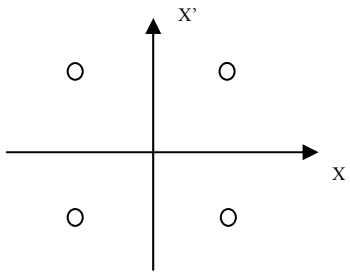


Fig. 7-14. An ordinary beam in phase space.

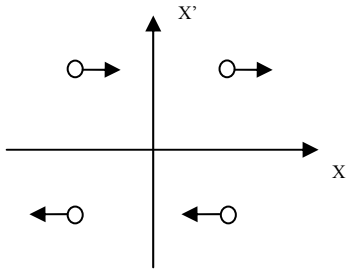


Fig. 7-15. The low charge beam moves in phase space when it propagates in free space.

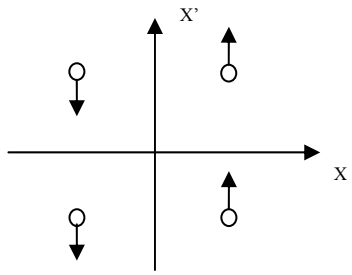


Fig. 7-16. The action of space charge will give the particles a new orbit angle.

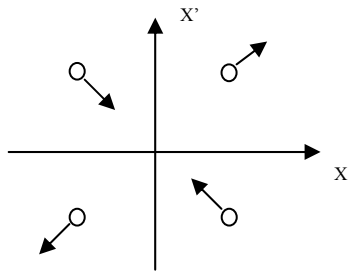


Fig. 7-17. Particle movement when drifting down the accelerator under the effect of space charge.

These forces are not linear and as the emittance is defined as the area of the ellips around all particles the emittance will increase by this action. BUT, if we look closely we can see that the particles in the second and forth quadrant are moving towards the origin and are thus damped. If we can make a beam which only consists of particles just along these arrows the whole beam will be damped. This kind of beam is a focused beam. By focusing the beam just the correct amount to make the arrows point towards origo we can damp the emittance. If the focusing is done in a solenoid this can be achieved in both horizontal and vertical direction. (The solenoid is only powerful enough at low energies and thus the method is only applicable up to around 5 MeV).
The space charge forces are, as mentioned, not linear and there will occur over focusing, over compensation and a beating of the emittance. Thus the focusing, the space charge and the further acceleration of the beam have to be matched to each other to actually produce a reduction of the transverse emittance.
But what about Liouvilles theorem which says that we can not make the emittance smaller?
We can see this as two effects. First the space charge forces are actually accelerating the particles, and then Liuoville is not valid. The second is that we can make the horizontal and vertical emittances smaller, but by doing so we will create more space charge forces longitudinally and thus the head of the beam will be accelerated and the tail decelerated. This is an increase in longitudinal phase space.

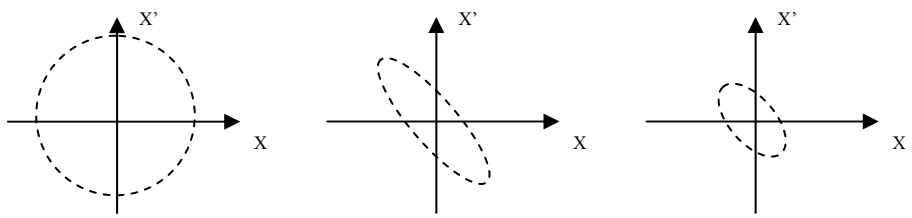


Fig. 7-18 A parallel beam (a) is focused by a solenoid (b) and continues under the effect of space charge (c).

7.4.2 Coherent Synchrotron Radiation

Coherent synchrotron radiation (CSR) is a more complicated effect. The charged particles (mainly electrons) can emit synchrotron radiation when bent by a bending magnet. The radiation can be in phase from all electrons if the wavelength of the radiation is longer than the electron bunch itself. (normally in the range of many micrometers or longer). This is called coherent radiation. When the electron turn in a bend the radiation originating from the tail of the electron bunch can hit the head of the bunch and as the radiation is in phase all electrons will experience the same field. This will give a slight transverse acceleration of those electrons which then will differ from the tail.

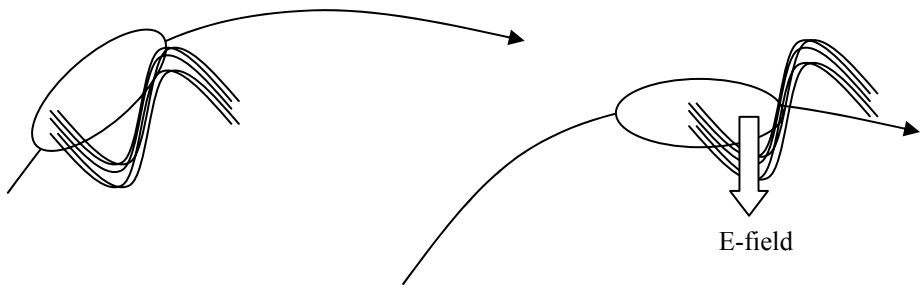


Fig. 7-19. Coherent synchrotron radiation from the tail of the bunch hits the head and kicks it sideways.

7.4.3 Compression

It is often of interest to make very short bunches. Either one is interested directly in a short bunch or one uses that if a long bunch is compressed into a short bunch the peak current increases.

The way to do this is to introduce an energy chirp along the bunch. This means that there is a small energy deviation from the head to the end of the bunch. To get this one lets the bunch pass through an RF accelerating structure (normally a linac) away from the perfect accelerating phase. As different phases will give different energies one can get an energy chirp.

If the energy of the bunch is low (almost non relativistic) the particles will have different energies and the quicker ones will catch up with the slower ones and a bunching occurs (ballistic bunching). This is seldom the case for an electron beam, as the electrons are relativistic already at 5 MeV. In this case the beam is made to pass a “dispersive section” where particles with different energy moves along different paths.

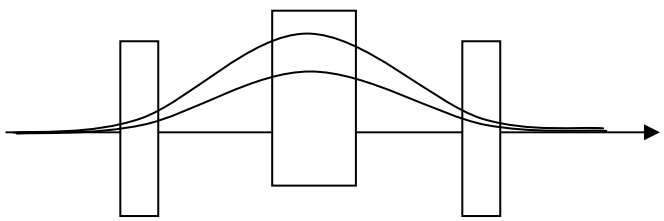


Fig. 7-20. A dispersive section of three bending magnets (“chicane”) to create buching. Two electron paths for two different energies.

The paths for a lower energy electron is longer and thus takes longer time. A problem here is that the path length difference is linear with the deviation in energy while the induced energy chirp is given by a sinus function in the accelerating structure. Thus there will not be a perfect compression of the bunch but the compressed bunch will take the shape of a “banana”. There are methods to overcome this, but none is very simple nor gives a perfect result. (Magnetic systems

that give non linear path differences and using several accelerating structures with different frequencies to linearize the energy chirp.)

7.5 Ion sources

To produce ions one proceed in different ways depending of if one wants to produce positive ions (remove electrons) or negative ions (add electrons).

Positive ions are in principle created from a gas which is transformed into a plasma, where electrons are stripped from the ions. A potential difference extracts the ions and a magnetic filter selects the ion mass of interest.

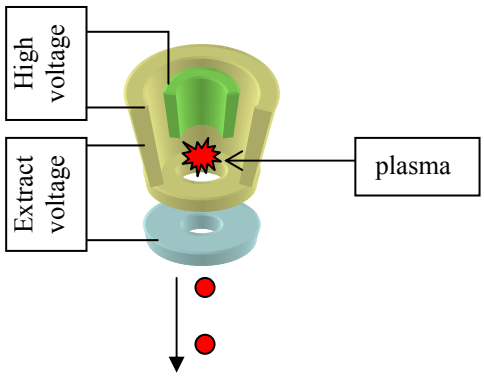


Fig. 7-21. A plasma ion source.

The plasma is formed between the two electrodes. The distance between the two is too small for a discharge to build up, except in the region below the inner electrode. A gas is injected inside the inner electrode, and this gas forms the plasma in the discharge region. Another electrode extracts the positive ions from the plasma, and a beam of ions is generated.

Several schemes with improved geometry, higher or lower discharge voltages etc. can, and have been, designed. There are also other ways to create the plasma by electron beams and high frequency electromagnetic fields.

Another common type of source, and storage place, for highly charged positive ions is the EBIS (Electron beam ion source). The idea is to create a place where the gas that one wants to ionize is enclosed and focused in a restricted region until sufficient ionisation has been achieved. One then opens the storage place and a beam of the desired ion is extracted.

A gas is injected into an enclosure which at each end has a electrostatic mirror ”reflecting back” the positive ions that approaches it. A high current electron beam passes along the mirror axis and it both focuses and ionises the gas. The ions stay in the enclosure until one of the mirrors is ”opened” and a beam of ions of the desired charge is extracted.

The method of producing negatively charged ions is a little different. An atom with positive electron affinity, where every atom happily accepts several electrons, is made to pass a gas of an alkali metal, such as cesium. It is convenient to start with a beam of low positively ionised atoms, as it is easily created and easily can make a beam. Such a beam passes the cesium gas, where the beam ions collects electrons, and end up as beam of negatively charged ions which can be further accelerated.

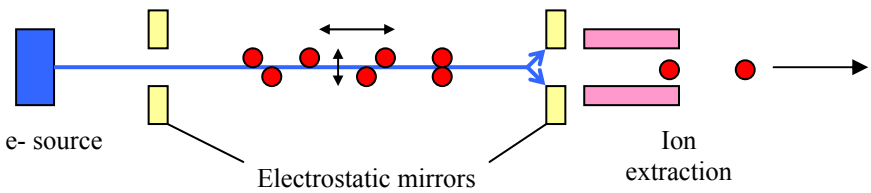


Fig. 7-22. EBIS ion source.

7.6 Amplifiers

To generate the powers needed to drive high frequency devices such as RF-guns, linacs and storage ring cavities special amplifiers are necessary. For a long time the principal device has been the klystron but lately two new types have come into use. One is the IOT (Induction Output Tube) which resembles the klystron and the other is solid state amplifiers. Other applications do not need as high powers, microwave ovens or radars in aircrafts or ships, and use more simple amplifiers such as the magnetron.

7.6.1 Klystrons

To generate the RF-power one needs in the accelerators one need RF amplifiers. The most common type is a so called klystron. The klystron is in a sence a mini electron accelerator, where one uses an electron beam to amplify a high frequency signal.

The principle is that one takes a uniform electron beam and disturbs the electrons longitudinally by a low power electric field at the desired frequency. Thus the electron energy will be modulated along the beam with a frequency equal to the input frequency. Electrons with higher energy will travel faster and will catch up with the
ones travelling slower, and thus the energy modulation will build up a density modulation. This density modulated electron beam passes an RF-cavity which can operate at frequencies around the desired one. Here they induce high electromagnetic fields in the RF-cavity. These fields depend most on the amount of electrons and the amount of density modulation, and thus one can achieve a high amplification of the initial signal.

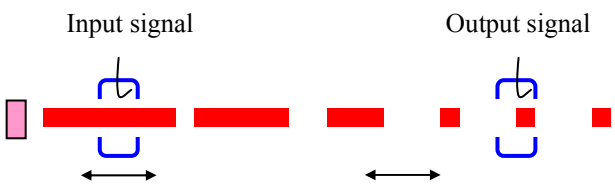


Fig. 7-23. The principle of a klystron. A uniform input beam gets modulated in the input cavity and leaves an amplified signal in the output cavity.

The klystron can be tuned within a limited frequency range, and thus adapt to different running conditions.

7.6.2 IOTs

The IOT (Induction Output Tube) is similar to the klystron. A round electron beam is created but when the klystron operates a DC source which is always emitting, the IOT operates a gridded source. In the klystron the bunching of the beam is done in a set of cavities, fed by the input signal, while in the IOT the input signal is fed to the grid of the triode which only emits when there is an input signal. Thus the bunching starts already at the electron emission. The beam is then accelerated and the output signal is collected in an output cavity (gap).
The result is a device which has a higher efficiency and lower power on the collector as there is only output when an input signal is present. The gain in this amplifier is lower than for a klystron though and it can not operate at higher frequencies (above ~2 GHz).

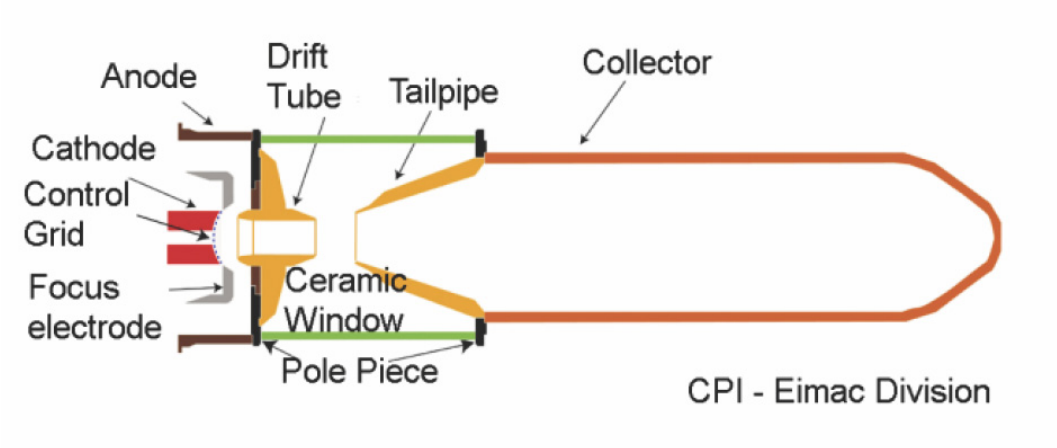


Fig. 7-24. The layout of an IOT by CPI-EIMAC.

7.6.3 Solid state amplifiers

A solid state amplifier is in principle limited in the amount of power it can provide especially if operated at high frequency. The way of solving this is by building a large number of small amplifiers that work together. One of the first large istallations of this technique is for the SOLEIL synchrotron in Paris, which started operation in 2006. 724 modules of 315 W each are combined into a 190 kW amplifier at 352 MHz. One of this kind of amplifier is needed to drive each of the four cavities in the storage ring.

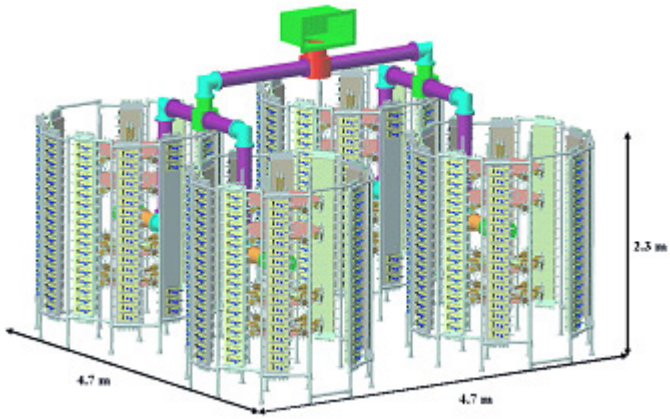


Fig. 7-25.A solid state amplifier assembly for Soleil, Paris.
(from Synchrotron Soleil, Soleil newsletter January 2005)

7.6.4 Magnetrons

The magnetron is another amplifier which is not very common in high energy accelerators as it can not provide extremely high powers. It can neither be tuned but within a very limited region. On the other hand it is simple, small and robust and is therefore used in some special applications ranging from microwave ovens to radiological equipment in health care to mobile systems.

The frequency of the magnetron is ”built into” the structure by a very special shape of the anode.

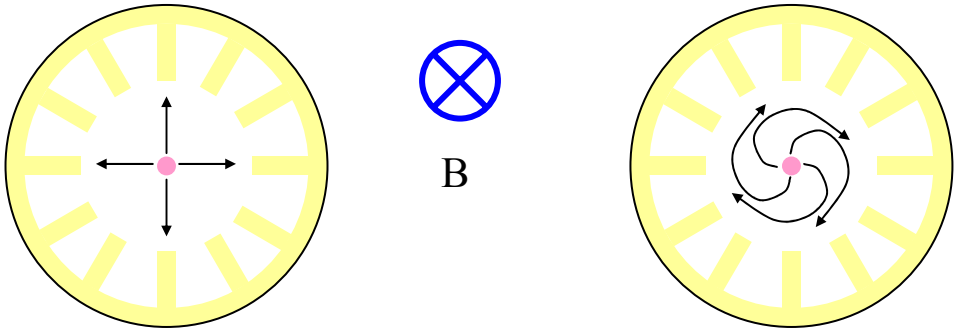


Fig. 7-26. A magnetron. Electron motion with and without the magnetic field.

Electrons are accelerated from the centre cathode towards the surrounding anode. A magnetic field is placed perpendicular to the electric field and the electrons start to move in circular orbits. To avoid hitting the anode the magnetic field and the electric field has to be balanced to each other.
The anode is formed to create a number of cavities into which the electrons build up an electromagnetic field. This field oscillates in the cavities and acts back on the electrons revolving in the magnetron. The field induces forces along the path of the electrons and a bunching of the revolving electrons occurs at the frequency of the cavity fields. As the electron beam now is bunched it induces even higher fields in the cavities, and these fields can be extracted by an antenna just as in the klystron output cavity.

7.6.5 Modulators

To drive a klystron, or an electron gun, one needs a large voltage with a large current which means very high energies. A typical klystron can deliver 30-40 MW. It is easy to understand that this power can not be supplied DC from the wall plug. The operation has to be pulsed. This pulsing is made in a modulator. A long "low voltage" pulse loads a capacitor bank (Pulse former net, PFN). The stored energy is quickly released during a short time by triggering a thyratron switch. The pulse passes a transformer in which the voltage is increased. This short pulse drives the klystron. The heating of the klystron gun can be connected in series in the klystron.

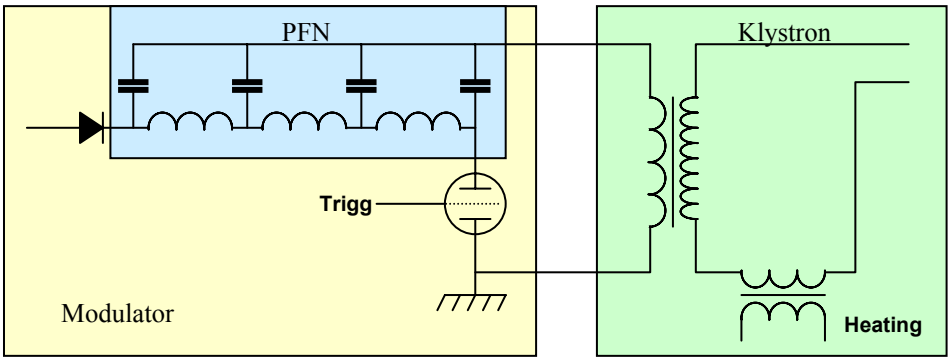


Fig. 7-27. A modulator with pulse former net and a pulse transformer.

7.7 APPENDIX: Waveguides

We have so far at many points assumed the existence of devices in which electromagnetic waves exist, oscillates or propagates. Mostly cavities have been mentioned, but before talking about them lets look at a device transporting electromagnetic waves at high frequencies: waveguides. In principle the waveguides are round or rectangular tubes in which the electromagnetic waves travel. Unlike in free space the walls will influence what happens in the waveguide.

The equation describing the propagation of electromagnetic waves is

$$\Delta\overline{E} - \frac{1}{c^2}\ddot{\overline{E}} = 0$$

Eq. 7-1

Let us assume that we can write the field as:

$$\overline{E}(\overline{r},t) = \overline{E}(\overline{r})e^{i\omega t}$$

Eq. 7-2

The equation of propagation can now be written:

$$\Delta\overline{E} + k^2\overline{E} = 0 \quad k = \frac{2\pi}{\lambda} = \frac{\omega}{c}$$

Eq. 7-3

If we now choose to look along the waveguide, along the tube, and let us call it the z-direction:

$$\frac{\delta^2}{\delta x^2}E_z + \frac{\delta^2}{\delta y^2}E_z + \frac{\delta^2}{\delta z^2}E_z + k^2 E_z = 0$$

Eq. 7-4

We can now assume a solution the field in the z-direction being in the form:

$$E_z = f_x(x)f_y(y)f_z(z)$$

Eq. 7-5

Using Eq. 7-5 in Eq. 7-4 generates:

$$f_x''f_yf_z + f_xf_y''f_z + f_xf_yf_z'' + k^2 f_xf_yf_z = 0$$

Eq. 7-6

By making the following definitions:

$$k_x^2 = -\frac{f_x''}{f_x} \quad etc. \quad \Rightarrow \quad k^2 = k_x^2 + k_y^2 + k_z^2 \quad \Rightarrow \quad k_z^2 = k^2 - \left(k_x^2 + k_y^2\right) = k^2 - k_c^2$$

Eq. 7-7

In Eq. 7-7 kc describes the modes of the wave in the transverse directions of the waveguide. This is really the difference from wave propagation in free space, where kc=0.
The final part of Eq. 7-7 can be rewritten as:

$$k_x = \frac{2\pi}{\lambda_x} \quad etc. \quad \Rightarrow \quad \frac{1}{\lambda_z^2} = \frac{1}{\lambda^2} - \frac{1}{\lambda_c^2} \quad \Rightarrow \quad \lambda_z = \frac{\lambda}{\sqrt{1 - \left(\frac{\lambda}{\lambda_c}\right)^2}}$$

Eq. 7-8

Already here we can see that for certain relations between λ and λ_c , λ_z will be complex. Let's see if we can understand what happens better.

By using the definition of kz in Eq. 7-7 we can write

$$f_z'' + k_z^2 f_z = 0$$
$$\Downarrow$$
$$f_z'' f_x f_y + k_z^2 f_z f_x f_y = 0$$
$$\Downarrow$$
$$\frac{\delta^2}{dz^2} E_z + k_z^2 E_z = 0$$
$$\Downarrow$$
$$E_z = E_{0z}(t) e^{ik_z z} = E_{0z} e^{ik_z z} e^{i\omega z}$$

Eq. 7-9

From the last line in Eq. 7-9 we can see that if k_z is complex, the wave will be damped along z, but if k_z i real we will have propagation of the wave.
From Eq. 7-8 we get that

- k_z is complex if $\lambda > \lambda_c$ which gives damping
- k_z is real if $\lambda < \lambda_c$ which gives propagation.

λ_c, which is given by some means from the transverse geometry of the waveguide, is called the cutoff wavelength describing which waves that can propagate in the waveguide. Too large wavelengths will induce damping and nothing will come through.

Another thing is that from Eq. 7-8 we get that lz, the wavelength along the waveguide, is always larget than l, the wavelength in free space.

We can also write the phase velocity of the wave inside the waveguide.

$$v_\varphi = \frac{\omega \lambda_z}{2\pi} = \frac{1}{\sqrt{1 - \left(\frac{\lambda}{\lambda_c}\right)^2}} \frac{\lambda \omega}{2\pi} = \frac{1}{\sqrt{1 - \left(\frac{\lambda}{\lambda_c}\right)^2}} c > c$$

Eq. 7-10

The phase velocity inside the waveguide for a propagating wave is larger than c.

7.7.1 A rectangular waveguide

Lets look at a rectasngular waveguide as an example. From Eq. 7-7 we can write

$$\begin{cases} f_x'' + k_x^2 f_x = 0 \\ f_y'' + k_y^2 f_y = 0 \end{cases}$$

Eq. 7-11

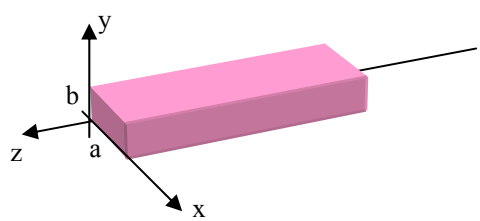


Fig. 7-28. A rectangular waveguide with the width a and height b.

Assume the solutions

$$\begin{cases} f_x(x) = A\sin(k_x x) + B\cos(k_x x) \\ f_y(y) = C\sin(k_y y) + D\cos(k_y y) \end{cases}$$

Eq. 7-12

As the electric fields perpendicular to the surface has to be zero at the surface gives B=D=0.

If the width of the waveguide is a and the height b, one gets:

$$\begin{cases} k_x a = m\pi \\ k_y b = n\pi \end{cases} \qquad k_c^2 = \left(\frac{2\pi}{\lambda_c}\right)^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

Eq. 7-13

We now have an expression for the cutoff wavelength for the waveguide and can judge wethere an available tube can carry the wave without damping or not.

One of the most simple solutions then takes the parameters m=1 and n=0. Painting this across the a cross section of the wave guide would look like:

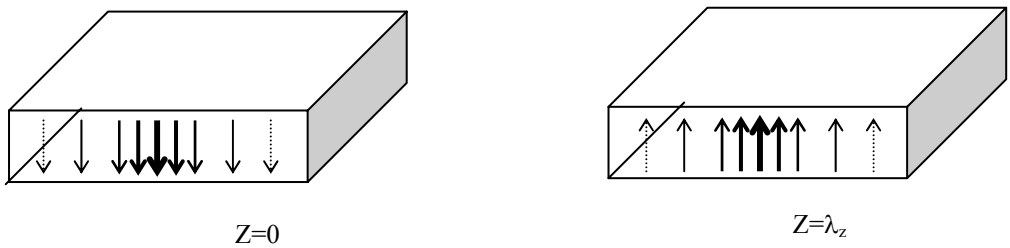


Fig. 7-29. The electric fields of a TE10 mode at two positions along the waveguide.

This solutions is called a TE10-mode, which means “Transverse Electric 1-0”.
What is not painted in the picture is that the electric field will give rise to a magnetic field.

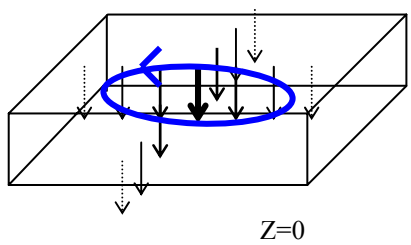


Fig. 7-30. The magnetic (blue) and electric fields of a TE10 mode in a waveguide.

The total expression for the electric field is given by

$$\begin{cases} E_x = 0 \\ E_y = E_0 \sin\left(\frac{\pi x}{a}\right) e^{ik_z z} e^{i\omega t} \\ E_z = 0 \end{cases}$$

Eq. 7-14

The magnetic flux density is also of interest and it can be achieved by using the Maxwels equation

$$\nabla \times \overline{E} + \mu \frac{\delta \overline{H}}{\delta t} = 0$$

Eq. 7-15

This gives the magnetic component as

$$\begin{cases} H_x = \\ H_y = 0 \\ H_z = -i \dfrac{E\pi}{\mu a \omega} \cos\left(\dfrac{\pi x}{a}\right) e^{-ik_z z} e^{-i\omega t} \end{cases}$$

Eq. 7-16

7.8 Cavities

Cavities ar in principle waveguides which are not open for the propagation of waves but only allows oscillating waves. The geometry makes it possible for different waves to be resonant and thus oscillate in the cavity. We will here actually find the resonant wavelengths, but rather give the results.

In a resonance cavity the length, *l*, of the cavity has to fulfill the criteria $l=q\lambda_z/2$. Incerting this into the expression for the limit wavelength one gets:

$$\frac{1}{\lambda_r^2} = \frac{1}{\lambda_c^2} + \left(\frac{q}{2l}\right)^2$$

Eq. 7-17

where λ_r is the resonance wavelength and q=1, 2, 3, ...

In a cylindrical cavity it can first be shown that one of the most common resonant wavelength for the structure is

$$\lambda_r = \frac{\pi D}{x_1} \qquad x_1 = 2.40483$$

Eq. 7-18

where x_1 is given by the first zero crossing of a besselfunction assuming a TM01 (transverse magnetic) wave and D is the diameter.

The technique of connecting the waveguides to the cavities, charging and picking up power from cavities is a complete subject of its own with ceramic windows, antennas, and different half and quarter wavelength pieces, crossings etc. which is peresented in text books on microwave technology.

8 Diagnostics

The subject of diagnostics deals with the large variety of devices and methods we have to measure different features of the particle beams in the accelerators such as the number of particles, their positon, the size and shape of the beam, the energy and its oscillations etc.

8.1 The beam transformer

The particle beam is a beam of charged particles and thus a "current" passing down the vacuum tube. Such a current will generate a magnetic field. The magnetic flux induces a current in a coil placed around the particle beam which is proportional to the charge (particles) passing by(Fig. 8-1). One obstacle is that the metal vacuum tube has to be interrupted and a ceramic part put instead.

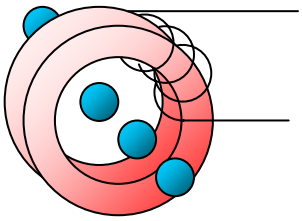


Fig. 8-1. A beam transformer

In practise the coil is connected like in Fig. 8-2.

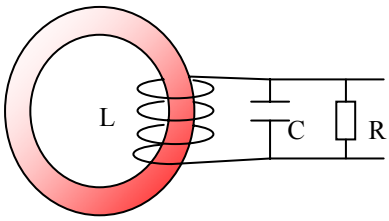


Fig. 8-2. The current coil connected.
L is the coil inductance, C is a stray capacitance and R the termination.

The response of such a circuit has a rise time $T_r = \sqrt{LC}$ and a decay time $T_d = L/R$. The iron in the coil will also damp high frequencies and thus it is difficult to operate above 100 MHz. Further it is not without problems to operate the device while there are several bunches coming through the device. Thus a number of schemes on how to connect the coils with feedback and integration exist, such as the DCCT (Direct Current Current Transformer)

8.2 The Faraday cup

The method given above is convenient as it does not destroy the particle beam. An easier way, though it destroys the beam, is to simply put a plate in the beam and lead away the charge. Unfortunately there will be a number of secondary electrons leaving such a plate and thus give an error to the result. To overcome this problem the collector plate is housed in a box and a negative potential pushes the secondary electrons back to the collector plate. This is called a Faraday cup (Fig. 8-3). It will thus collect the total charge in the beam.

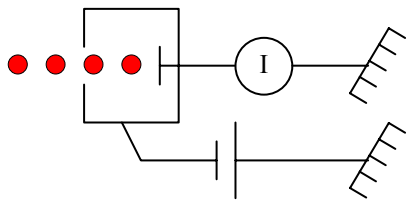


Fig. 8-3. The Faraday cup.

8.3 Position measurement by pick-ups

A pick-up is something placed close to the beam that produces a signal when a bunch of charged particles pass. In contrast to the beam transformer above it is placed in one position and not around the beam.

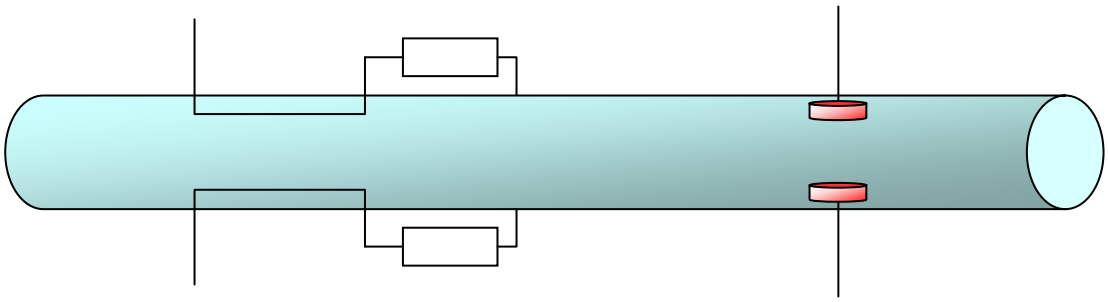


Fig. 8-4. A stripline pick-up and a button pick-up.

The signal in a stripline (Fig. 8-4) is built up when the electron bunch passes the entrance and the exit of the strip. The magnetic field around the moving charge induces a pulse in the part of the stripline which is crossed by the field. The pulse travels in both directions, one to the

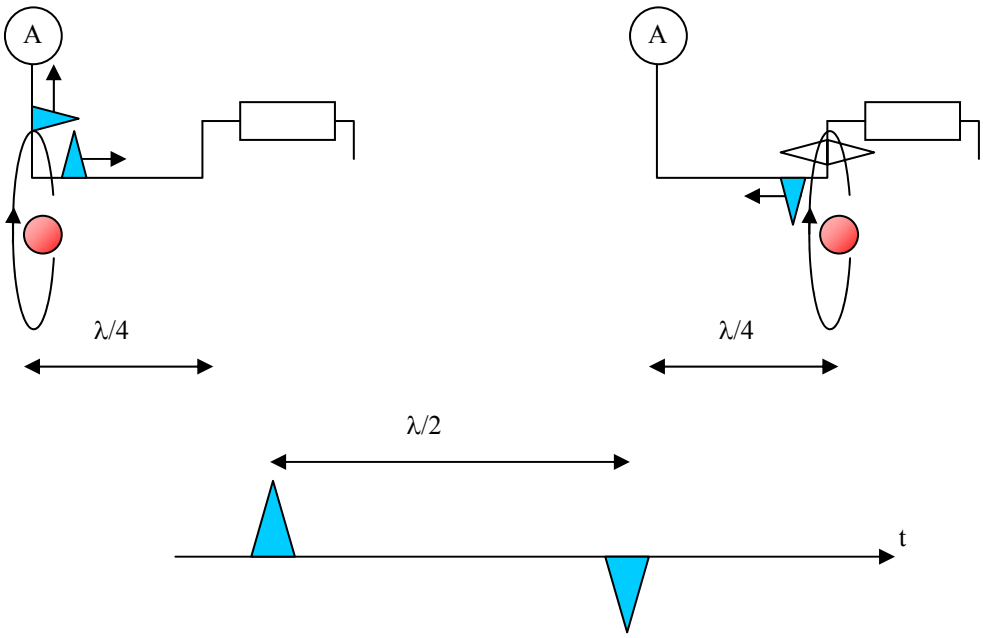


Fig. 8-5. The signal build-up in a stripline. (λ is the distance between two bunches in the accelerator)

detector and one hitting the resistance is reflected back. When the pulse reaches the resistance the charged particle has reached the end of the strip and induces a new pulse. The pulses at the end add up, and the net result is a pulse train with period λ arriving to the detector.

The pickup is a much simpler device as it gives one signal at the passage of the particle bunch. Both these devices give signals which are proportional to the distance of the particles to the detector. By placing four pickups around the circumference of the vacuum tube a total image of the position of the beam can be achieved by taking the sum and difference signal from the different pickups (Fig. 8-6).

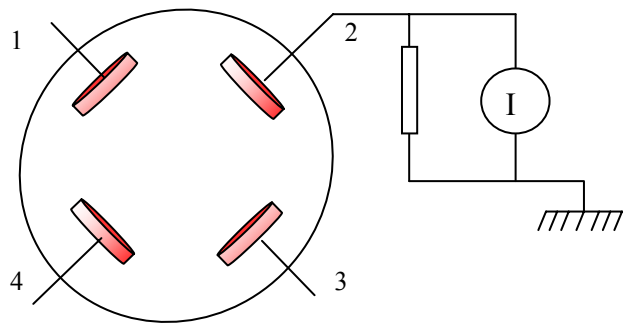


Fig. 8-6. Placement of four button pick-ups to measure the beam position.

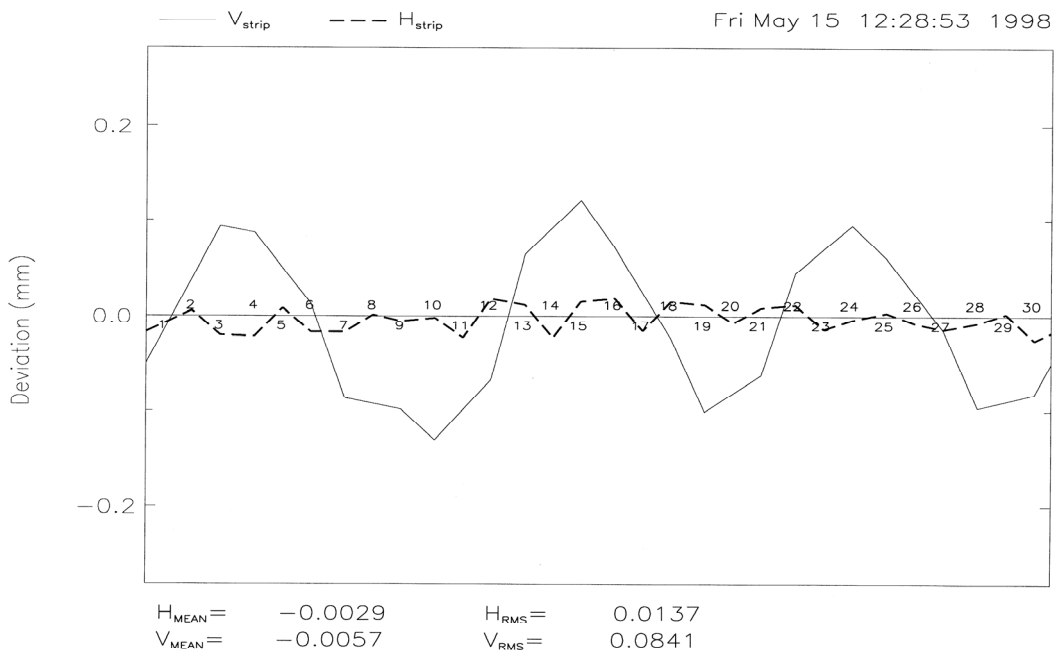


Fig. 8-7. A total measurement of the beamposition in MAX II with a vertical kick artificially introduced.

In an accelerator such sets of four pickups are placed at several locations. By reading all their signals (Eq. 8-1) a total picture of the particle beam position around the machine is obtained (Fig. 8-7).

$$\Delta x \propto \frac{(I_2 + I_3) - (I_1 + I_4)}{\sum I}$$

Eq. 8-1

8.4 Synchrotron radiation pick-ups

The synchrotron light emitted from electrons or positrons in a bending magnet or undulator in the accelerator can be detected. As this radiation is emitted in the perfect tangential of the beam some information about the position can be found.

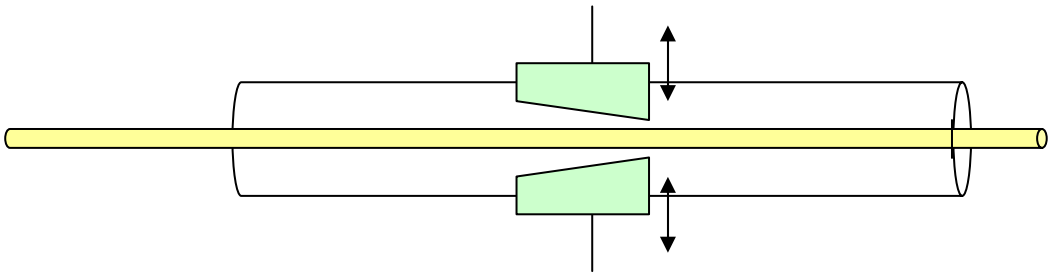


Fig. 8-8. Photoelectric pick-up.

Photoelectric pick-ups (Fig. 8-8) can be placed down the line in a beamline. They are either fixed or moved closer to the light beam for the measurement. By the photoelectric effect a current can be measured proportional to the amount of light hitting the pick-up. By taking the difference signal between two opposing pick-ups the position of the light beam can be resolved. This partly destroys the photon beam.

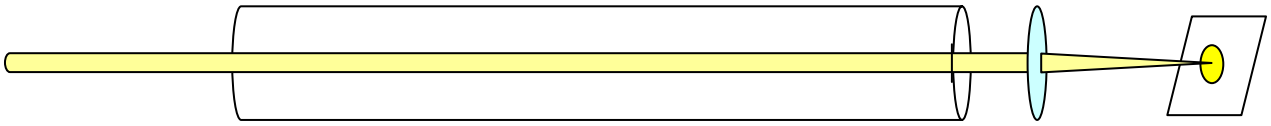


Fig. 8-9 The semiconductor pickup.

The lense (Fig. 8-9) makes a true image of the source point of the radiation on the semiconductor pick-up, often a CCD array or a camera. The source is of course the particle beam and will thus give information about both the position and the size and shape of the particle beam. With knowledge of the accelerator beam optics, a measure of the emittance of the beam can also be obtained.

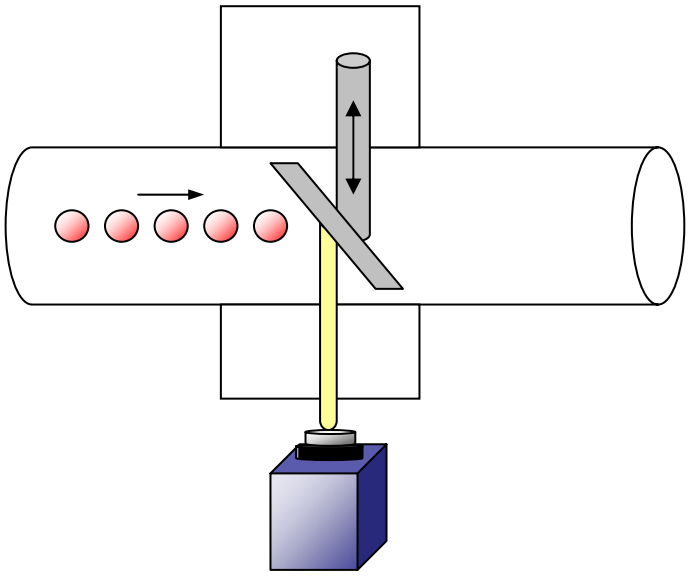


Fig. 8-10. The flourescent screen.

8.5 Brehmsstralung

Another method to find the particle beam in a straight section of an accelerator is to detect the brehmsstalung emitted by the particles hitting rest gas in the vacuum tube. This radiation, consisting of high energy gamma, is well colimated in the straight forward direction. It will give an estimate of the average position of the beam in the straight section, but no detailed information.

8.6 Scintillator screens

A scintillator screen (Fig. 8-10) is a plate covered with some material that shine visible light when hit by the particle beam. A common material is a glass plate with ZnS on it. The screen is moved into the particle beam, which is destroyed. The spot where the beam hits the screen emits light which can easily be detected by the eye or a TV camera. Other materials can also be used such as YAG:Ce crystals (yttrium-aluminium-garnet doped with cesium) to improve the resolution of the screen.

8.7 Scrapers and wires

An even more direct method of measuring the position and the size of the particle beam is to "scrape off" parts of the beam. Four blades are moved in towards the beam and the intensity in the beam is measured. When a blade comes to the edges of the beam particles will rapidly be lost from the beam and a decrease in current can be measured. This can then be repeated for all four directions.

A related method is to use a thin wire which is rapidly moved through the beam.

8.8 Mirror current detector

In the vacuum chamber surrounding the charged particle beam a mirror current flows with the same magnitude but the opposite direction. By measuring this current the particle beam current can be achieved.

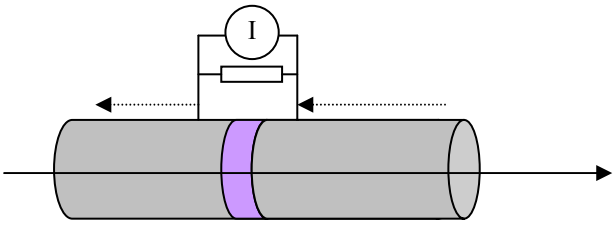


Fig. 8-11. A mirror current detector

8.9 Q-measurement

The betatron tunes of a machine are a very important parameter to measure. A method to do this is by exciting the beam at a given frequency and studying the responding oscillation amplitude. This is done by a Spectrum Analyser which in principle is a scanning radio receiver.

The frequencies measured are composed of the revolution frequency of the storage ring, F_{rev}, and the frequency of the RF system, F_{RF}. The whole ring will have place for a maximum number of bunches, n, and thus $F_{RF} = n\ F_{rev}$. In practice the F_{RF} will be the strongest signal measured.

An individual particle bunch goes around the ring with the F_{rev} frequency, but it also oscillates with the betatron frequencies which has the values

$$F_{Q,vert} = (M \pm Q)_{vert} F_{rev}$$

Eq. 8-2

where M is an integer and Q the fractional part (M,Q) of the betatron tune.

A similar expression can be written for the horizontal direction. These oscillation frequencies of the bunch are added to the main frequency and the detected frequency becomes:

$$F_m = F_{RF} \pm F_{Q,vert} \pm F_{Q,hor}$$

Eq. 8-3

where F_m is the measured frequency. By only using one pickup the integer part of the betatron oscillation can not be detected and thus the result becomes

$$Q = \left| \frac{F_m - F_{RF}}{F_{rev}} \right| = \left| \frac{F_m - F_{RF}}{\frac{F_{RF}}{n}} \right|$$

Eq. 8-4

The signal will look like Fig. 8-12. If one would look closer there are also other oscillations in the beam, such as synchrotorn oscillations which can be detected.

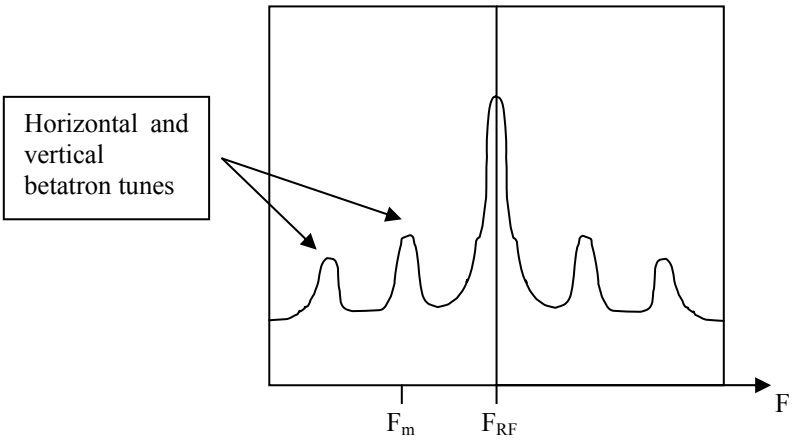
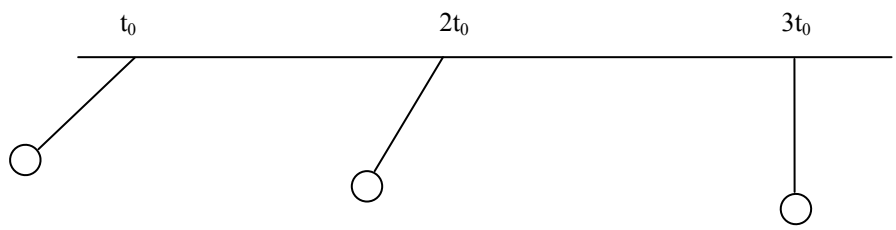
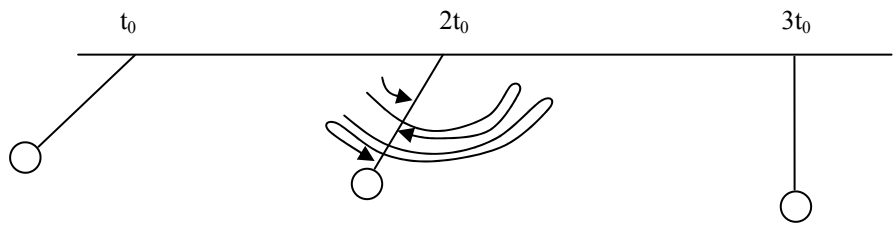


Fig. 8-12. The response measured by a spectrum analyser

How does this work?
Assume you have a pendulum which you only see the position of every now and then.



During the time from t0 to 2t0 you can not know exactly how many oscillations it has made.



The angle traversed by the pendulum will be: α or $2\pi-\alpha$ or $2\pi+\alpha$ or $2*2\pi-\alpha$ etc.
The oscillation can be written as $A*\sin(\omega t)$ and thus: $\omega t = \alpha$ or $2\pi-\alpha$ or $2\pi+\alpha$ or $2*2\pi-\alpha$ etc.
The electronbunches makes one turn in the ring with the revolution frequency $\omega_{REV} = 2\pi/t_0$

Rewriting gives

$$\omega = \frac{\alpha}{t_0} = \Delta\omega$$

$$\omega = \frac{2\pi}{t_0} - \frac{\alpha}{t_0} = \omega_{REV} - \frac{\alpha}{t_0} = \omega_{REV} - \Delta\omega$$

$$\omega = \frac{2\pi}{t_0} + \frac{\alpha}{t_0} = \omega_{REV} + \frac{\alpha}{t_0} = \omega_{REV} + \Delta\omega$$

$$\omega = 2\frac{2\pi}{t_0} - \frac{\alpha}{t_0} = 2\omega_{REV} - \frac{\alpha}{t_0} = 2\omega_{REV} - \Delta\omega$$

$$\omega = \ldots$$

By measuring the frequency $\Delta\omega$ and knowing the time for one revolution t_0 we can find the angle α. The tune is defined as the number of oscillations in one turn and $\alpha/2\pi$ gives us the fraction of oscillations.

$$Q = \frac{\alpha}{2\pi} = \frac{\Delta\omega\, t_0}{2\pi} = \Delta\omega\,\frac{t_0}{2\pi} = \frac{\Delta\omega}{\omega_{REV}} = \frac{\Delta f}{f_{REV}}$$

8.10 Feedback

Feedback in an accelerator is not directly a diagnostics tool but rather a system that measures some aspect of the beam and then tries to correct for it.
One basic system would be to cure transverse oscillations in the particle beam (Fig. 8-13). A pick-up monitor will sense the position of the bunch and an excitation gap will damp the oscillation.

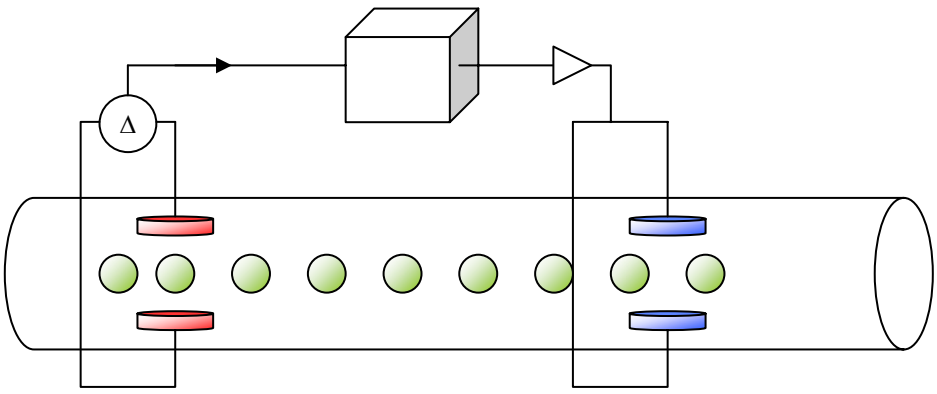


Fig. 8-13. A transverse feedback system.

Another system would be a system working in the longitudinal direction. The particle bunches also make synchrotron oscillations longitudinally. These oscillations also show as energy oscillations and by measuring the position in a dispersive region the amount of energy difference can be detected. This difference can be fed to the phase of an accelerating cavity and thus adjust the energy oscillation of the particle bunch.

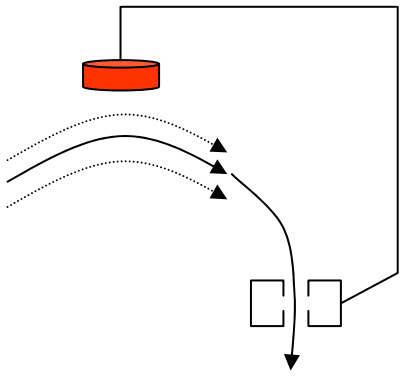


Fig. 8-14. Longitudinal (energy) feedback. Sensing the energy deviation in a point with dispersion and feeding the signal to a cavity which adjusts the energy.

8.11 Alignment

The accelerator itself, as well as some of the measuring devices above, has to be positioned very precisely. Normally each component has to be placed within a fraction of a mm relative to each other. To achieve this a number of techniques are used.

8.11.1 Invar wire

An *invar wire* is a wire which has almost no expansion or contraction due to change in temperature. It is fabricated in a pre defined length of a special steel alloy. By stretching the wire by a certain force a very precise length measurement can be obtained. A precision of 0.01 mm is possible.

8.11.2 Theodolite

To measure angles and also heights a theodolite (Fig. 8-15) can be used. It is in principle a very precise looking-glass with scales for both rotation angle and tilt angle. The precision in angle obtainable is around 0.05 mRad.

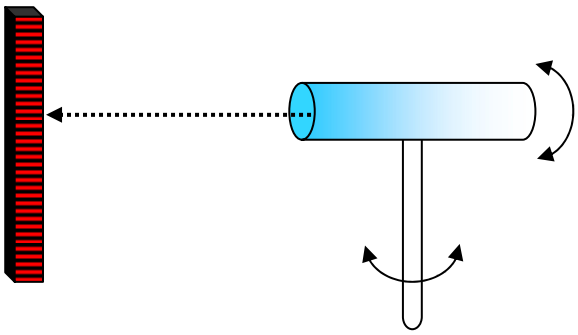


Fig. 8-15. The theodolite.

8.11.3 Water levelling systems

To adjust different objects in level to each other a water levelling system (Fig. 8-16) can be used. It uses the same principle as when you use your garden hose to level a new fence: the water level at the two ends is always the same. In this method a refined way of measuring the level of the water is used. A laser beam enters the water surface at an angle and is reflected up from an object below the water surface to hit a detector. If the water level changes, the position of the light spot on the detector changes. By this method a precision of 0.01 mm can be achieved.

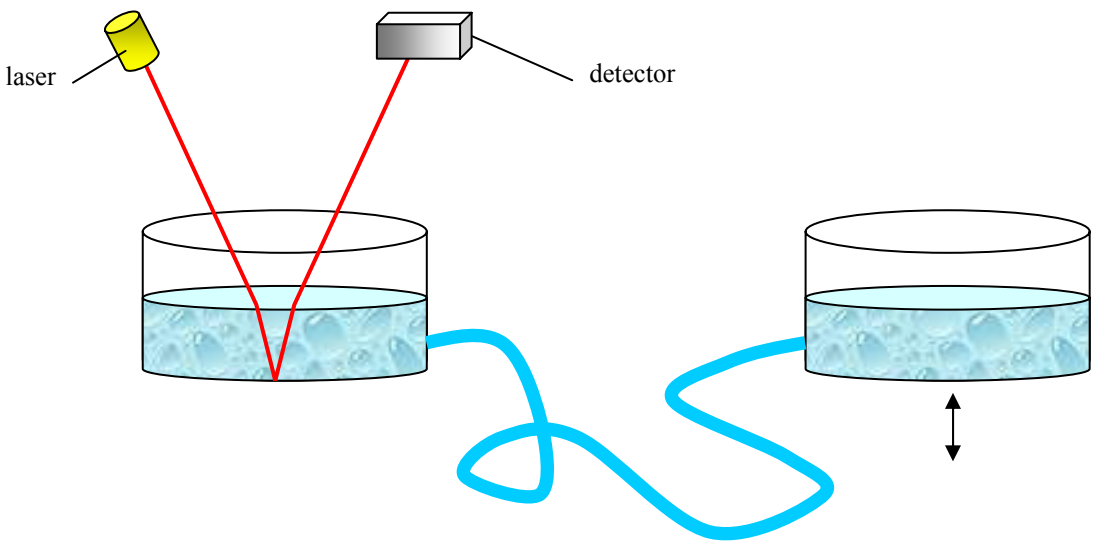


Fig. 8-16. The water levelling system

9 Injection and extraction

Injecting a beam of particles into an accelerator might sound simple but it is not entirely the case. If we think of a round accelerator into which we direct a beam we need a magnet which just steers it into the correct path. After one turn in the accelerator the beam will be directed out from the accelerator by the same magnet.

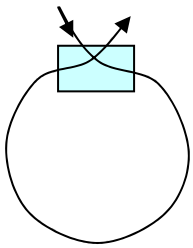


Fig. 9-1. Non successful injection

We can then in principle make a magnet which is only "on" for one turn in the accelerator and then switched off. In an accelerator as MAX I which is 32 m in circumference one turn takes 100 ns, and the rise time of the magnet should be less than 10% of this time to be able to be useful which means 10 ns. Even if it in principle could be made there are easier methods.
Another obstacle is that one often wants to inject particles in several steps: accumulate them in the ring. Thus the beam injected earlier must remain in the machine when the new arrives.
We can use special magnets to steer the beam to be injected, magnets which move around the already injected beam and damping mechanisms in the beams.

9.1 Damping

Despite having previously stated that damping effects were beyond the scope of this course we here have to make a short dip into the subject.

An electron loosing energy by radiating a photon, emitting light (synchrotron radiation), recives an impulse in the opposite direction of the photon. This direction is distributed around the backward direction but within a cone with angle 1/g (see the chapter on synchrotron radiation). An electron can not loose too much energy because it will then be lost out from the accelerator system due to too low energy. Thus the energy lost is replaced while passing an RF-cavity. The electrons will get an impulse in the forward direction by the electric fields in the cavity, but this impulse is always in the straight forward direction and proportional to the energy loss experienced by the electron.

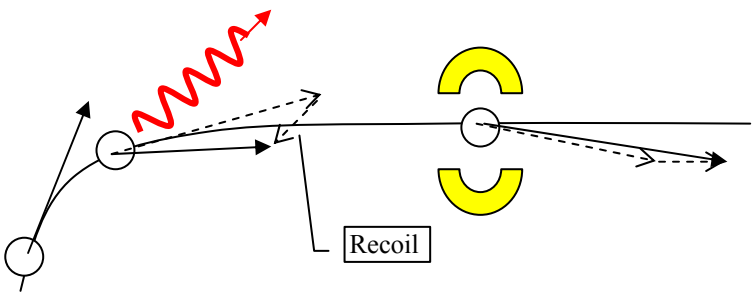


Fig. 9-2. Damping mechanism. The electron moves along the path - It emits a photon and recoils - Energy is restored and the angle "damped".

An electron that initially travels with a small inclination will also loose energy by light emission. This loss will be on average in the straight backward direction of the electron, but always replaced

in the "correct" forward direction. Thus the "wrong inclined impulse" will after a while be replaced by a "correct straightforward impulse". This effect will damp the transverse size of the electron beam as all "inclined impulses" will sooner or later be replaced by "straight impulses".

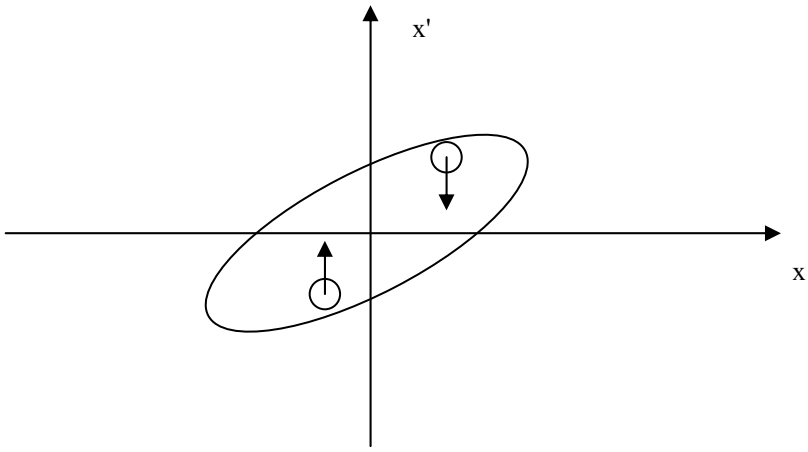


Fig. 9-3. How a particle move in phase space due to damping.

An electron loosing energy by light emission will on average not change its direction. As the size of the ellipse decreases the emittance decreases and thus also the size of the beam as it scales with the emittance.

There are of course mechanisms that will blow up the beam and it will after a while find an equilibrium where it does neither grow nor decrease more. If one at such a time puts in an electron which is a bit outside all the other electrons it will start to damp into the ensemble of electrons already in the machine.

This process can be used for the injection process of new particles!
The damping described here is only relevant for light particles which emit significant amounts of synchrotron radiation such as electrons or positrons. Thus injecting other heavier particles has do be done using techniques not dependent of damping.

9.2 General considerations

While injecting a beam into a machine we do not only have to match the position of the new beam to the machine and the old beam, but we do also have to match phase space.

The newly injected beam will be damped in a storage ring down to the equilibrium emittance, but it can not have an emittance that is significantly larger than that while coming from the injection channel. Thus the particle source, the preaccelerator and the transportlines have to match the emittance of the storage ring. We have of course the same problem at all interfaces between different parts of the accelerators.

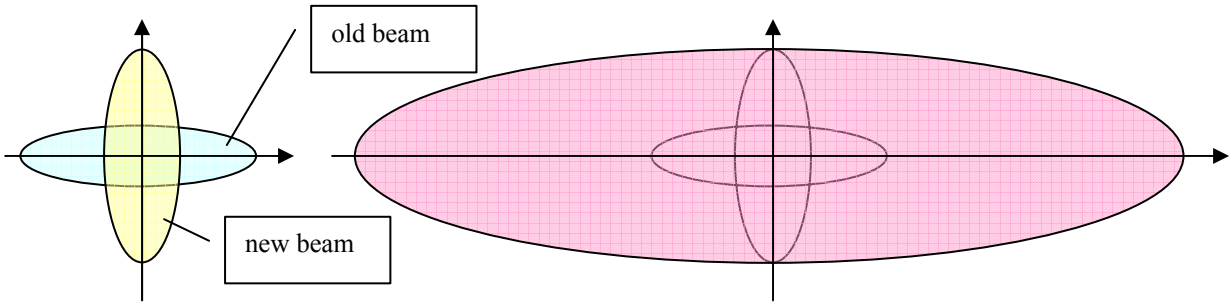


Fig. 9-4. Two beams are injected with similar emittance but different orientation of the phase space. The resulting emittance follows the phase ellipse of the old beam, but the size has to cover all particles!

It is not enough to match the emittance at the injection. We also have to have a reasonable match between the phase ellipses (Fig. 9-4). Again, damping can help us out, but if damping is weak or we have large mismatches it does not work efficiently.

9.3 Injection

9.3.1 Simple injection

In an electron storage ring the electron beam follows the closed orbit in the centre of the machine. The phase area that this beam fills is damped and in principle the machine can normally also take particles which are outside this phase area. A beam can thus be injected close to the closed orbit, and after a while it will be damped into that orbit.

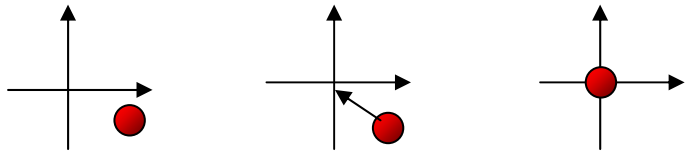


Fig. 9-5. A beam is injected and damped into the closed orbit.

In practise this is a difficult method as the damping normally is too weak to handle the angles needed in a real machine.

9.3.2 Electron injection

To make the process more efficient the basic technique is improved. First one uses a septum magnet (one type of septum magnet is shown in Fig. 9-6) which can direct the new beam closer and with a smaller angle to the closed orbit.

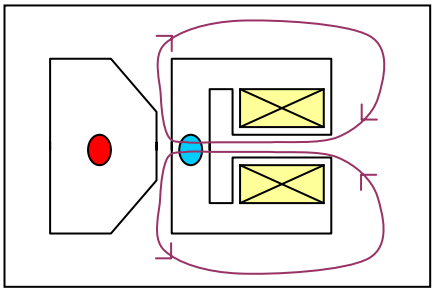


Fig. 9-6. A septum magnet. The already stored beam to the left on the closed orbit. The new beam is directed from below (or above) to a parallel path close to the closed orbit.

The injected beam comes in a narrow channel and feels a horizontal magnetic field (Fig. 9-6). Due to the shape of the septum the magnetic field does not penetrate into the chamber for the stored beam and thus only the new beam is bent. The wall between the two chambers is very thin.

To make the injection easier the already stored beam can be moved close to the wall between the two chambers. Thus the total phase space made up by the two beams in common will be smaller and the damping more efficient.

Fig. 9-7. Excitation of kicker magnets and rapid switch off before one turn is made.

After one turn the beam will come back to the septum magnet, and it must then pass in the injection channel. To achieve this one can either very quickly move the stored beam (Fig. 9-7) or one can use the effect that by choosing a tune of the machine as a high multiple one has a number of turns to play with (Fig. 9-8).
If the tune of the horizontal betatron oscillation is Q,X it will take 1/x turns until the beam is back in the same position. (it will arrive at the position Q times every turn, but that is not at the septum place). During this time damping will occur and there is plenty of time to slowly move the previously stored beam back to its original position.

When the beam including the new beam is back at its original position we can wait until it has damped to a small size and then repeat the injection process.

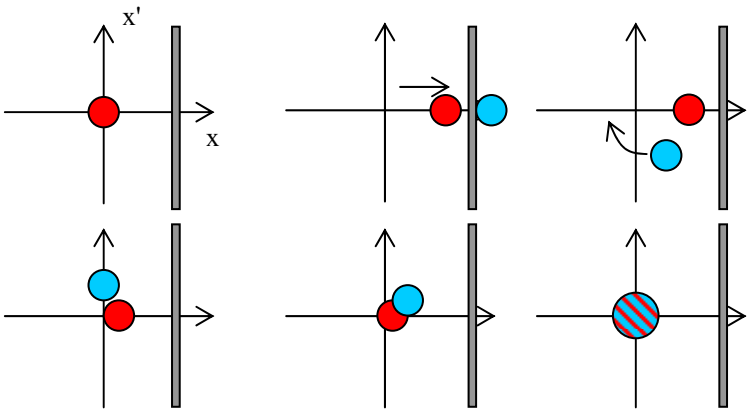


Fig. 9-8. Multi turn injection. The stored beam is moved close to the septum. The new beam is injected and rotates in phase space and damps down in size.

The movement of the previously stored beam is made by bumper and kicker magnets. The bumper magnets are "slow" magnets moving the beam closer to the septum wall and the newly injected channel. The movement is not enough but a large part. The last movement can be made by a quick magnet, the kicker, which is not capable of making large movements. This magnet can not use ordinary iron and coils but has to rely on faster systems (as an example Fig. 9-9).

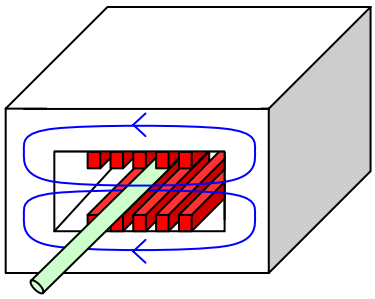


Fig. 9-9. A kicker magnet with a ferrite frame.

9.3.3 Heavy particle injection

A heavy particle will not experience damping as it does not emit any considerable amount of synchrotron radiation. Thus the beam will not shrink over time until an equilibrium is achieved. An injected beam of heavy particles will remain in the phase space position and occupy an similar phase space as the injector accelerator uses.

One can fill the desired phase space both in time and "size" by injecting a number of beams close to the original path, thus building up the total beam (Fig. 9-9).

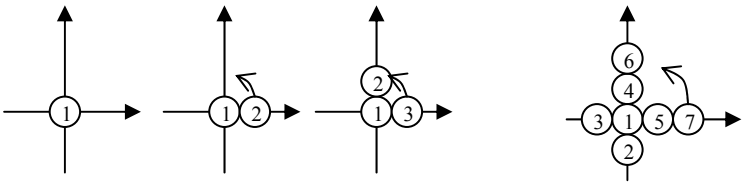


Fig. 9-10. Filling the phase space with heavy particles.

Another method which can be used when injecting for example protons is to inject a beam of negative hydrogen ions, H-. By a foil in the accelerator the electrons are stripped from the negative ion and a positive hydrogen ion, a proton is achieved. When the beam in the next turn approaches the septum it has the opposite charge and will be bent in the opposite direction removing the need for a fast kicker.

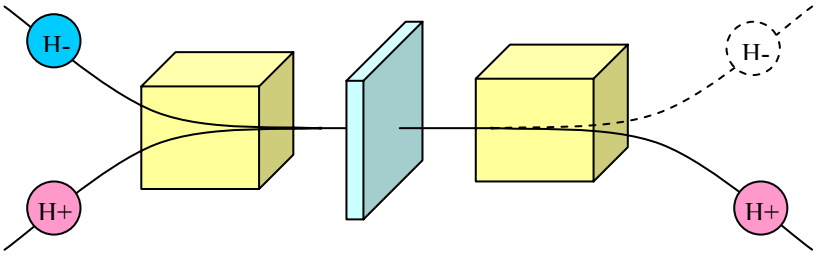


Fig. 9-11. Injection of protons by use of a stripping foil.

9.4 Extraction

The basic ideas for the extraction are the same ones used for injection. If one has negative ions in the machine, the method by a stripping foil can be achieved and no fast kickers are needed. A second method kicks the beam rapidly over the septum magnet into the extraction channel. Very short rise times and, if only parts of the beam are to be ejected, very short pulses are needed. This is tricky for high energy machines, but is the main technique used for extraction from one ring-accelerator to another. A third method, not similar to anything above, is called resonance extraction.

9.4.1 Resonance extraction

By changing the optics of the accelerator to a point close to a betatron resonance, the particles will start to move out from the centre. A common choice is a third order resonance, but also other order can be used. It can be shown that the beam phase space takes the form of a triangle (Fig. 9-12) with resonant particles moving out along the legs. '

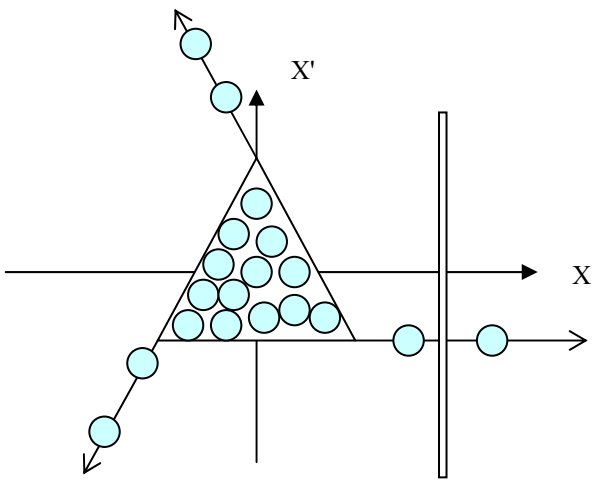


Fig. 9-12. Third order resonance extraction.

As the particles also rotate around the phase space one can achieve that very few particles actually hits the septum magnet and the efficiency can be very high (<1 % losses). The extraction time can also be made very long if one only moves closer to the resonance very slowly.

10 Colliders and other applications

So far this text has taken examples almost only from the world of synchrotron radiation accelerators, but in technical details been fairly general. Below I will briefly travel through a number of other applications for particle accelerators. Many of them have been around for a longer time than the large synchrotron accelrators today.

10.1 Science

Under the very general headline "science" the machines which are in the same category as the synchrotron radiation machines fit. These machines are used for applications in physics, chemistry and biology.

10.1.1 Colliders

The family of colliders is the very essence of "noble price machines". Many of the modern prices in physics have been won as results of activities at large colliding particle beam accelerators. The topic here is to analyse the "fundamental" particles around us: quarks, gluons … you name them and to learn enough to be able to build and test theories on their birth, life and death. To be able to produce any of these particles extreme energies are necessary. At the same time very high fluxes are needed as the probability to create a particle might be very small. The accelerator takes the role of producing intense high energy beams and to collide them with either a fixed target or another particle beam. After this a work which is as big comes about and that is to produce a detector which can detect the particle in question and to analyse its qualities.

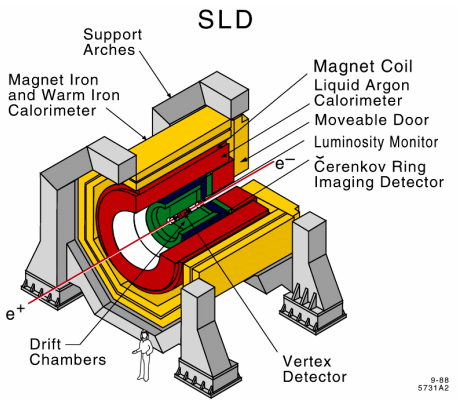


Fig. 10-1. The SLD detector at SLAC, Stanford built to perform studies of polarized Z particles produced in collisions between electrons and positrons (http://www-sld.slac.stanford.edu/sldwww/sld.gif)

In principle there is no difference between an accelerator working for high energy physics and other accelerators. In the easiest set up a beam of particles, might be electrons, is directed onto a fixed target (like a piece of metal). The beam interacts with the target and a number of new particles are created and spread out in a fan.

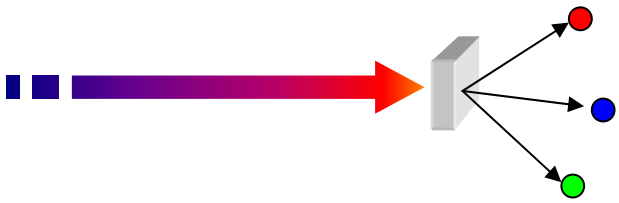
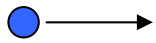


Fig. 10-2. Colliding with a fixed target

10.1.1.1 Why collide beams?

The principle of using a fixed target is an easy solution but not the best if one wants to go to high energies. Assume that the collision is between two identical particles. The available collision energy is:
$W_{col} = 2\ W_{cm} - 2\ W_o$
Where cm is the centre of mass.

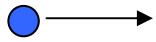
In the lab-system the collision is given by:



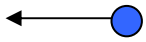
$W_{lab} = W_{kin} + W_o$
$P_{lab} = mv$

$W_{lab} = 0 + W_o$
$P_{lab} = 0$

And in the center of mass system by:



W_{cm}
P_{cm}



W_{cm}
$-P_{cm}$

Let us now see how the centre of mass energy (giving the available collision energy) depends on the energy in the lab system (the energy of the accelerator).

The rest energy is given by: $Wo^2 = W^2 + (cP)^2$

In the lab system the total energy is given by: W = Wlab + Wo
And the momentum by: P= Plab + 0

In the center of mass system the total energy is given by: W=2 Wcm
And the momentum by: P=Pcm-Pcm=0

Expressing the rest energy squared in the two systems gives:

$$(W_{lab} + W_0)^2 - (cP_{lab})^2 = (2W_{cm})^2$$

Eq. 10-1

The energy of the moving particle in the lab system is given by:

$$W_{lab}^2 = W_0^2 + (cP_{lab})^2$$

Eq. 10-2

Substituting the momentum from Eq. 10-2 into Eq. 10-1 gives

$$2W_0^2\left(\frac{W_{lab}}{W_0} + 1\right) = 4W_{cm}^2$$

Using the relativistiv energy, γ, instead

$$\sqrt{\frac{\gamma_{lab} + 1}{2}} = \gamma_{cm}$$

For large g one can write:

$$\sqrt{\frac{\gamma_{lab}}{2}} = \gamma_{cm}$$

Eq. 10-3

In the case of one particle hitting a fixed target, the available collision energy goes as the square root of the particle energy. While if two identical particles hit head on, the lab system and the centre of mass system are identical and thus the collision energy scales linearly with the energy in the lab system.

Thus, experiments using two colliding beams are much more "energy efficient" than fixed target experiments.

10.1.1.2 Luminosity and beam quality

The quality of an accelerator for collision experiments does not only depend on the energy, which of course has to reach some minimum value for the reaction of interest to occur with some probability. What one really is interested in is the probability, or density, in time and space, of a head-on collision. This is called *luminosity, L* ($m^{-2}\ s^{-1}$).
The rate a certain reaction occurs is thus
$R = \sigma L$
Where σ is the cross section for the reaction of interest.
In a colliding beam accelerator the luminosity is given by

$$L = fn\frac{N_1 N_2}{A}$$

Eq. 10-4

where f is the revolution frequency, n the number of bunches, N the number of particles in each bunch in the opposing beams and A the cross section area of the beams (assumed to be equal).

To increase the luminosity it is of course important to generate a large number of particles but it is equally important to create a small interaction area. This is not trivial due to several reasons. A small beam of charged particles tend to blow up due to space charges. Bending a beam heavily induces emission of synchrotron radiation. By using a large radius of the machine this problem can be reduced, but still one has to bend the beams onto each other at the collision point and any synchrotron radiation will show it self as background on the detectors. Further the two beams will interact with each other bending and focusing depending on their densities and relative distances. And finally the beam has to be handled after the interaction point not to hit the detector.

10.1.2 Examples on colliders

There are a number of laboratories operating huge particle accelerators for high energy physics. Among the most well known is the joint European laboratory CERN outside Geneva on the border between Switzerland and France. Another important site is SLAC at Stanford University south of San Fransisco in the US. Further there is DESY in Hamburg, Germany, Fermilab outside Chicago in the US and KEK in Japan and several more.

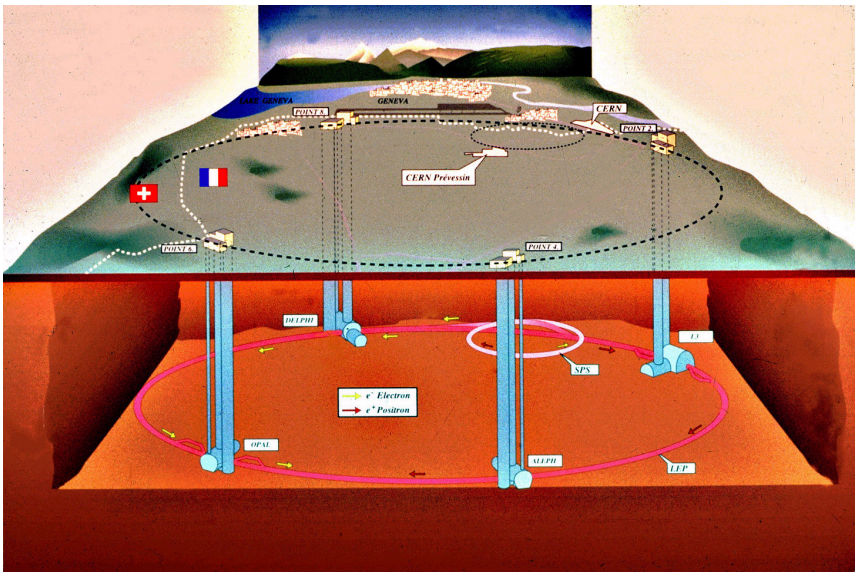


Fig. 10-3. CERN on the border of Switzerland and France. (SLAC homepage)

10.1.2.1 CERN

(Centre Européenne pour la Recherche Nucleaire) started as a joint project in the 50ies and has operated and is still operating a switchyard of several accelerators. A number of smaller pre-accelerators and rings for accumulating anti particles and so forth supply the main accelerators, the SPS and the LEP for experiments and in the future the LHC. All these accelerators are built in tunnels, like the London underground, which traverses the border between Switzerland and France.

SPS is a proton synchrotron which has been used extensively for high energy physics but is now used also as injector for the LEP and in the future for the LHC.

LEP (Large Electron Positron collider) is the largest accelerator in the world with a circumference of 27 km. It accelerates electrons and positrons in opposite directions and can create collisions at an energy of 100+100 Gev.

LHC (Large Hadron Collider) is a project in construction. The idea is to use the existing 27 km tunnel for LEP and replace the conventional magnets with superconducting ones. Thus collisions between two proton beams at energies around 7-on-7 TeV can be achieved in a number of crossing points around the machine. It can also collide beams of heavy ions such as lead with a total collision energy in excess of 1,250 TeV.

CLIC

On the design tables at CERN, and many collegues, are the drawings of a 0.5-5 TeV electron-positron collider. The machine would be two large linacs operating at 30 GHz. (the normal frequency today is 3 GHz) The wavelength is only 3 mm and thus the cavity sizes on the same dimension. The linac should be normal conducting in contrast to other high energy ideas today which aims for superconducting linacs. A problem with a 30 GHz structure is to generate the RF-power. CERN proposes a scheme where another electron beam, called drive beam, is accelerated and the traverses a linac in which it is slowed down and 30 GHz RF-power is extracted. Despite the small cavity sizes the overall size will be quite big! (see Fig. 10-4)

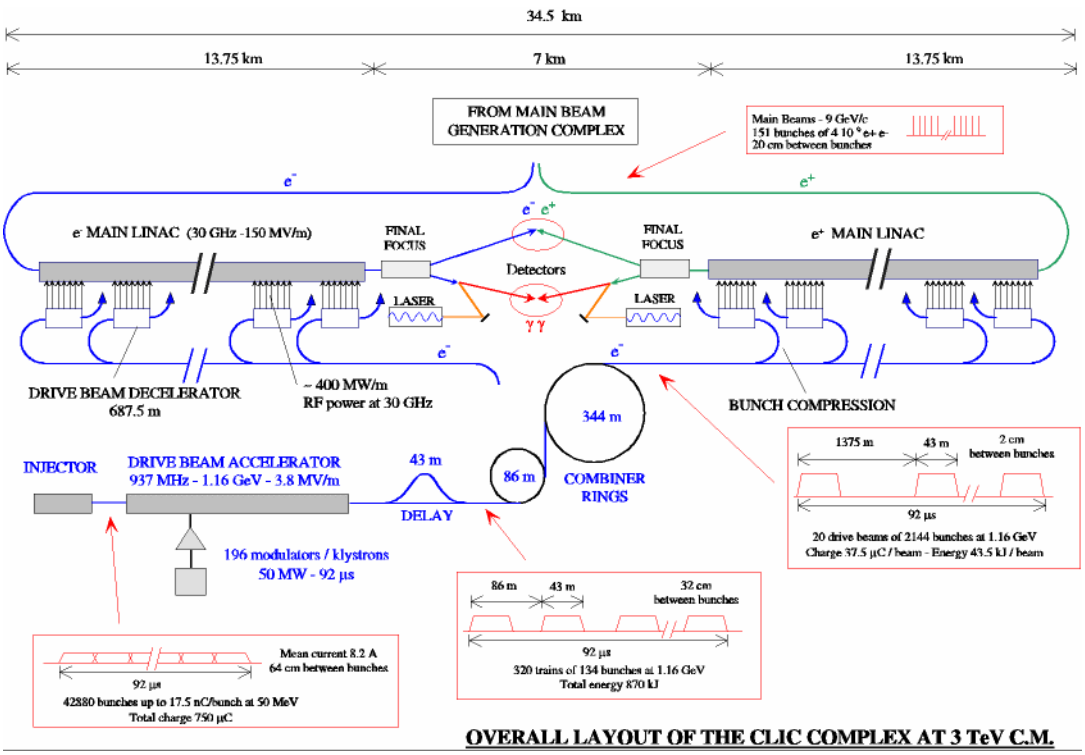


Fig. 10-4. Sketch of the 3 TeV case of CLIC. (CERN homepage)

10.1.2.2 SLAC

(Stanford Linear Accelerator Centre) operates the longest linac in the world a 3 km electron-positron machine delivering up to 50 GeV beams. The electrons and positrons travel at 180^0 phase difference through the machine. The electron beam is used both in the actual experiments and in the process of producing positrons. This is done by letting the electrons after a part of the linac collide with a target (a metal) and thus positrons are produced. These are collected, accelerated and transferred to a damping ring. When enough positrons have been accumulated one also accumulates electrons at the same energy in another damping ring. Damping here means that the beam emittances are damped. Especially the emittance of the positrons produced at the target is initially very large. Finally the two particle trains are extracted simultaneously and accelerated down the linac at 180^0 phase difference. The beams thus created can be used for two different machines:

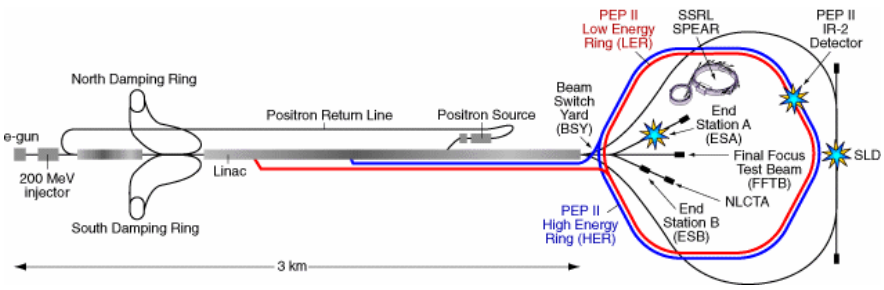


Fig. 10-5. The Stanford accelerators: linac, damping rings, PEP, Collider and the separate SSRL-SPEAR synchrotron radiation storage ring. (SLAC homepage)

SLC (Stanford Linear Collider) (and SLD in figure) At the "beam switch yard" the electrons and positrons from the linac turn right and left respectively and follow an arc to finally hit each other head on at the collision point. The machine uses an elaborate focusing system that reduces the sizes of the colliding beams down to dimensions much smaller than a human hair. This machine has not only produced elementary particles, it has also produced Nobel prizes.

PEP is a storage ring system in a tunnel about 800 meters in diameter which takes both electrons and positrons from the linac. PEP can produce electron-positron collisions up to center-of-mass energies of about 30 GeV. This machine is currently upgraded into what is called: Asymmetric B Factory or PEP-II, colliding 9 GeV electrons with 3.1 GeV positrons.

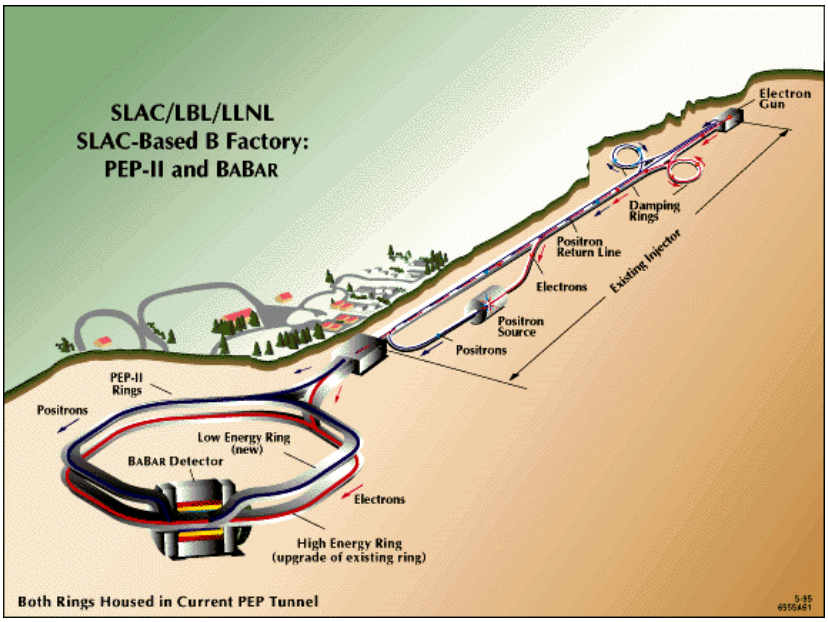


Fig. 10-6. The upgraded PEP-II facility at SLAC, Stanford. (SLAC homepage)

TESLA

In Hamburg there is a large accelerator laboratory, DESY, which operates accelerators both for nuclear physics and synchrotron radiation. There is also a development program for a 500 GeV super conducting linac to produce collisions between electrons and positrons. 33 km of superconducting structures are designed to be dug into the ground north-west of Hamburg.

For the development of the project DESY and its collaborators is now running the project TTF, Tesla Test Facility wher the superconducting cavity structures are tested and a SASE FELl is built.

Fig. 10-7. The size of TESLA is 33 km long. (DESY homepage)

10.1.3 Free electron lasers

The history of applications for Free Electron Lasers (FEL) really took of in the 80ies when the idea of "Star Wars" was presented in the US. The idea was to create an absolute protection against

nuclear missiles sent of by the USSR towards the US. One solution would be to build a "super laser" which would hit the missile in space and destroy it. Development of long wavelength extremely powerful lasers, with in accelerator business bad beam quality, took of but ran into problems. There were technical problems but also severe political and diplomatic problems as the idea violated the international agreements signed by the US and the USSR. Both had adopted to a principle called "MAD", Mutual Assured Destruction, which would secure that if anyone started a nuclear war the other one would have the possibility to strike back. They thus signed the ABM treaty (Anti Ballistic Missile) in which they agreed not to deploy a system that would secure the homeland from enemy missiles, which was just the idea of "Star Wars". There were also other treaties which not worked against the FELs but other solutions, such as agreements not to station nuclear weapons in space and so forth.

Since the end of the 80ies the scientific applications of FEL are dominating and a couple of "user facilities" are operated around the world in different wavelengths. It is still difficult, though possible, to built FELs at shorter wavelengths and thus the existing user facilites operate in the um-mm range. (among others are: Stanford, Santa Barbara, Vanderbilt and Duke in the US, FELIX and CLIO in Europé and facilities in Japan).
Below 10 um there are plenty of ordinary lasers and thus no use of building an FEL until one comes down to 2-300 nm. To reach into this wavelength region one so far has gone either for a Storage ring FEL or a SASE FEL. Both have advantages and drawbacks but it seems quite sure that for the very short wavelengths only the SASE type of FEL is a possible solution today. The dream right now is to build an X-ray laser with wavelengths close to 1 Å. Two main project work in this direction: TESLA in Hamburg and LCLS in Stanford.

FELIX
At the FOM research institute outside Utrecht in Holland there is since a number of years a user facility operating two infra-red FELs. These are based on two electron linacs giving energies up to around 50 MeV.

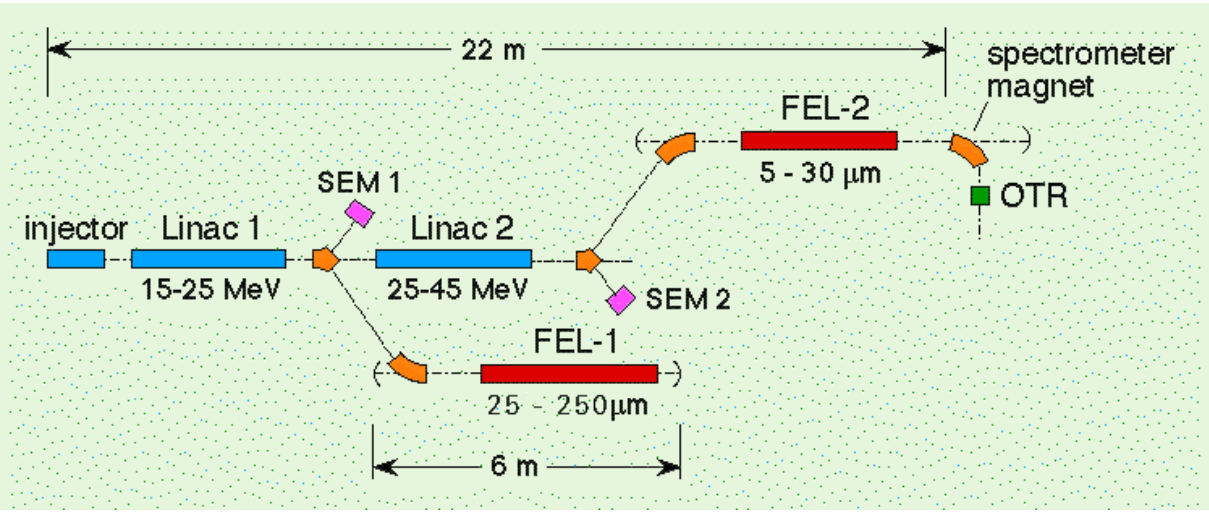


Fig. 10-8. The FELIX facility (homepage).

CLIO and Super-ACO
At LURE in Orsay south of Paris very much of the development in Europe on long wavelength FELs has been done. An Infra-red system using a linac is operated as a user facility, CLIO, and also a storage ring FEL on the ring Super-ACO takes external users.

TESLA and TTF
In Hamburg there is the project TESLA under design. As a first step the TTF (Tesla Test Facility) is beeing built and on this source a SASE FEL. The TTF has been lasing at 109 nm which is a large step for the proof of the SASE principle using an electron energy of 230 MeV.

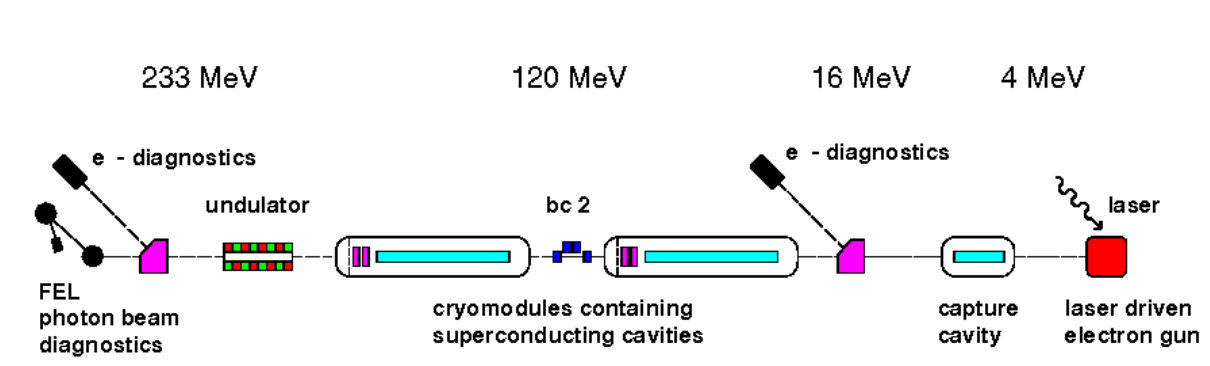


Fig. 10-9. Layout of the first TTF system (DESY homepage).

The TESLA project (see above) is not funded at the moment but with the available beam energy of this machine an X-ray laser is planned. Only a smaller part of the 33 km accelerator aimed for high energy physics will be used for this purpose. The dimensions of such a system are huge. Only the "swtich yard" to house the undulators and to direct the laser beams to different experimental stations take the size of a railway junction with a length close to 1 km.

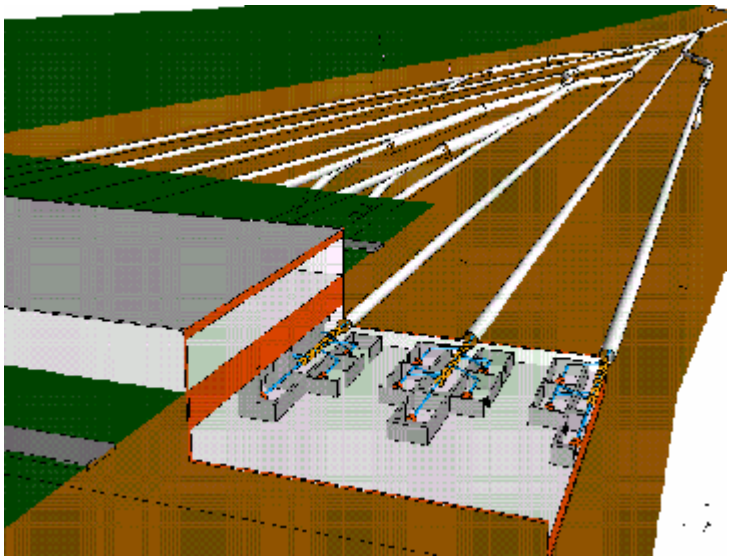


Fig. 10-10. The Expremintal site for the TESLA X-ray FEL. (The accelerator itself is outside the drawing) (DESY homepage).

LCLS

At Stanford another X-ray FEL is under design. It will use 1 km of the already existing linac to extract a 15 GeV electron beam to a 100 m long undulator to produce 1.5 Å radiation.

10.1.4 Ion accelerators

All physicists or chemists are not interested in elementary particles, they might as well want to study larger elements like ions or even molecules. These heave particles also have their family tree of laboratories and in fact in Sweden they are in majority of the large facilities.

10.1.4.1 ELISA and ASTRID

At the university of Århus in Denmark there is one storage ring which regularly switch between electron and ion operation. A more special tool is the ELISA storage ring which is aimed for low

energy ions. It is not a magnetic machine but an *electrostatic* storage ring developed in Århus. Magnets are replaced by plates which form the electric field which directs the particles. The overall size is just a couple of meeters.

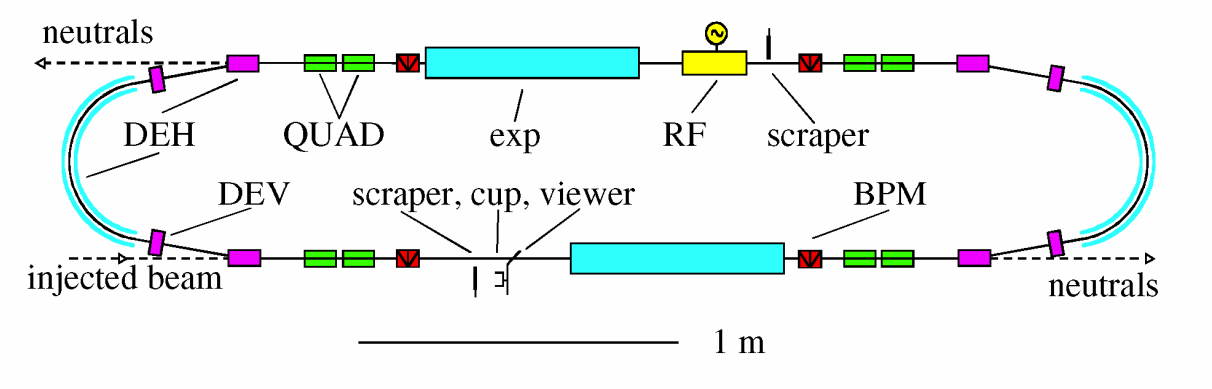


Fig. 10-11. The ELISA storage ring in Århus.

10.1.4.2 The Svedberg laboratory

In Uppsala the The Svedberg laboratory operates a Cyclotron and a storage ring, Celcius, for heavy particles.

A zoo of particles can be produced in a number of different particle sources and thes can be transferred to the Celcius storage ring.

Table 10-1. Particles routinely accelerated at the The Svedberg laboratory.

p	H2+	d	D2+	3He2+
4He1+	alfa	10B5+	12C4+	12C5+
12C6+	13C4+	14N5+	14N6+	14N7+
16O2+	16O4+	16O5+	16O6+	16O7+
16O8+	16O8+	17O5+	20Ne4+	20Ne6+
20Ne9+	40Ar9+	40Ar11+	40Ar12+	84Kr15+
129Xe16+	129Xe21+	129Xe25+	129Xe27+	129Xe29+

10.1.4.3 The Manne Siegbahn Laboratory

MSI in Stockholm operates the Cryring facility which is also a storage ring

The CRYRING facility in Stockholm consists of a small acceleration and storage ring with electron cooling, a preaccelerator and two injectors. Highly charged ions are produced in an electron beam ion source and singly charged ions of atoms and molecules in a plasmatron source. An RFQ is used as the intermediate accelerator.
The scienctific program is focused on atomic and molecular collision physics using the low energy beam directly from the EBIS as well as the cooled circulating beam in the ring. The electron cooler, which also serves as an electron target, has an expanded electron beam, which results in a substantially improved resolution in electron recombination experiments.

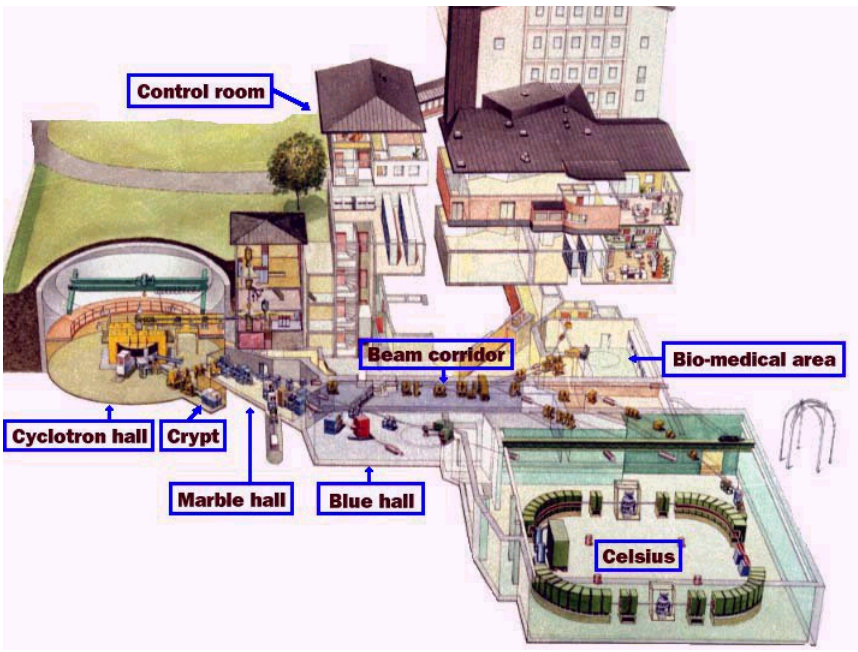


Fig. 10-12. Layout of the The Svedberg Accelerators in Uppsala.

10.2 Medical applications

10.2.1 Radiotherapy

Radiotherapy is a technique to fight cancer tumours in the body. The tumour is irradiated by one or several types of radiation killing the tumourous cells which are more sensitive. The effect of this treatment is excellent and around 2/3 of all cancer patients undergo some kind of radiotherapy, but there are a few problems to come around.

- The radiation has to be delivered to the cancer tumour only, and not to the surrounding tissue.
- The human body is very irregular and the radiation has to be "shaped" accordingly.
- The radiation is produced in accelerators which are complicated devices, but has to be operated extremely reliably.

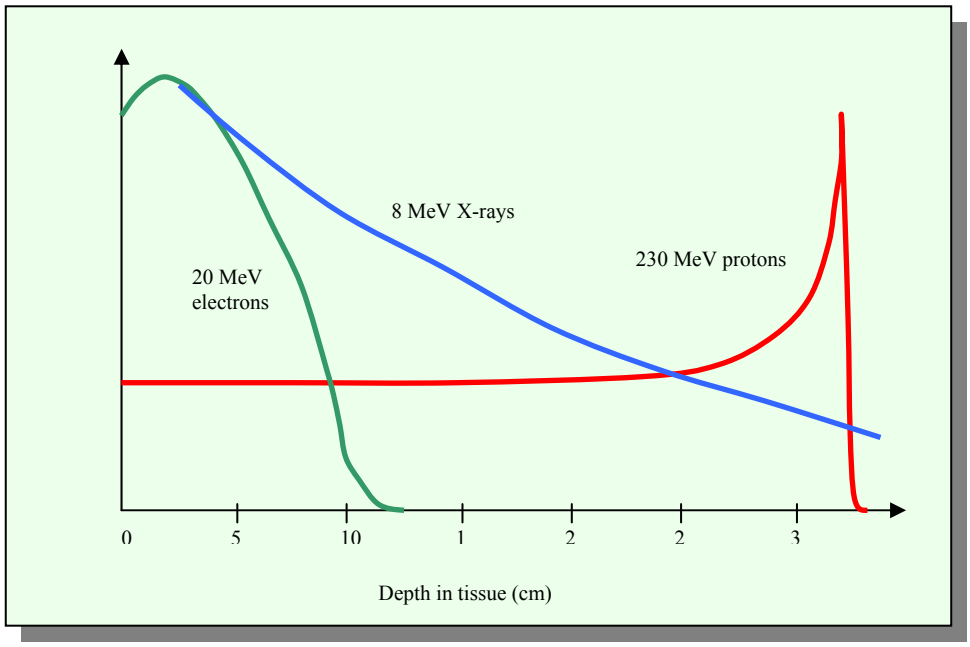


Fig. 10-13. Penetration depth for different particles in tissue.

To reach a tumour well inside the body one would like to use radiation that is absorbed at a certain depth.
Both electrons and X-rays are absorbed in a fairly large volume starting at the tissue surface (see fig). Protons on the other hand are more ideal as the majority of the energy is deposited at a certain depth in the tissue. By introducing some hydrogen rich material in front of the patient the absorption peak (Bragg peak) can be moved to smaller depths and thus be positioned precisely in the tumour.

The most common accelerator for protons is the cyclotron and a 200 MeV cyclotron is not a small device. Thus this method is more complicated and expensive than using electrons and not very much used, despite the better quality, though under development.

The most common irradiation is done by electrons or X-rays produced by an electron accelerator. An electron gun and a small (around 20 MeV) linac is mounted into a rotatable arm, called a gantry. The electron beam is bent and focused onto a target diffusing the beam to uniformly cover a larger are. The target can also be used to produce X-rays from the electrons. The uniform beam passes a collimator which forms the radiation after the tumour.
The complete gantry, including accelerator, can be rotated around the patient to allow radiation from several directions.

Fig. 10-14. Elekta SLi series linear accelerators, with a patient below the gantry. The accelerator is partly behind the back wall. (Eelekta homepage)

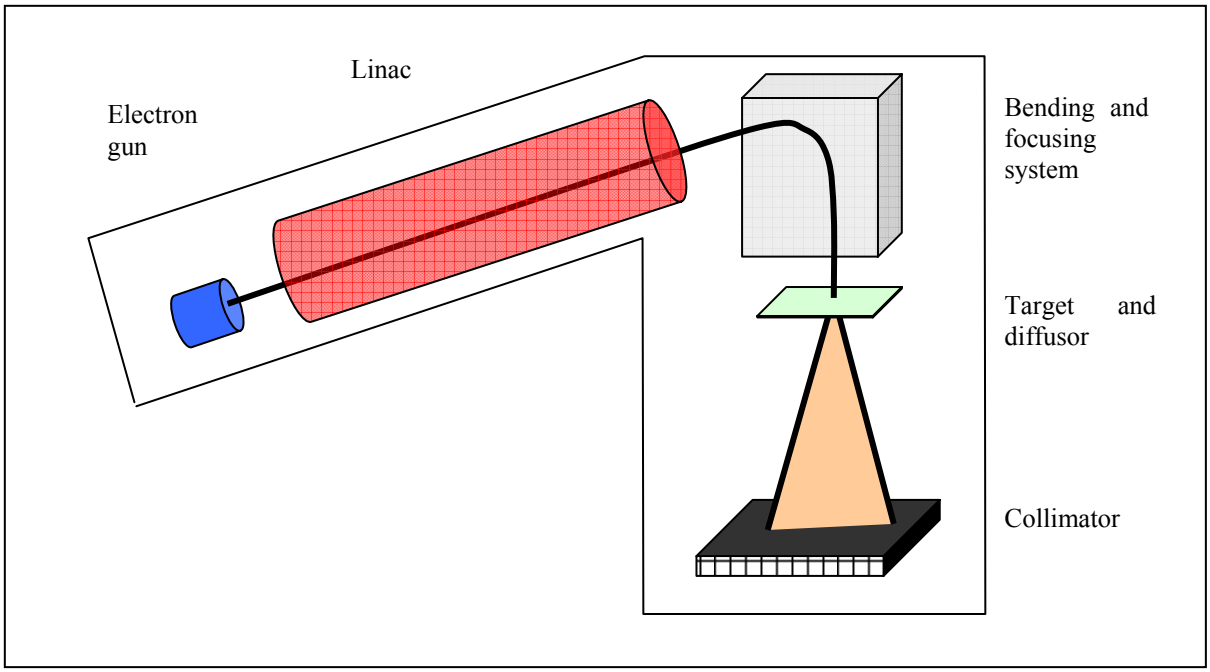


Fig. 10-15. Schematic view of the components in an electron linac for cancer treatment.

10.2.2 Isotope production and PET

PET stands for positron emission tomography and is a method to make "pictures" of cross sections of the human body. A radioactive isotope is injected into the patient. The isotope is "mounted" onto a molecule which one knows will concentrate into certain parts or processes in the body. The isotope emits positrons and when annihilated with an electron two photons are emitted. By detecting these two photons the exact position in space from which the emission took place can be achieved. The most popular PET radioisotopes, 11C, 13N,15O and 18F , can be produced by low-energy cyclotrons (< 20 MeV).

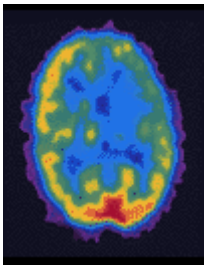


Fig. 10-16. Metabolism in the brain. (picture from IBA, http://www.iba.be/)

10.3 Industrial applications

10.3.1 Radiation processing

Sterilization

A number of products can be sterilized by radiation such as medical products and pharmaceuticals. A 10 MeV electron beam can sterilize products without removing them from the transport package. The demands on the beams are almost only on particle flux while other qualities are of minor interest.

Modification of polymers

Pulp treatment is another method of breaking down wood fibers into cellulose.

Food irradiation is a controversial method of prolonging the durability of foods. Traditionally this has been made by radiation from a Cobalt-60 source, but can also be done by an electron / X-ray source.

10.3.2 Lithography

In the semiconductor industry there is a strong development on making the patterns of the circuits smaller. The method used is called lithography which in principle works by shining light through a mask containing the pattern and making a picture of the mask on the chip substrate. The problem is that one is getting close to the diffraction limit using visible light. In the eighties one started to look towards generating X-rays in accelerators, which then could be used for even smaller patterns. Oxford Instruments has built a very compact superconducting storage ring for IBM, Helios, and Sumitomo in Japan also has superconducting solution, the Aurora. Both these concepts work very well, but unfortunately the ordinary lithography techniques did also develop. Thus the interest of running a complex accelerator is not as big at the moment.

10.3.3 Neutron camera

Ordinary X-rays can "look through" a number of objects, but show mainly heavier objects and metals. Is one instead of using photons use neutrons one will detect not necessarily the heavy materials but the hydrogen rich. One such use would be to detect plastic explosives in security checks, another to see corrosion and plastic materials. One commercial way of producing neutrons is to use a cyclotron producing protons, 10-15 MeV, and collide them with a beryllium target.

11 Vacuum

11.1 Basics

Vacuum is in many senses a situation where most of the molecules normally present around us are absent. The pressure, or lack of vacuum, is defined as the force per unit area that the gas introduces on a surface.

$$P = constNM\langle v^2 \rangle$$

Eq. 11-1

where P is the pressure, N the number of molecules per cm^3, M the mass of the molecule and $<v^2>$ the means square of the molecule velocity.
The pressure P is given in Pascal (1 Pa = 1 N m^{-2}). Other units often used are bar and torr.

1 mbar	100 Pa	0,75 torr
1 torr	1,33 mbar	133 Pa

The atmosphere around us has a pressure of roughly $1*10^5$ Pa = 1 bar. The different pressures present in vacuum technology are divided into ranges:

10^5 - 100 Pa	Low vacuum
100 - 0,1 Pa	prevacuum
0,1 - 10^{-5}	High vacuum
$<10^{-5}$	Ultra high vacuum (UHV)

A few important statements about vacuum are:

- The total pressure P is the sum of the partial pressures, the pressure of each individual gas.
- The *amount* of gas is given in pV (pressure * volume).
- The **gas flow**, Q, is the amount of gas per time, Q = p dV/dt
- The molecular flow $\varnothing_m$ = number of gasmolecules per time = dN/dt.
- Pumping speed is given in **volume flow:** S= dV/dt
- Conductance is the gasflow/pressure fall, C=Q/(p2-p1)

In an ideal gas the gas law is valid (Eq. 11-2).

$$P \cdot V = n \cdot R \cdot T = \frac{m}{M} R \cdot T = N \cdot k \cdot T$$

Eq. 11-2

Where R = 8314 J/kmol is the Gas constant, k = 1.38 10^{-23} is the Boltzmanns constant, T is the temperature in Kelvin, n is the number of kmol of the gas, m is the mass of the gas and M the molecule mass.

11.2 Pressure and temperature

The pressure that a gas makes on a vessel can be seen as the impulse each molecule imposes when colliding with the wall. The velocity of the particles is proportional to the temperature of the gas and the amount of collisions is dependent of the number of molecules. Thus the pressure depends on both the temperature and the number of particles.
The movement of the molecules in a gas is said to be viscous at pressures above 1 Pa and molecular below 1 Pa. In a viscous flow the molecules move like a liquid but in the molecular flow they move independent of each other. In most practical problems regarding vacuum systems in accelerators we can assume the flow to be a molecular flow. Depending of the pressure a

molecule can move long distances before making a collision with another molecule. We here talk about the free mean path length (Eq. 11-3) which is the distance between two collisions.

$$l = \frac{kT}{\sqrt{2}\pi d^2 p}$$

Eq. 11-3

Where l is the mean free path length and d is the molecule diameter.

11.3 Conductance, gasflow, pumping speed

The conductance for a vacuum system is defined as

$$C = \frac{Q}{P_1 - P_2}$$

Eq. 11-4

i.e. the gas flow through a system for a certain difference pressure.
Which can be rewritten into

$$\frac{1}{C} = \frac{P_1}{Q} - \frac{P_2}{Q} = \frac{1}{S_1} - \frac{1}{S_2}$$

Eq. 11-5

where S is the volume flow.
When different systems are connected the conductance is given by

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_{21}} + \frac{1}{C_3} +$$

In a circular tube the conductance for molecular flow is given by

$$C = \frac{4\pi r^3}{6L}\sqrt{\frac{8RT}{M\pi}} \approx 1000\frac{r^3}{L}\left[m^3/s\right]$$

Eq. 11-6

where r is the radius and L the length. By convention the conductance are given for N_2 with M=28.

An aperture has the conductance

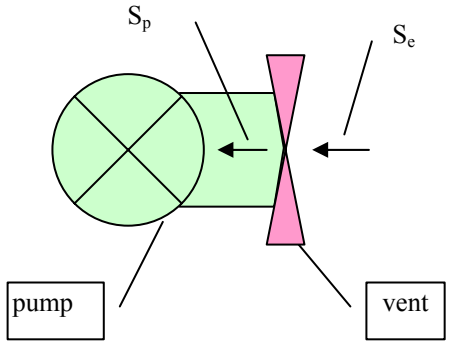
$$C = A\sqrt{\frac{RT}{2\pi M}} \approx 119A\left[m^3/s\right]$$

Eq. 11-7

where A is the area. It should be noticed that an aperture conductance only should be taken into account while going *into a smaller* vessel.

Example:
The capacity of a pump is given to 1000 l/s (this is a volume flow). This pump is connected through a circular vent with a radius of 3 cm. What is the effective pumping speed of the pump?

The conductance of the vent is: $C = 119\pi 0.03^2 = 0.336\, {m^3}/{s} = 336\, {l}/{s}$ l/s



The pumping speed, Sp, is 1000 l/s and we look for the effective pumpin speed S_e.
By Eq. 11-5 it is possible to write

$$\frac{1}{C} = \frac{1}{S_p} - \frac{1}{S_e} \Rightarrow \frac{1}{336} = \frac{1}{S_e} - \frac{1}{1000} \Rightarrow S_e = 251 l/s$$

11.4 Adsorption, desorption, cleaning and baking

To achieve and maintain vacuum in a system one not only need a vessel without any leaks, and good pumps, one also needs a clean inner surface of the vessel.

Inevitably there will be molecules stuck to the surface in the vessel and in a bad situation these molecules will leave the surface (desorption) and eventually stick to some other part. Before obtaining a vacuum the air molecules will stick to the surface(adsorption).

While producing the vacuum system care has to be taken to what materials are used (stainless steel, aluminium, copper) and the different parts has to be carefully cleaned. In the mounting process the care has to be maintained not to expose the surfaces to "dirt". In accelerators most parts has to be able to reach the UHV region, and thus just a fingerprint from you lunch is unacceptable!

The desorption of molecules from a surface can be enhanced by increasing the temperature of the vessel wall, it will also be enhanced if exposed to strong radiation, such as synchrotron radiation. The desoprtion or outgassing due to temperature follows

$$\frac{dN}{dt} = -constNe^{\left(\frac{-E}{kT}\right)}$$

Eq. 11-8

where N is the surface coverage of molecules (molecules per area unit), E is the binding energy, k Boltzmanns constant and T the absolute temperature.
If N is very high the amount of desorbed molecules is high.

This relation opens up for a few possibilities. By cooling down the system the amount of desorbed gas will decrease and by heating up the system more gas will be desorbed. If a complete vacuum system does not reach the desired vacuum, which is almost always the case in UHV, one can heat the system up. The remaining molecules will thus be desorbed and can be pumped away. After this process the surface coverage has gone down and less molecules will be desorbed even at ordinary temperatures. Thus the pumps will suffice to decrease the pressure further. This method is called *baking* or *bakeout*.

11.5 Pumping systems

A large number of pumping systems exist to fulfil different desires while constructing, achieving and maintaining vacuum. No single pump is able to cover the range from atmospheric pressure down to UHV.

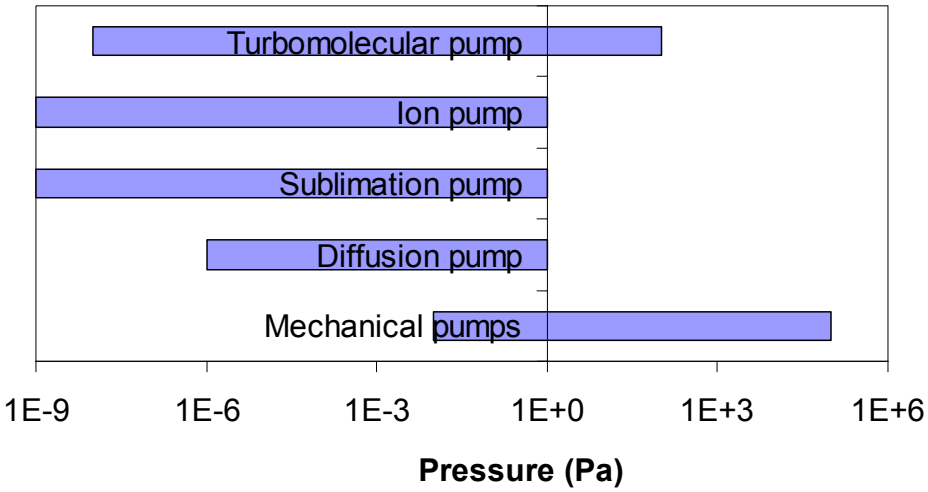


Fig. 11-1. Working ranges of some vacuumpumps.

11.5.1 Mechanical pre vacuum pump

There exist a number of different mechanical pumps, which all work roughly by the same principles. The gas is let into the pump, compressed and leaves through a vent on the high pressure side (Fig. 11-2).

Other pumps are the Leybold-Heraus and the Roots pump.

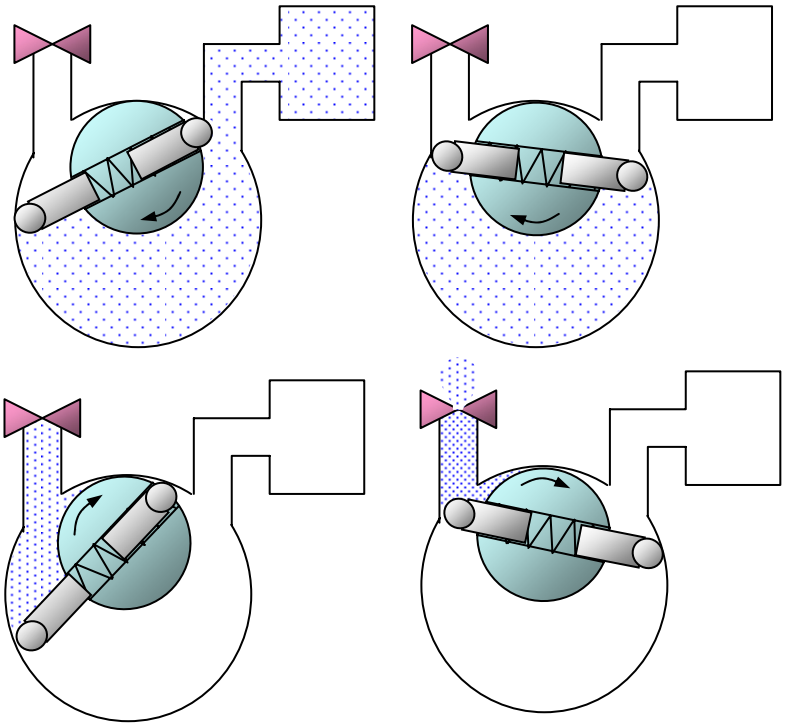


Fig. 11-2. The Gaedes pump.

11.5.2 Diffusion pump

In a diffusion pump (Fig. 11-3) a pump oil is heated up to an oil steam and injected into the pump through nozzles, where they attain very high velocities (up and above the speed of sound). These high mass high velocity particles collide with the gas that one wants to pump out, and the gas molecules are compressed in the direction of the pump oil steam. The gas can then be pumped away by a mechanical pre vacuum pump.
The hot oil hits the walls of the pump, which are cooled, where it condensates and the oil flows back to the bottom of the pump and is reused in the process.

This pump is an effective pump to rapidly reach low pressures. A disadvantage with this pump is the presence of oil in the system, which can/will diffuse into the vessel that one want to evacuate.

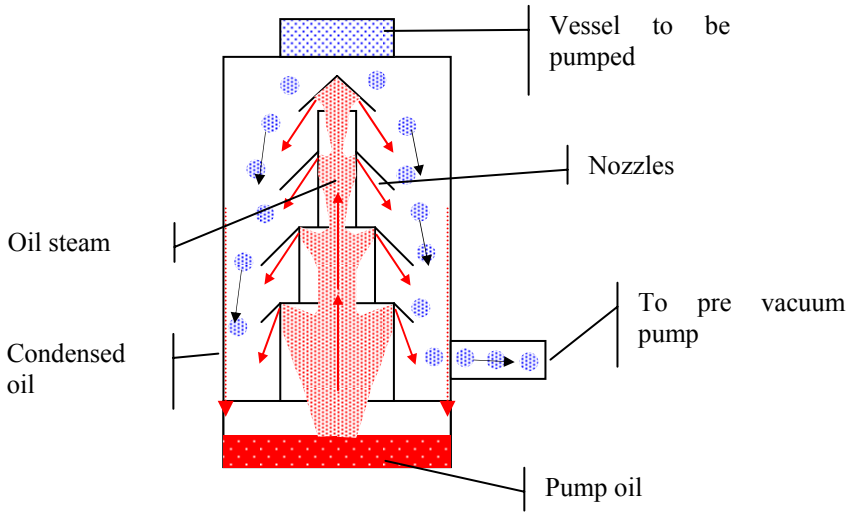


Fig. 11-3. The diffusion pump.

11.5.3 Titanium sublimation pump (TSP or getter pump)

This kind of pump works through chemical reactions instead of mechanical action on the gas molecules. Titanium (Ti) is deposited on the inner surface of the pump. Ti is a very reactive substance which acts with the constituents in the gas and forms different Ti compounds. The Ti-surface of the pump will be saturated after some pumping time, and the pumping speed will decrease. A new layer of Ti can then be deposited from a Ti source installed in the pump. Ti is heated up to a Ti gas and then deposited on the inner walls of the pump.

11.5.4 Sputtering ion pump

The ion pump also uses Ti plates but they are combined with an electric and a magnetic field (Fig. 11-4). A cathode and an anode are placed in the pump, and a voltage of several kV are put between them. Electrons are emitted from the cathode by fieldemission and accelerated towards the anode. Due to a perpendicular magnetic field, the electrons will spiral through the pump. In this motion they collide with gas molecules in which atoms are transformed to positive ions and new electrons produced. The ion is accelerated towards the cathod which they hit and are buried in the Ti material while Ti atoms are sputtered out from the cathod and deposited on all parts of the pump. The Ti atoms will then help in the pumping process, just like in the TSP.

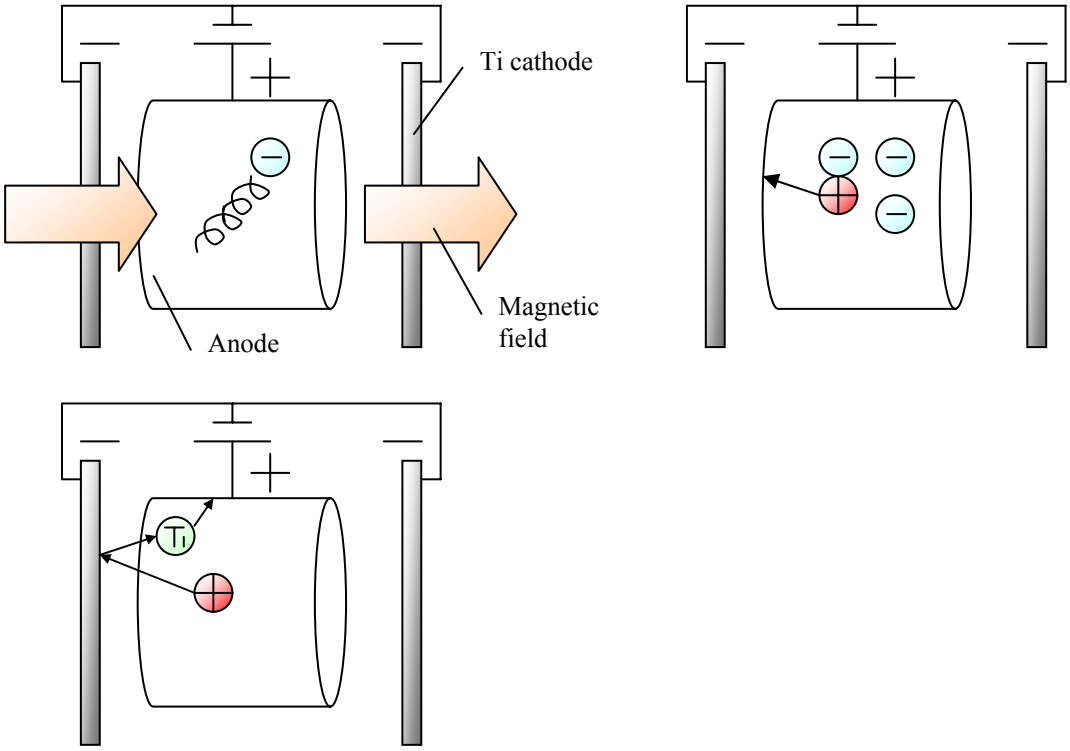


Fig. 11-4. Sublimation ion pump.

11.5.5 (turbo)Molecular pump

In the turbo molecular pump a number of rotors with inclined blades, like a propeller, rotate at extremeley high velocities (20-60 000 turns per minute). The gas molecules collide with the rotor blades and an impulse is given to the molecule. At then other end of the pump the molecules are pumped away by a pre vacuum pump.

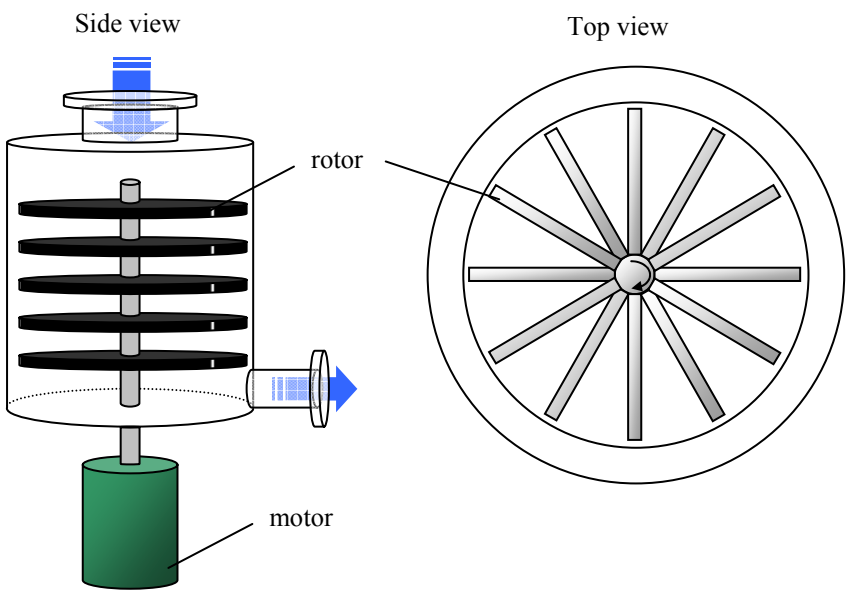


Fig. 11-5. The turbo molecular pump.

This pump is one of the most common to pump down a vacuum system for the first time. It operates over a large pressure range, but it is most efficient to pump heavy molecules. Thus

hydrogen will be one of the dominating rest gases in a system pumped by a Turbo Molecular Pump.

11.6 Vacuum meters and mass spektra

A number of different devices exist to measure the pressure in a vessel. As with pumps none is good for the whole pressure range.

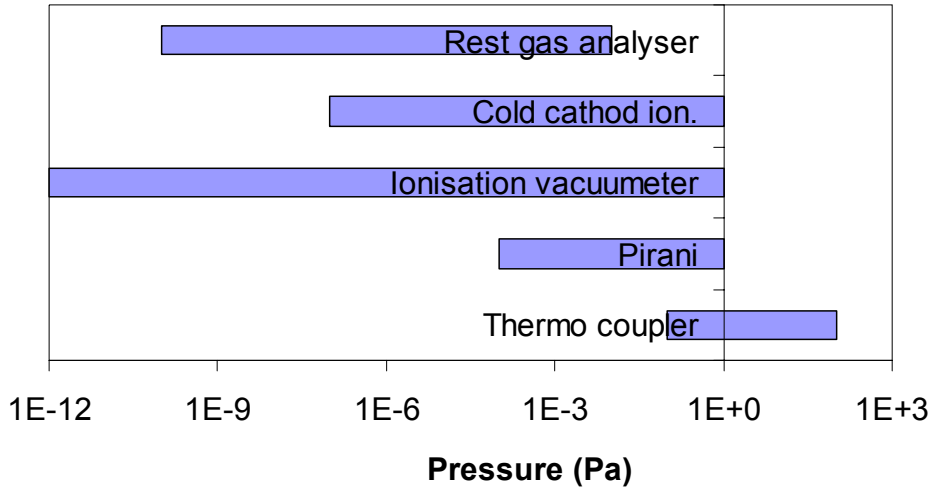


Fig. 11-6. Working ranges of some pressure gauges.

11.6.1 Thermo coupler and Pirani vacuumeter

The principle behind the thermocoupler is that the heat is transported away from a wire inside the vessel at different speeds at different pressures. Different ways exist on how to measure the temperature of the wire, thus the two different names. In the first type a thermocoupler measures the temperature, which is translated into pressure. And in the Pirani vacuummeter the wire is coupled in a balanced "bridge coupling".

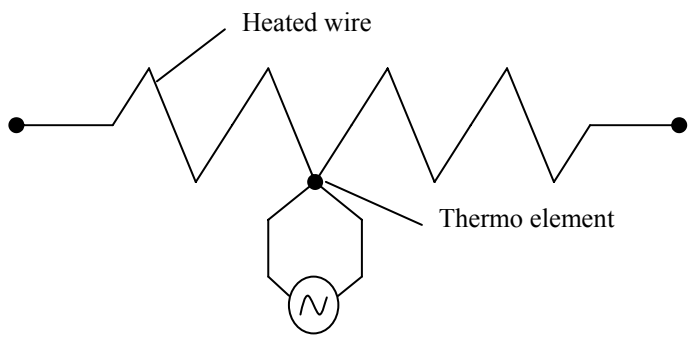


Fig. 11-7. The thermo coupler.

11.6.2 Ionisation vacuumeter

The operation principle of this device is rather similar to the ion-pump described above. Electrons are created on a heated filament they are accelerated towards the grid, but most of them miss it and will move back and forth until and they collide with molecules in the gas. The atoms in the gas are ionised and attracted to the collector which they hit and collect an electron to once again become neutral. The amount of electrons leaving the cathode is measured as a current, which then is proportional to the pressure.

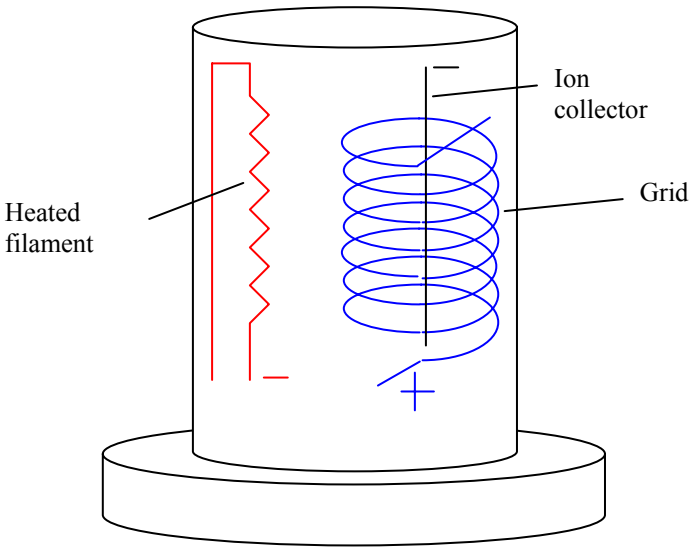


Fig. 11-8. Ionisation vacuumeter.

11.6.3 Cold tube vacuummeter (Penning vacuummeter)

This kind of pressure monitor is very similar to an ionpump. It consists of an anode and a cathode placed in a magnetic field. Electrons in the device are attracted to the positively charged anode. Due to the magnetic field they move in a spiral path which improves the possibility of collisions between the electron and the molecules. Each such collision will give rise to an ion and some electrons. The ions are attracted by the anode and when hitting it they collect the missing electrons. This current is proportional to the number of molecules in the chamber.

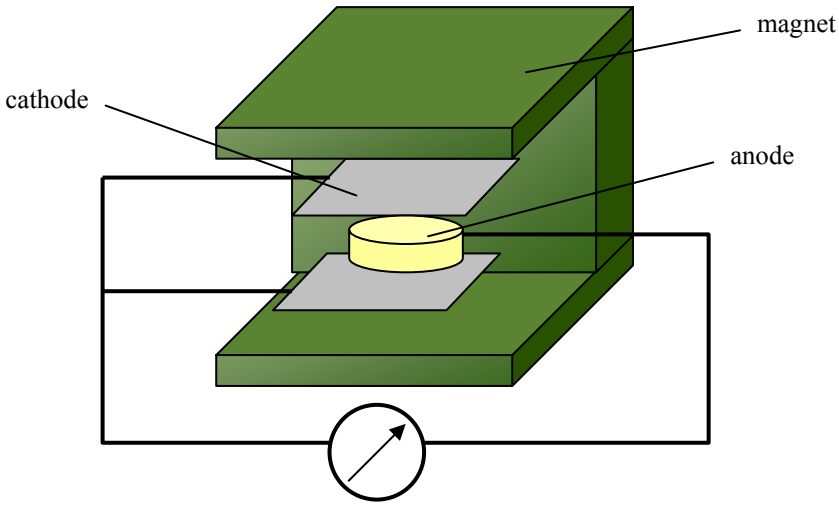


Fig. 11-9. Cold cathode ionisation vacuummeter.

This kind of vacuummeter is gaining in popularity and as it can cover most of the intersting pressue range it is become something of a standard device in UHV installations.

11.6.4 Rest gas analyser and the mass spectrum

The rest gas analyser (RGA) or the partaial pressure meeter gives information of the pressure or the amount of molecules of certain masses (partial pressures). Two slightly different devices exist: the magnetic RGA and the quadrupole RGA. Both these devices uses the fact the molecules with different mass move differently in magnetic and electric fields.

In the magnetic RGA the gasmolecules are first ionised and then accelerated by a voltage. As the ions pass a magnet they are bent, but the angle with which they bend is dependent of both the magnetic field and their mass. By varying the magnetic field a certain mass number of ions may pass over to the detector, and the amount of ions that hit are measured as a current.

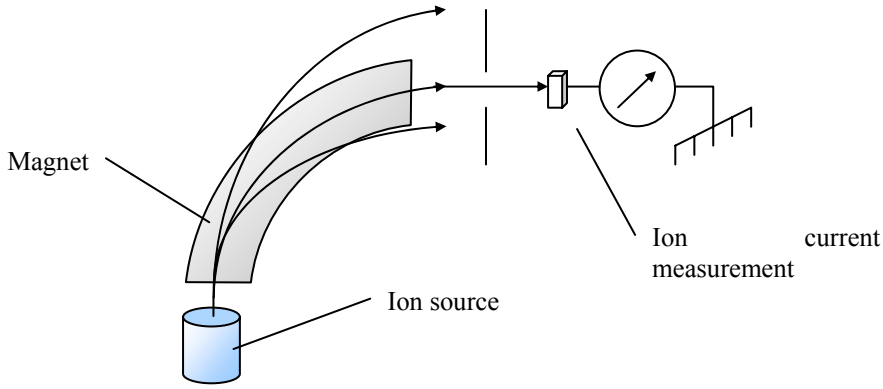


Fig. 11-10. Magnetic RGA

In the quadrupole RGA, which has become the more used device, the different masses of ions are selected by a quadrupole shaped field created by four rods. The ions are created and accelerated just as in the magnetic RGA, but then they pass through a channel in the middle of the rods. These are powered by an RF voltage and the ions move in a spiral path through the device. At certain voltages on the rods a selected mass number can be made to pass.

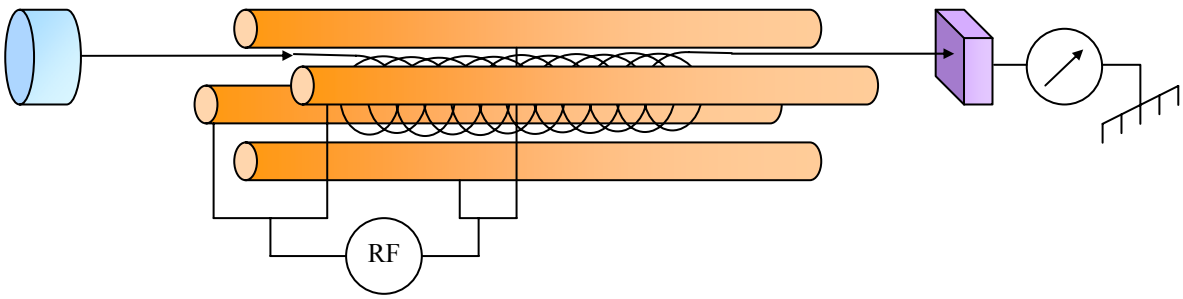


Fig. 11-11. A quadrupole RGA

	2	4	14	15	16	17	18	19	20	26	27	28	29	30	32	35	37	40	44
H_2	100																		
H2		100																	
CH_4			11	80	100														
NH_3					92	100													
H_2O						25	100												
F								100											
Ne									100										
N2												100							
CO												100							
C_2H_6										19	30	100	21	26					
O_2															100				
Cl																100	30		
Ar									20									100	
CO_2												13							100

Table 11-1. Relative strenght for molecules in a RGA spectrum.

By use of an RGA a spectrum over the different masses in the gas can be achieved, also the amount of each component is achieved. To interpret such a spectrum one needs knowledge not only of the mocleule masses but also some hint of the ionisation possibilities. In practise one uses a table (Table 11-1) where most molecules show up at 2 or 3 masses due to the fact that they form ions with a choice of charges. One more complication is that at a certain mass number there might exist several types of molecules.

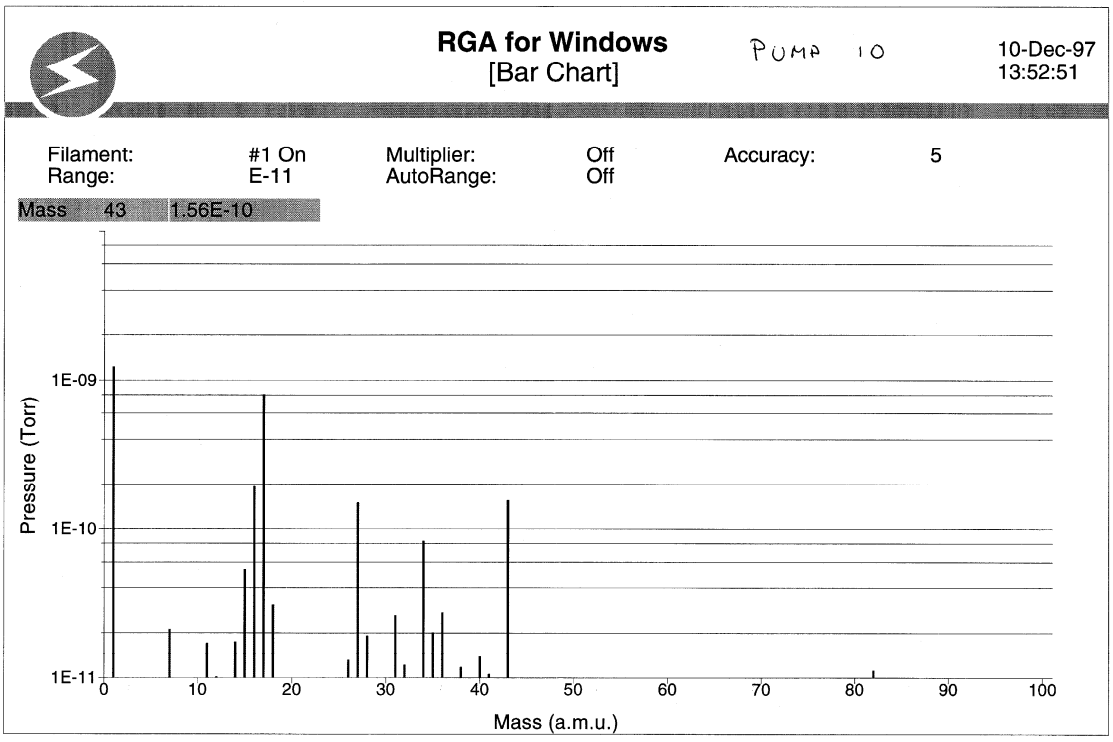


Fig. 11-12. A rest gas spectrum from a Turbo Pump at MAX-lab.

11.6.5 Sealing techniques

A vacuum is normally built out of a number of different details: tubes, chambers, windows, vents, feed throughs, manipulators etc. To produce each item special welding and soldering techniques should be applied, but they are not the topic of this overview. While mounting the different pieces together in the lab sealings have to be put between them. Normally each component has a flange welded onto it. The purpose of the flange is to house the seal and to allow for robust tightening. The flanges are conveniently produced in a few standard sizes.

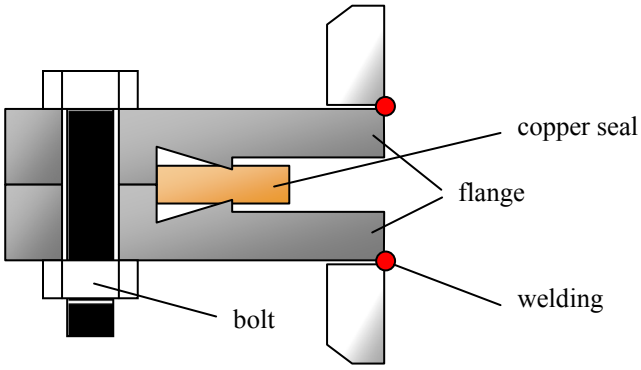


Fig. 11-13. The conflat flange

In general two types of sealings are available, either rubber gaskets or metal gaskets. The metal is normally Cu, Au, Al or In where copper is the most widely used. The rubber seals are normally not be used in UHV situations even though some special materials can be baked up to 200° C.
A typical Cu seal is the Conflat-flange (Fig. 11-13). It consists of flanges with circular edges between which the copper seal is placed. The edges makes grooves in the copper and a very tight seal is produced.

11.6.6 Air leak testing

An assembled vacuum system might seem perfect to the eye but very often contains leaks. Most of them will be extremely small and impossible to find without special measuring techniques. A very common used method is by a Helium leak analyser. It consists mainly of an RGA which only detects He ions. The analyser is connected to the leaking vacuum system. Helium gas is then carefully "blown" onto the system from the outside. If the gas penetrates the vacuum system the analyser will very quickly register an increase in the He amount. One can thus get a good indication on where the leak is. By careful use and some luck the leak can be localised to within a few cm.

12 Synchrotron radiation, undulator, wiggler and FEL.

In the earlier chapter we have looked at what happens if a charged particles is inserted into an electric or a magnetic field. Now it is time to turn the problem around. If a charged particle is experiencing a force, what happens? The Lorentz equation once again can help us (Eq. 12-1).

$$\overline{F} = m\overline{a} = q\overline{E}$$

Eq. 12-1

A force will give an acceleration and that will generate an electric field parallel to the acceleration.

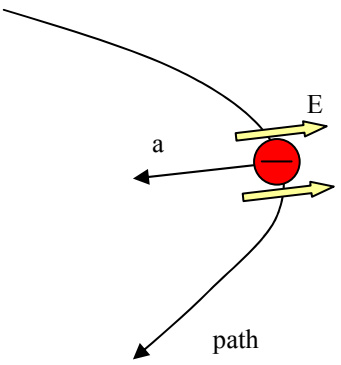


Fig. 12-1. The electric field from an accelerated electron.

The electric field is propagating out in space in a direction perpendicular to the E-field vector, thus generating a form called a donought.

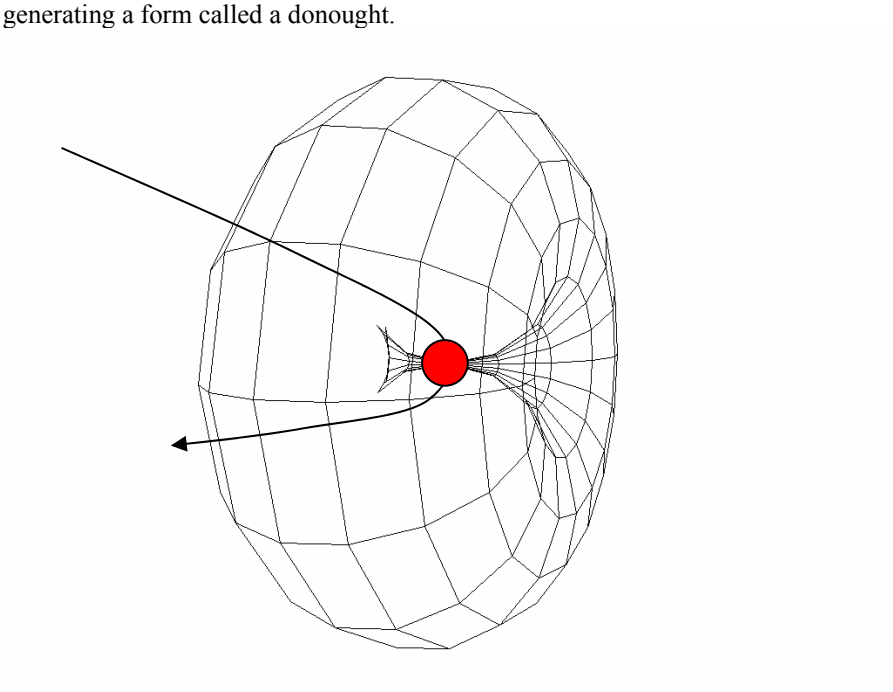


Fig. 12-2. The electric field shape from an accelerated electron.

The picture drawn of the electric field is valid in the electron restframe (=if you travel along with the electron) or at low electron velocities (much lower than the speed of light). If the electron travels at relativistic speeds the shape of the electric field as seen in the laboratory system is quite different. The whole shape is dopplershifted in the forward direction.

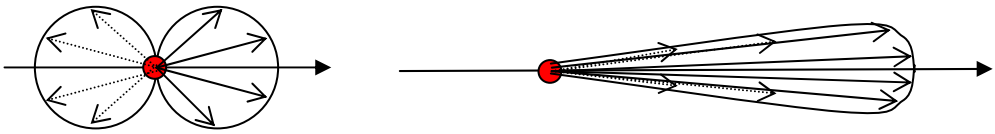


Fig. 12-3. Relativistic doppler shift of the electric field emission shape.

The "radiation cone" for the emission in the laboratory frame has an opening angle of approximately 1/γ (where γ is the relativistic parameter).

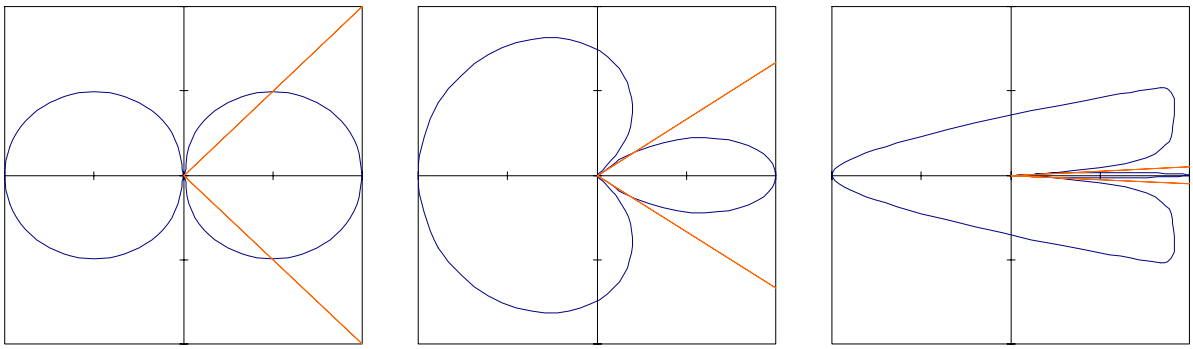


Fig. 12-4. Doppler shift of the emission cone for γ=1 / 1.5 / 20 (from top).
The angle indicated is 1/γ.

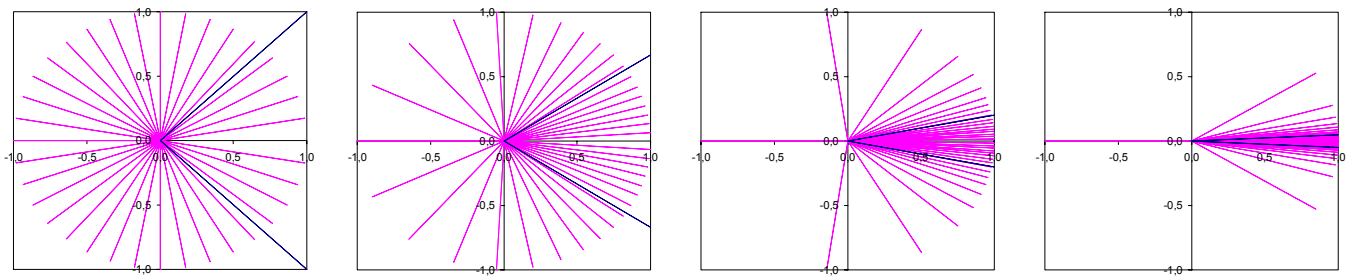


Fig. 12-5. Doppler shift of the emission cone for γ=1 / 1.5 / 5 / 10 (from side).
The angle indicated is 1/γ.

12.1 Wavelength of the radiation

The next question that comes to mind is to decide which wavelength the radiation has. As the particle follows a circular path during the acceleration and emits light in rougly a cone with opening angle 1/γ a spectator will see a pulse of radiation when the particle is in a proper angle to the observer.

A pulse of radiation can be expanded by fourier transform, but lets not do the calculation but just look at what it means.

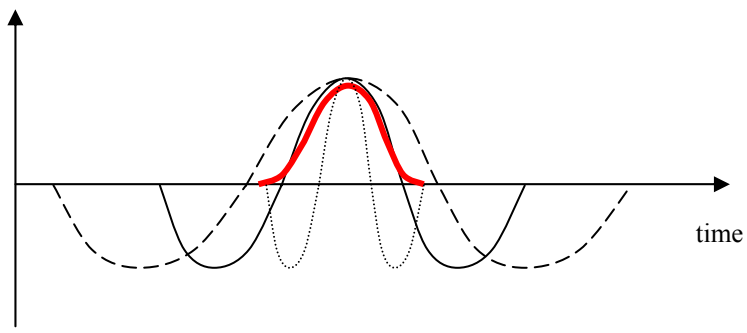


Fig. 12-6. Fitting an oscillation to a pulse. The thick red line is the emitted pulse of radiation. solid line cut off, dashed line long wavelength, dotted line too short wavelength.

If we fit an oscillation to the emitted pulse of radiation we can see that there is one wavelength of the oscillation that fits perfectly. If wee choose a longer wavelength there is nothing saying that this can not be exited. On the other hand if we choose a shorter wavelength it can not be exited as the fields have opposite signs at certain times, and it is thus suppressed. We can call the perfect match for the cut-off wavelength below which there is little radiation.

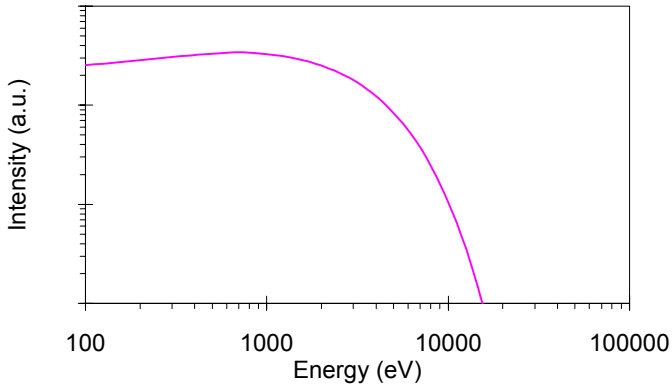


Fig. 12-7. Synchrotron radiation spectrum from a bending magnet (E=1.5 GeV, r=3.5 m).

Lets now try to find the length of the pulse and then continue to get an estimate for the cut-off wavelength.

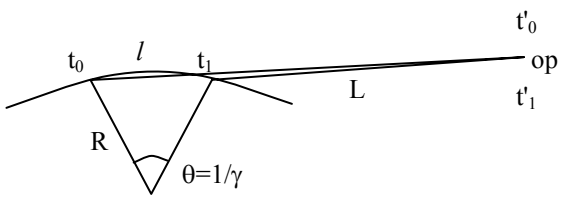


Fig. 12-8

At the observation point (op) one starts to see the radiation from point t_0 and ends to see it from t_1. The distance between these points is given from the radius (R) and the energy (γ). The leading edge of the pulse will arrive to the observer at time t'_0 and the trailing edge at t'_1, thus giving the length of the pulse.

$$t_1 - t_0 = \frac{l}{\beta c}$$

$$t'_0 - t_0 = \frac{L+l}{c}$$

$$t'_1 - t_1 = \frac{L}{c}$$

$$l = R\theta$$

Eq. 12-2

$$\Delta t = t'_1 - t'_0 = \frac{L}{c} + t_1 - \left(\frac{L+l}{c} + t_0\right) = \frac{l}{\beta c} - \frac{l}{c} = \frac{l}{c}\left(\frac{1}{\beta} - 1\right) = \frac{l}{c}\left(\frac{\gamma}{\sqrt{\gamma^2 - 1}} - 1\right)$$

Eq. 12-3

For the example in Fig. 12-7 the energy was 1.5 MeV (γ≈3000) and the R=3.5 m. This gives Dt=2.2e-19 s

This is equivalent to a wavelength of λ=0.13 nm or a photon energy of E_γ=9.4 KeV.

If we go back to Fig. 12-7 we see that at this energy the intensity has dropped by 1.5 orders of magnitude from the maximum value and above this energy the intensity drops even more rapidly.

Often one does not talk about a cut-off in the synchrotron radiation spectrum, but a "critical wavelength" (or energy). It is defined as half the power is radiated above and half below this wavelength. In practise it is close to the maximum of the spectrum.

$$\lambda_c = \frac{4\pi R}{3\gamma^3}[m]$$

Eq. 12-4

In the case above the λ_c= 0.54 nm or E_c=2.3 KeV.

Associated with this is of course the power radiated from an electron. It is given by

$$P = \frac{2q^2 c\gamma^4}{3R^3} = \frac{2e^2 c}{3R^3}\frac{E^4}{(m_0 c^2)^4}$$

Eq. 12-5

In this context it can be interesting to see how much energy that one electron radiates over one turn in an accelerator. That energy has to be replaced at every passage of the accelerating cavity.

$$\delta E[KeV] = 88.5\frac{E^4[GeV]}{R[cm]}$$

Eq. 12-6

12.2 Flux and Brilliance

To find the accurate expression for the actual flux of photons at a certain wavelength is a cumbersome job. (see J.D.Jackson, Classical Electrodynamics which is a bible in the area) Without posting the formulas we shall look at a few things regarding the radiation.

One often talks about the photon flux from a synchrotron radiation source. What is meant by this can be a little bit arbitrary, but the most common definition is:

Flux = Photons per (second *and* 0.1% bandwidth *and* 1 mRad horizontal opening angle *and* 1 mA current in the accelerator) = photons/(s * 0.1%BW * mRad * mA)

The flux is a handy unit as it tells you how many photons you actually have available at your experimental station. On the other hand it is not a good unit when you compare the quality of different (synchrotron) sources as it is very important what the source actually looks like. An attempt to find such a dimension to compare different sources is the brilliance (or brightness). It is defined as

Brilliance = Flux per (source size *and* solid angle of radiation) = flux/(mm^2 * $mRad^2$)

12.3 Polarization

In the above text and pictures the direction of the E-field vector is always drawn parallel to the acceleration, that means in the plane of trajectory of the particle. The E-field vector defines the polarisation of the radiation and thus it is in the plane of the trajectory.
This is a truth with modification! If we observe the radiation from a point above or below the plane of the particle trajectory the polarisation is not the same.

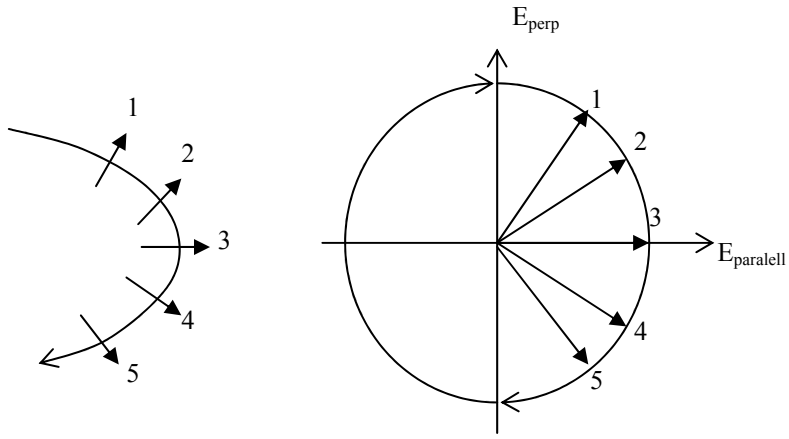


Fig. 12-9. Looking on the emission off plane (left). The direction of the first emitted radiation is different from the last thus giving a circular(elliptical) emitted radiation (right).

The distribution of radiation polarised in another plane than in the trajectory plane is given in Fig. 12-10.

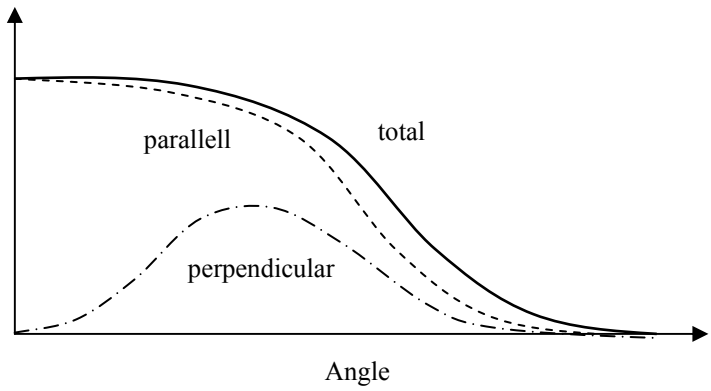


Fig. 12-10. Intensity distribution of different polarisations.

12.4 Interference

So far the description has only been made for single particles, and we have not mentioned the possibility of several particles interacting together to build up the radiation. If they would do so we could talk about interference or coherence. The question then becomes, will the electrons interact, all of them or two-by-two or not at all?

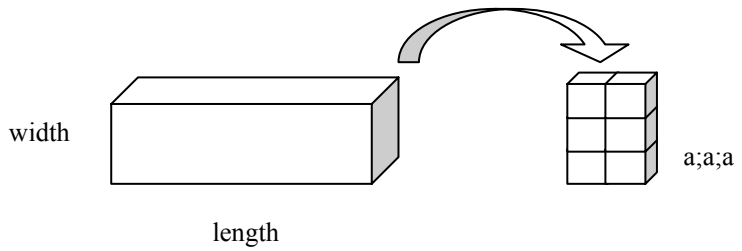


Fig. 12-11. A bunch consists of electrons which each have a small box for itself.

Assume that we have an electron beam of 100 mA with 100 ps (= 3cm) long and 1 cm wide pulses in a 500 MHz system.

$$Pulse\ length = 0.03m$$
$$RF - frequncy = 500MHz$$
$$\#e \Big/ bunch = \frac{0.1[A]}{500*10^6[Hz]e} = 1.25*10^9\ electrons/bunch$$

The total volume of the bunch is

$$V_{bunch} = 0.03*0.01*0.01 = 3*10^{-6}m^3$$

And the volume available per electron is

$$V_{electron} = \frac{3*10^{-6}}{1.25*10^9} = 2.4*10^{-15} \quad \Rightarrow \quad a = 1.3*10^{-5}$$

If one assumes that interference effects can occur if the distance between the particles is less than the radiation wavelength one gets a condition

$$a < \lambda \quad \Rightarrow \quad \lambda > 1.3*10^{-5}$$

At wavelengths above 13 μm there starts to be interference effects between pairs of electrons, but to make the whole bunch radiate coherently needs much longer wavelengths! If we are using radiation in the visible, vacuum ultraviolet or X-ray there is no interference effects and no significant coherence due to the electron source, if we do not make some special tricks!

12.5 Insertion devices

An insertion device is a device which is inserted into the storage ring (clever huh?). It is not part of the original lattice of the storage ring and the use of it is in almost all cases to improve the generation of synchrotron radiation. There are two main types of devices, the ones that does not utilise interference effects and the ones that does. Often the first types are called wigglers and the second type undulators, but the border between the two is fairly arbitrary, and you might here other ways of separating the two.

The insertion devices are magnet structures placed onto the storage ring in a place where the electron beam normally follows a straight path, in what we often call a straight section.

12.5.1 Wiggler

By a wiggler one means a device which emits bending magnet like radiation. In its most simple form it consists of three magnets: one strong in the middle and two weaker beside it (Fig. 12-12). The electron beam will make one small wiggle and emit radiation mainly in the middle magnet. This concept is used if one wants to introduce a very strong magnetic field, much stronger than the bending magnets to reach higher photon energies. Compare the critical wavelength in Eq. 12-4 which decreases with the bending radius.

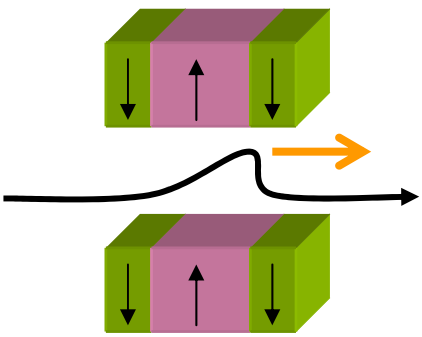


Fig. 12-12. A three pole wiggler.

This kind of wiggler can often be made of superconducting magnets to be able to reach field much higher than the limit due to iron saturation (1.5-2 T), thus these devices can have a peak field in the range of 5-7 T.

The other wiggler type, the multi-pole wiggler, consists of a row of magnets which produce a field which is altering indirection periodically and causes the electron beam to make several "wiggles" through the device.

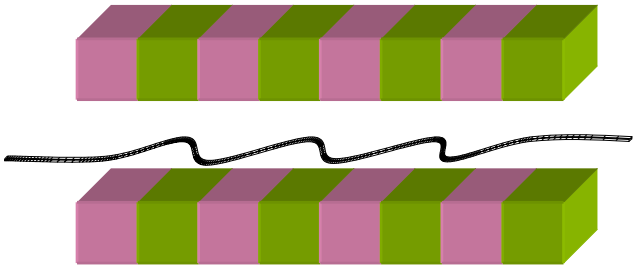


Fig. 12-13. A wiggler magnet

The light generated in a multi-pole wiggler magnet has the same characteristics as light from a bending magnet. There is no interference in the emission and thus the light generated at each individual bend just sums up to a total radiation as from an equal number of bending magnets. One can say that the amount of photons is multiplied by the number of poles in the wiggler.

12.5.2 Undulator

The next type of insertion device is the undulator. The principle is very much the same as for the multipole wiggler, but in the undulator we have interference. In the discussion above we concluded that there is no interference between different electrons. In the undulator we have interference between the light emitted by the *same* electron but at different places. The light emitted in one "wiggle" of the undulator will interfere with the light emitted by the same electron but in the coming wiggles. If there is positive interference there will be an "amplification" and vice versa.

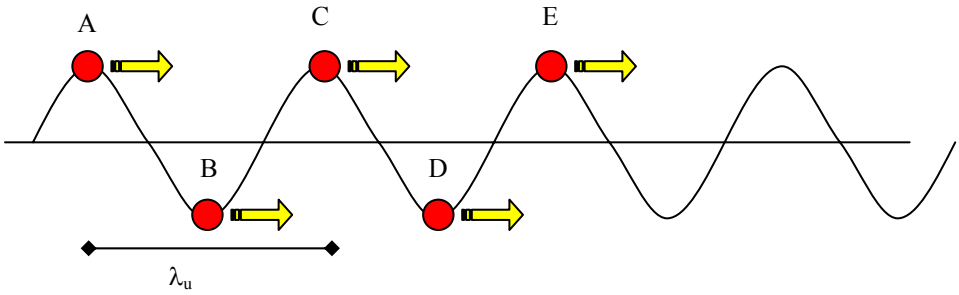


Fig. 12-14. The electron path and light emission in an undulator

Lets look at the emission of synchrotron radiation in an undulator. The path of the electron can be assumed to be almost a sine-function (Fig. 12-14). At a number of point separated by the period length (typically a few cm) in the undulator the electron will emit radiation. From the discussion about bending magnet radiation we know that the E-field has a direction opposite the acceleration in the emission point, and that the radiation will propagate in a narrow cone in the forward direction.

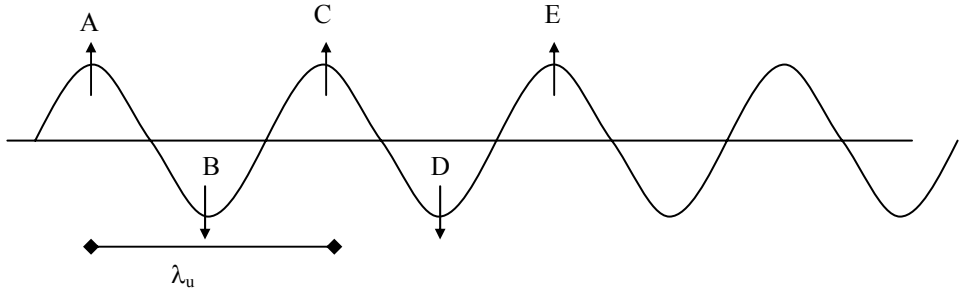


Fig. 12-15. The E-field produced by the electrons.

We can see the radiation from each pole as a "short burst of E-field". When the electron travels along the undulator it will first emit radiation at point A. It will then continue to point B and emit radiation again. The radiation emitted at A have propagated in the straight forward direction with the speed of light. As the electron travels slightly slower than the speed of light (very slightly) and makes a detour due to the sine-formed path, it arrives to B after the radiation from A. The process continues and when the electron emits again at C the radiation from A and B has passed a little ahead again.

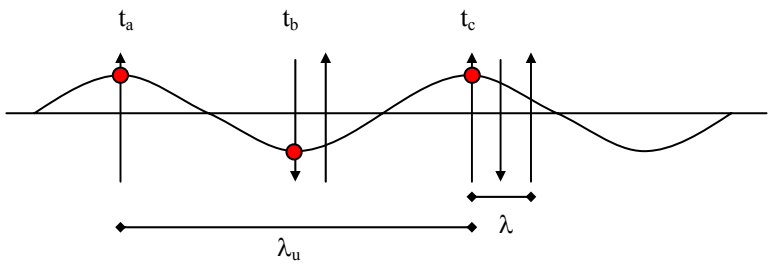


Fig. 12-16. E-field from an electron in an undulator at different times.

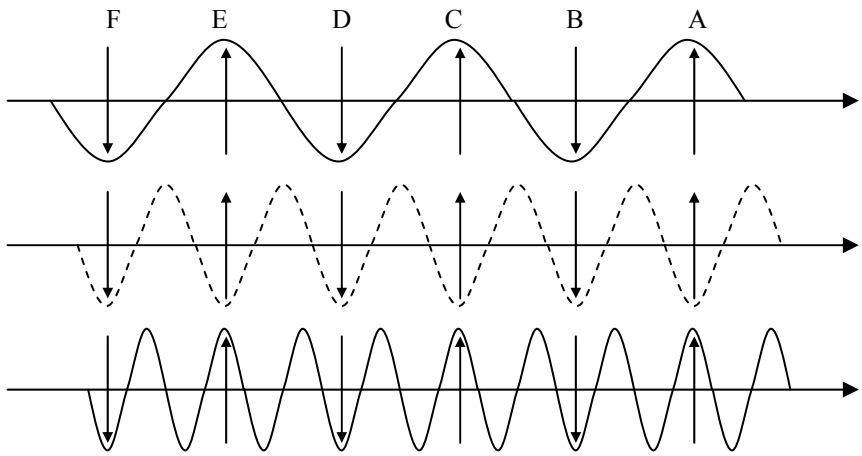


Fig. 12-17. Fitting frequencies to the emission by an electron in an undulator.

The observed radiation will be a train of pulses separated by a fixed distance. When one looks for the wavelength of the radiation one can say that one fits a sine-function to it. This is done in Fig. 12-17 and the result shows that a sine with the same period as the burst fits fine. A sine-function with one third of the period also fits, but the one with half the period does not. The conclusion is that sine-functions with odd multiples of the period fits and the others don't.

The wavelength is, as stated above, given by the time difference between the passage of a period in the undulator by the radiation and the electron.

$$\lambda_n = \frac{\lambda_u}{n2\gamma^2}(1+\frac{K^2}{2})$$

Eq. 12-7

where K is the undulator parameter (K-factor, undulator strength ... a dear child has many names!) given by:

$$K = \frac{eB_0\lambda_u}{2\pi m_o c}$$

Eq. 12-8

And n is the harmonic number and thus only takes the odd-integer numbers.

The picture of the undulator painted above is correct if one looks directly on the electron beam axis. If one starts to accept radiation at angles away from the axis two things happen. The first is that the radiation wavelength increases, and the second thing is that the even harmonics start to show up. The wavelength off axis is given by:

$$\lambda_n = \frac{\lambda_u}{n2\gamma^2}(1+\frac{K^2}{2}+\theta^2\gamma^2)$$

Eq. 12-9

From the pictures above we can also see that the radiation is linearly polarised in the plane of oscillation in the undulator.

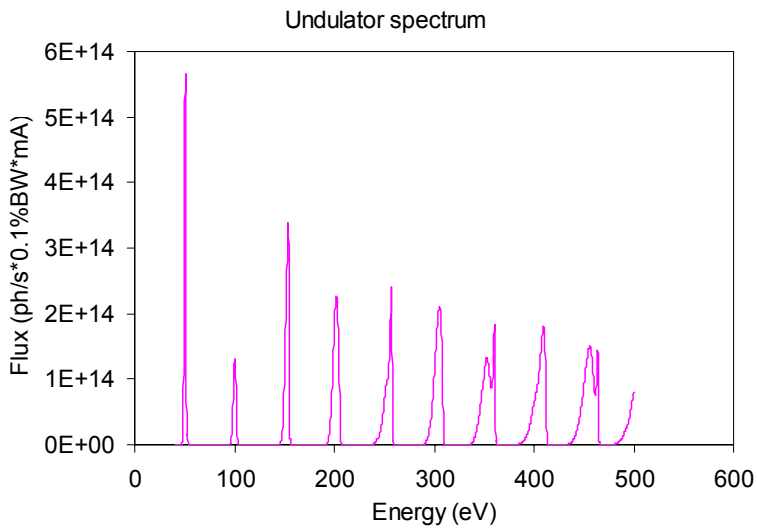


Fig. 12-18. Undulator spectrum (E=1.5 GeV, K=4, λ_u=0.046m, N=50, 0.35x0.35 mRad)

Another point of interest is the width of a peak in the undulator radiation spectrum. It is defined by the number of periods in the undulator. From each period there comes a burst of photons, and by taking this train one gets a number of periods of an E-field equal to the number of periods in the undulator. A pulse with N periods has a linewidth of $d\lambda/\lambda=1/N$, or for a higher harmonic of the undulator the linewidth becomes $d\lambda/\lambda=1/nN$. Thus sharper lines at higher harmonics.

12.6 Free electron laser

The Free electron laser, or FEL, is a device using undulators but capable of generating coherent radiation. One can see three different operation modes for this technique: Harmonic generation, FEL with optical cavity and "self amplified spontaneous emission", SASE. The complexity and the demands on the accelerator are different as well as the wavelength regions one can operate in.

All three techniques use the idea of mixing an electron beam passing the magnetic field in an undulator with the electromagnetic field of a light wave. If some special criteria are fulfilled an energy exchange can occur between the light wave and the electrons. As a consequence of this

interaction energy can be transferred coherently from the electron beam kinetic energy to the electromagnetic field of the light wave.

12.6.1 Harmonic generation

Harmonic generation uses an external coherent light source to illuminate the electron beam in the undulator. The electrons become redistributed in longitudinal direction with bunches at distances equal to the wavelength of the light. Further down the undulator they can then radiate coherently, which in an undulator also occurs at higher harmonics. The overall effect can be seen as generation of higher harmonics of the initial coherent laser source.

The lorentz equation

$$\frac{d(ym_{0c})}{dt} = q\bar{v}\bar{E}$$

says that to have a change in energy of the electron due to an electric field (light wave) we need to have an electron velocity parallel to the E-field vector. If we shine a light wave along an electron beam, to reach a long interaction length, the E-field vector is perpendicular to the velocity of the electron. In an undulator the situation changes as there is a velocity perpendicular to the ordinary path. If one then tune the oscillation of the electron in the undulator to the oscillation of the E-field an energy change can occur:

$$\frac{dE}{dt} = qv_y \sin\left(\frac{2\pi s}{\lambda_u}\right) E_y \sin(\omega t - ks + \phi)$$

Eq. 12-10

If we look at a part of the electron bunch which currently have a large velocity, v_y, perpendicular to the ordinary trajectory (Fig. 12-19).

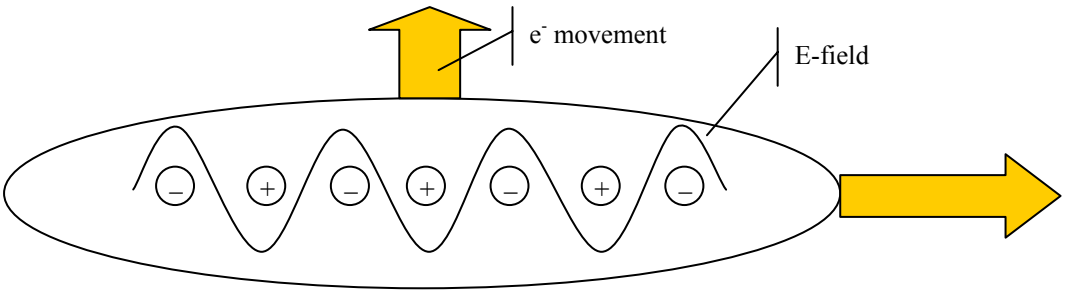


Fig. 12-19. Electrons moving sideways in an undulator change energy due to an E-field.

The electrons having the same phase as the E-field loose energy and the electrons 180 degrees out of phase gain energy in the interaction with the E-field. The E-field will move faster than the electrons, but the wavelength has to be chosen such that at the next period of the undulator, the E-field has advanced 360 degrees, and the same condition occurs. This resonance criteria is exactly the relation for the wavelength of the undulator.

Now there are some electrons loosing energy and some gaining energy along the electron bunch. In between the electrons are not changing their energy. If one lets this bunch pass along the undulator the electrons which have lost energy will tend to move a tiny amount slower, and they will also take a slightly longer path as they are bent more by the magnets in the undulator, they thus will sack behind. The electrons gaining energy will do just the opposite and hurry in advance.

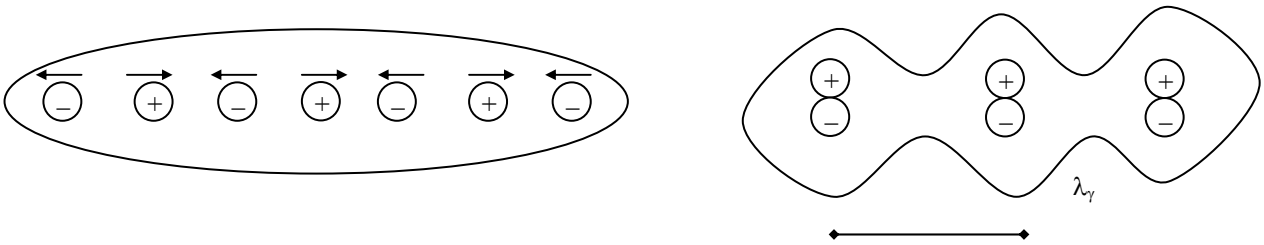


Fig. 12-20. Bunching.

Thus by waiting a proper distance an electron bunch with a density varying with the light wavelength can be achieved. When this bunch then radiates towards the end of the undulator the electrons can radiate coherently as they are spaced with the wavelength of the radiation. The number of photons thus does not follow the number of electrons, N, but N^2. The coherent radiation will also be emitted in the higher harmonics of the undulator radiation, but the degree of coherence will rapidly drop.

12.6.2 FEL

A Free electron laser uses the same basic principle of bunching of the electron beam as in harmonic generation. But there are two distinct differences. The first is that one normally does not use a laser to start the bunching, but the ordinary undulator light is reflected back by two mirrors and hits a later coming electron bunch and thus the device chooses by itself a wave on which to bunch on. The second point is that contrary to the harmonic generation, where one only manipulated the source and then let it radiate as it liked, one here continues the interaction between the electrons and the E-field.

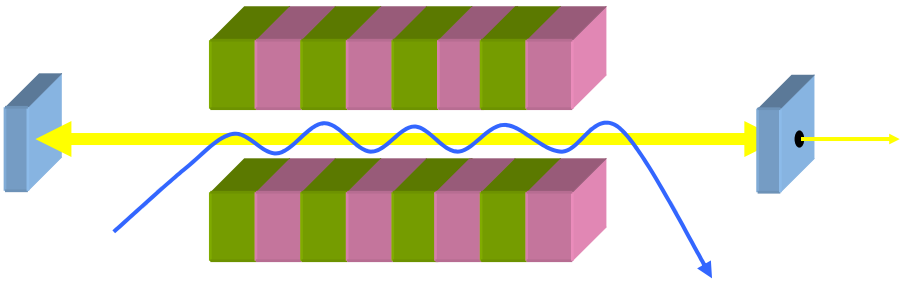


Fig. 12-21. A FEL with mirrors.

If one takes a bunched beam, as in Fig. 12-20, and lets it interact with an E-field which is in such a phase that all electrons are decelerated energy can be directly transferred from the kinetic energy to the E-field.

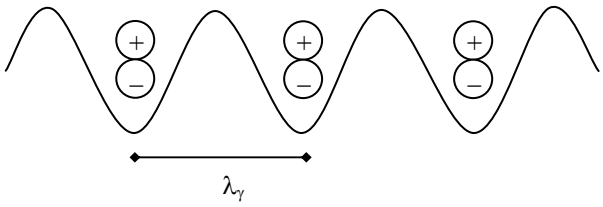


Fig. 12-22. Phase relation in an FEL.

As the kinetic energy of the electrons is very large a large amount of energy can be transferred and the amplification (laser action if you wish) can be huge. In practise one often wants to do this in one single undulator, and to be able to first achieve the bunching, when the waves are in phase, and later the amplification, when the phase has moved 90 degrees, the amplification will lie at a slightly different wavelength than the undulator radiation, just to make this 90 degree phase lag after, say, half the undulator.

The method of FEL using mirrors has limitations as it is difficult to find suitable mirrors at shorter wavelengths. A couple of facilities producing FEL light in the infra red (1um - 1mm) exist today and a few lasers which aim at wavelengths down to 200 nm are built as experimental facilities. Some of these shorter wavelength FELs are built using electron beams in storage rings while the majority of FELs in the infra red use electrons from linacs, which are only used once and then disposed.

12.6.3 SASE

The Self amplified stimulated emission (SASE) principle is rather new. The technique has been demonstrated to operate well in the infra red and a couple of test have been done at slightly shorter wavelengths.

The idea is to use the FEL principle but without mirrors and instead let the electron beam generate some radiation in the beginning of the undulator which down the line lets the bunching build up and towards the end the bunch is ready for FEL action giving energy away to the E-field. It is thus a single pass device and no mirrors are needed. On the other hand the undulators tend to grow very long. A goal for the coming ten years is to reach close to 1 Å which would need an electron beam of several GeV and an undulator of more than 100 m. But advantages of the principle are strong though, as there is no direct limitation on how far one can go, just that it gets more difficult the shorter the wavelength.

Two large projects are proposed to push for SASE FEL close to 1 Å: the LCLS at Stanford in the USA and the TESLA in Hamburg, Germany.

13 An introduction to WINAGILE

WINAGILE is a program to simulate and evaluate the optics of rings and transport lines. It is written at CERN (Phil Bryant, CERN http://bryant.web.cern.ch/bryant/) to be used in their courses, CERN Accelerator School. The program is free to download and runs under WINDOWS 3.x, WINDOWS 95 and WINDOWS NT and XP.

The instructions given below are aimed at the work with the construction duty in the course Acceleratorfysik, 5p. Please note that the program can do significantly more things than described here.

13.1 How to start

The WINAGILE package consists of the files:
winagile.exe
winagile.hlp
___.lat (several)

They are given either in compressed form or as files ready for installation.

To extract the files open the file manager (utforskaren) copy the files to a suitable directory (ex. *c:\programs\winagile* and *c:\programs\winagile\lattices* respectively) and double click on the files in their new directories.

13.2 Layout of the program

Winagile is built about a few Windows, and the program jumps between these windows fairly quickly. When a new window is opened, the previous is closed, though you can return to it later. Until you catch how these "jumps" are made it might see confusing.

You start in the "**Main Window**" from where you first continue to an "**Lattice Editing**" window. In this window you enter the different elements of your accelerator. You go back to the Main Window and then to a "**Rings and Matched sections"** (or **Transfer Lines**) window which also performs the basic calculations about your machine. You can also open a "**Kick file Editing**" window and different "**Graph"** windows.

On top of these you can open sub-windows to fex. make Tune fittings (Two Parameter Fitting).

The Help function explains most details of the program, and will in many cases be more useful than the text below.

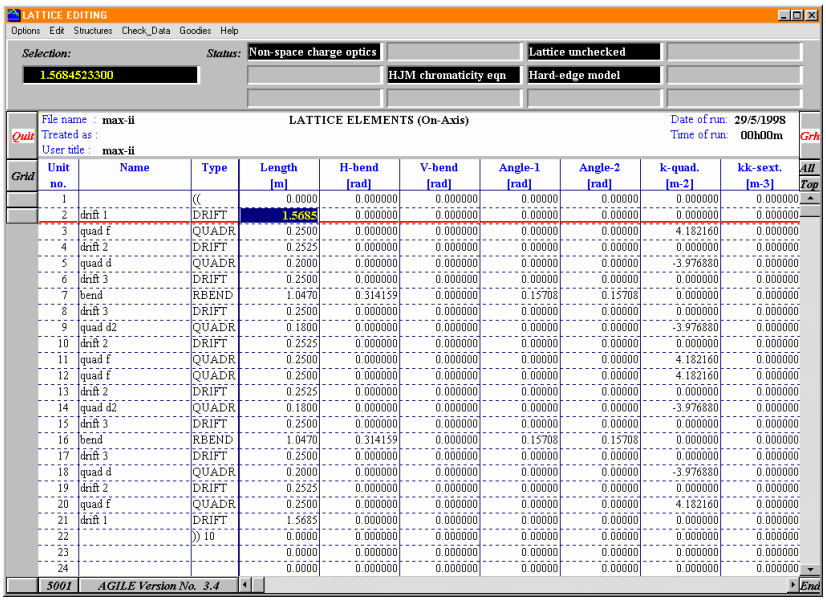
One note is that WINAGILE tends to mess up the colours Windows, but to my experience everything is restored when you close WINAGILE.

13.3 How to run

To run WINAGILE, double click on the file: *c:\programs\winagile\winagile.exe* in the Filemanager (you should choose the directory **you** installed the program in).

For a new lattice:	File → New
For an old lattice:	File → Old
	File → Edit

You have now switched to LATTICE EDITING (Fig. 13-1).
Start input by double-clicking a field and input necessary parameters.



Unit no.	Name	Type	Length [m]	H-bend [rad]	V-bend [rad]	Angle-1 [rad]	Angle-2 [rad]	k-quad. [m-2]	kk-sext. [m-3]
1		((	0.0000	0.000000	0.000000	0.00000	0.00000	0.000000	0.000000
2	drift 1	DRIFT	1.5685	0.000000	0.000000	0.00000	0.00000	0.000000	0.000000
3	quad f	QUADR	0.2500	0.000000	0.000000	0.00000	0.00000	4.182160	0.000000
4	drift 2	DRIFT	0.2525	0.000000	0.000000	0.00000	0.00000	0.000000	0.000000
5	quad d	QUADR	0.2000	0.000000	0.000000	0.00000	0.00000	-3.976880	0.000000
6	drift 3	DRIFT	0.2500	0.000000	0.000000	0.00000	0.00000	0.000000	0.000000
7	bend	RBEND	1.0470	0.314159	0.000000	0.15708	0.15708	0.000000	0.000000
8	drift 3	DRIFT	0.2500	0.000000	0.000000	0.00000	0.00000	0.000000	0.000000
9	quad d2	QUADR	0.1800	0.000000	0.000000	0.00000	0.00000	-3.976880	0.000000
10	drift 2	DRIFT	0.2525	0.000000	0.000000	0.00000	0.00000	0.000000	0.000000
11	quad f	QUADR	0.2500	0.000000	0.000000	0.00000	0.00000	4.182160	0.000000
12	quad f	QUADR	0.2500	0.000000	0.000000	0.00000	0.00000	4.182160	0.000000
13	drift 2	DRIFT	0.2525	0.000000	0.000000	0.00000	0.00000	0.000000	0.000000
14	quad d2	QUADR	0.1800	0.000000	0.000000	0.00000	0.00000	-3.976880	0.000000
15	drift 3	DRIFT	0.2500	0.000000	0.000000	0.00000	0.00000	0.000000	0.000000
16	bend	RBEND	1.0470	0.314159	0.000000	0.15708	0.15708	0.000000	0.000000
17	drift 3	DRIFT	0.2500	0.000000	0.000000	0.00000	0.00000	0.000000	0.000000
18	quad d	QUADR	0.2000	0.000000	0.000000	0.00000	0.00000	-3.976880	0.000000
19	drift 2	DRIFT	0.2525	0.000000	0.000000	0.00000	0.00000	0.000000	0.000000
20	quad f	QUADR	0.2500	0.000000	0.000000	0.00000	0.00000	4.182160	0.000000
21	drift 1	DRIFT	1.5685	0.000000	0.000000	0.00000	0.00000	0.000000	0.000000
22		)) 10	0.0000	0.000000	0.000000	0.00000	0.00000	0.000000	0.000000
23			0.0000	0.000000	0.000000	0.00000	0.00000	0.000000	0.000000
24			0.0000	0.000000	0.000000	0.00000	0.00000	0.000000	0.000000

Fig. 13-1. The LATTICE EDITING window.

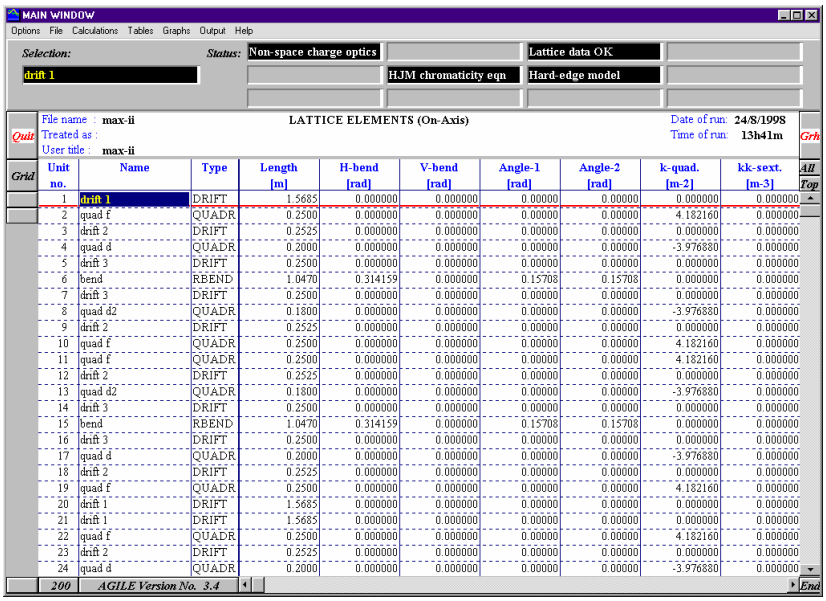
1. The name is arbitrary and invented by you.

2. The Type of magnet you enter should be one of the following:
   DRIFT Drift Space or straight section
   SEXTU Sextupole lens
   QUADR Quadrupole lens
   SBEND Sector bend with variable edge angles
   MPOLE Multipole thin lens of any order up to decapole (normal and skew components)
   SKEWQ Skew quadrupole
   SOLEN Solenoid with round ends or endplates with slots.
   RBEND Rectangular (SBEND with edge angles equal to half the bending angle)
   THINQ Thin quadrupole lens.

   Depending of which element you choose you should fill in the appropriate column (length etc.)
   If you have parts of the accelerator which are repeated you can try the options under "**Structures**".

3. Check the lattice editing by choosing
   Check_data → Check without decompression
4. And finally end the lattice editing by
   Check_data → Check with decompression
   (Observe that any repeated structures now will be expanded, which is a little confusing).

You should now have returned to the MAIN WINDOW (Fig. 13-2).



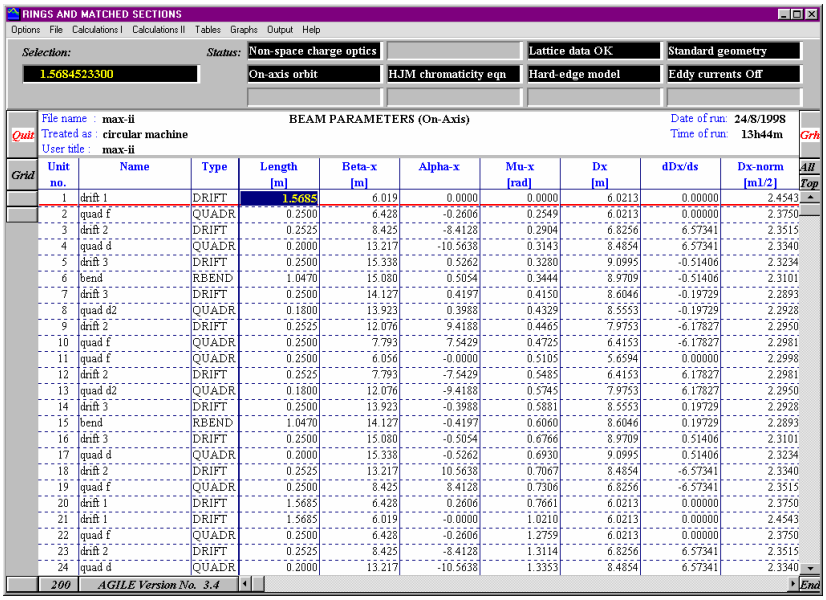
Unit no.	Name	Type	Length [m]	H-bend [rad]	V-bend [rad]	Angle-1 [rad]	Angle-2 [rad]	k-quad. [m-2]	kk-sext. [m-3]
1	drift 1	DRIFT	1.5685	0.000000	0.000000	0.00000	0.00000	0.000000	0.000000
2	quad f	QUADR	0.2500	0.000000	0.000000	0.00000	0.00000	4.182160	0.000000
3	drift 2	DRIFT	0.2525	0.000000	0.000000	0.00000	0.00000	0.000000	0.000000
4	quad d	QUADR	0.2000	0.000000	0.000000	0.00000	0.00000	-3.976880	0.000000
5	drift 3	DRIFT	0.2500	0.000000	0.000000	0.00000	0.00000	0.000000	0.000000
6	bend	RBEND	1.0470	0.314159	0.000000	0.15708	0.15708	0.000000	0.000000
7	drift 3	DRIFT	0.2500	0.000000	0.000000	0.00000	0.00000	0.000000	0.000000
8	quad d2	QUADR	0.1800	0.000000	0.000000	0.00000	0.00000	-3.976880	0.000000
9	drift 2	DRIFT	0.2525	0.000000	0.000000	0.00000	0.00000	0.000000	0.000000
10	quad f	QUADR	0.2500	0.000000	0.000000	0.00000	0.00000	4.182160	0.000000
11	quad f	QUADR	0.2500	0.000000	0.000000	0.00000	0.00000	4.182160	0.000000
12	drift 2	DRIFT	0.2525	0.000000	0.000000	0.00000	0.00000	0.000000	0.000000
13	quad d2	QUADR	0.1800	0.000000	0.000000	0.00000	0.00000	-3.976880	0.000000
14	drift 3	DRIFT	0.2500	0.000000	0.000000	0.00000	0.00000	0.000000	0.000000
15	bend	RBEND	1.0470	0.314159	0.000000	0.15708	0.15708	0.000000	0.000000
16	drift 3	DRIFT	0.2500	0.000000	0.000000	0.00000	0.00000	0.000000	0.000000
17	quad d	QUADR	0.2000	0.000000	0.000000	0.00000	0.00000	-3.976880	0.000000
18	drift 2	DRIFT	0.2525	0.000000	0.000000	0.00000	0.00000	0.000000	0.000000
19	quad f	QUADR	0.2500	0.000000	0.000000	0.00000	0.00000	4.182160	0.000000
20	drift 1	DRIFT	1.5685	0.000000	0.000000	0.00000	0.00000	0.000000	0.000000
21	drift 1	DRIFT	1.5685	0.000000	0.000000	0.00000	0.00000	0.000000	0.000000
22	quad f	QUADR	0.2500	0.000000	0.000000	0.00000	0.00000	4.182160	0.000000
23	drift 2	DRIFT	0.2525	0.000000	0.000000	0.00000	0.00000	0.000000	0.000000
24	quad d	QUADR	0.2000	0.000000	0.000000	0.00000	0.00000	-3.976880	0.000000

Fig. 13-2. The MAIN WINDOW.

13.4 To make a calculation

In the MAIN WINDOW choose:
Calculations → Ring or matched section

Basic data about the lattice you wrote before is calculated and displayed in the RING OR MATCHED SECTIONS window (Fig. 13-3).



Unit no.	Name	Type	Length [m]	Beta-x [m]	Alpha-x	Mu-x [rad]	Dx [m]	dDx/ds	Dx-norm [m1/2]
1	drift 1	DRIFT	1.5685	6.019	0.0000	0.0000	6.0213	0.00000	2.4543
2	quad f	QUADR	0.2500	6.428	-0.2606	0.2549	6.0213	0.00000	2.3750
3	drift 2	DRIFT	0.2525	8.425	-8.4128	0.2904	6.8256	6.57341	2.3515
4	quad d	QUADR	0.2000	13.217	-10.5638	0.3143	8.4854	6.57341	2.3340
5	drift 3	DRIFT	0.2500	15.338	0.5262	0.3280	9.0995	-0.51406	2.3234
6	bend	RBEND	1.0470	15.080	0.5054	0.3444	8.9709	-0.51406	2.3101
7	drift 3	DRIFT	0.2500	14.127	0.4197	0.4150	8.6046	-0.19729	2.2893
8	quad d2	QUADR	0.1800	13.923	0.3988	0.4329	8.5553	-0.19729	2.2928
9	drift 2	DRIFT	0.2525	12.076	9.4188	0.4465	7.9753	-6.17827	2.2950
10	quad f	QUADR	0.2500	7.793	7.5429	0.4725	6.4153	-6.17827	2.2981
11	quad f	QUADR	0.2500	6.056	-0.0000	0.5105	5.6594	0.00000	2.2998
12	drift 2	DRIFT	0.2525	7.793	-7.5429	0.5485	6.4153	6.17827	2.2981
13	quad d2	QUADR	0.1800	12.076	-9.4188	0.5745	7.9753	6.17827	2.2950
14	drift 3	DRIFT	0.2500	13.923	-0.3988	0.5881	8.5553	0.19729	2.2928
15	bend	RBEND	1.0470	14.127	-0.4197	0.6060	8.6046	0.19729	2.2893
16	drift 3	DRIFT	0.2500	15.080	-0.5054	0.6766	8.9709	0.51406	2.3101
17	quad d	QUADR	0.2000	15.338	-0.5262	0.6930	9.0995	0.51406	2.3234
18	drift 2	DRIFT	0.2525	13.217	10.5638	0.7067	8.4854	-6.57341	2.3340
19	quad f	QUADR	0.2500	8.425	8.4128	0.7306	6.8256	-6.57341	2.3515
20	drift 1	DRIFT	1.5685	6.428	0.2606	0.7661	6.0213	0.00000	2.3750
21	drift 1	DRIFT	1.5685	6.019	-0.0000	1.0210	6.0213	0.00000	2.4543
22	quad f	QUADR	0.2500	6.428	-0.2606	1.2759	6.0213	0.00000	2.3750
23	drift 2	DRIFT	0.2525	8.425	-8.4128	1.3114	6.8256	6.57341	2.3515
24	quad d	QUADR	0.2000	13.217	-10.5638	1.3353	8.4854	6.57341	2.3340

Fig. 13-3. The RING OR MATCHED SECTIONS window

Further results can be found in the GRAPHS menu:
Graphs → Plan view geometry
Graphs → Twiss parameters
Graphs → Normalised dispersion

13.5 Kick and Closed orbit

To introduce a kick in the lattice you can proceed in several ways. First you have to define the kick, which is done in a KICK FILE EDITING window. A kick is introduced at the beginning of any chosen element in your lattice.

1. To get to the KICK FILE EDITING window from the MAIN WINDOW choose:
   File → Edit and enter kicks → Dipole
2. And from the RING OR MATCHED SECTIONS window choose:
   Calculations 1 → closed orbit → (choose "prepare dipole kick data" (Fig. 13-4))

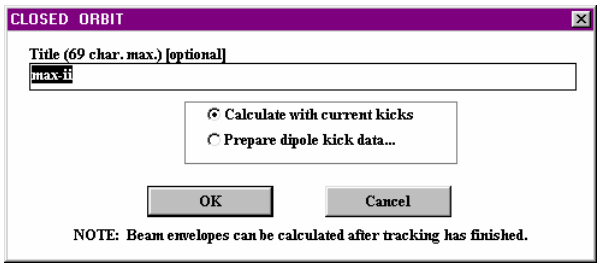


Fig. 13-4. Sub-window to start closed orbit calculations.

3. To proceed from the KICK FILE EDITING window choose:
   Options → Back to main window
4. Go to the RING OR MATCHED SECTIONS window by:
   Calculations → ring or matched sections
5. Caculate the Closed orbit with a kick by:
   Calculations 1 → closed orbit → (choose "calculate with current kicks" (Fig. 13-4))
6. See the result by:
   Graph → Closed orbit

13.6 Tune fitting

To fit a tune choose from the RING OR MATCHED SECTIONS window:

1. Calculations I → Fit tunes/phase advances
2. Change the value of the tunes you desire to calculate, and mark which elements that are allowed to change to achieve your new desired tunes (Fig. 13-5).
3. Compute the new values, and if you desire, accept them, or reset original values.

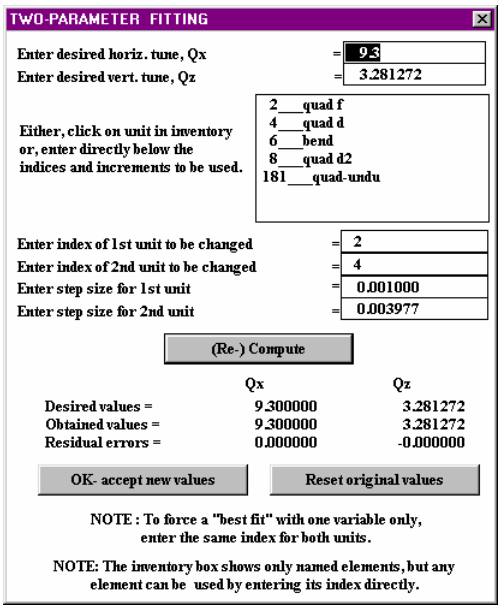


Fig. 13-5. Sub-window for tune fitting.

14 References