Status of Normal Conducting Coupler development at INFN-LNL

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Overview

- INFN-LNL Commitment in High-Intensity Linacs: TRASCO RFQ, IFMIF RFQ, ESS DTL
- Coupler design, development and tests for TRASCO RFQ
- Coupler design, development and tests for IFMIF RFQ
- Coupler design for ESS DTL
- Conclusions

The RFQs developed at LNL: TRASCO

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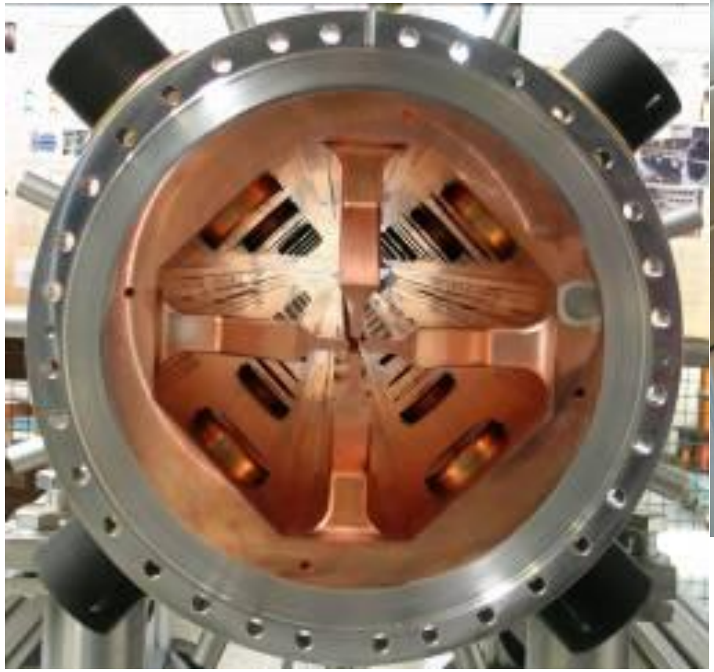
Completed modules of the RFQ

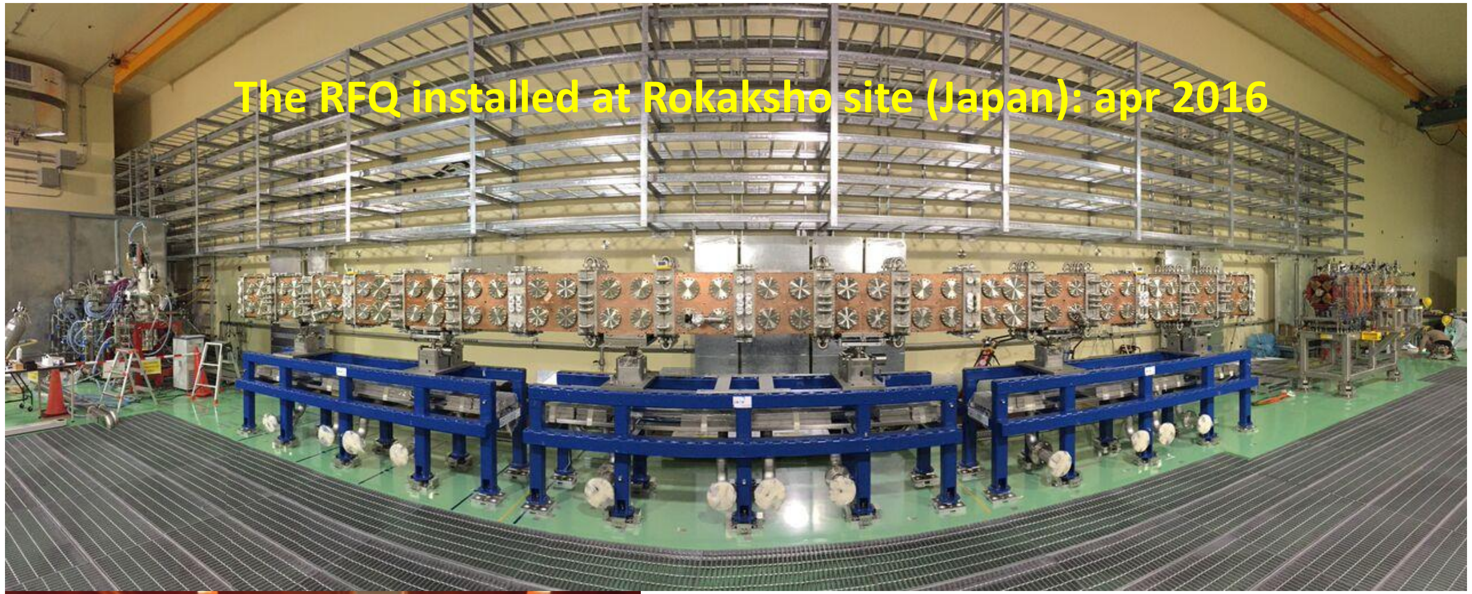
	TRASCO
status	Constructed
particle	p
f [MHz]	352.2
l [m]	7.13
l/λ	8.4
R_0[mm]	2.93-3.07
r/R_0	1
Ib [mA]	30
V[kV]	68
W [MeV/u]	5
E.M. segments	3
Mech. modules	6
N. of Couplers	8 (loop type)
Q_0	8000 (20% margin)
RF power [kW] (CW)	800 (20% margin)

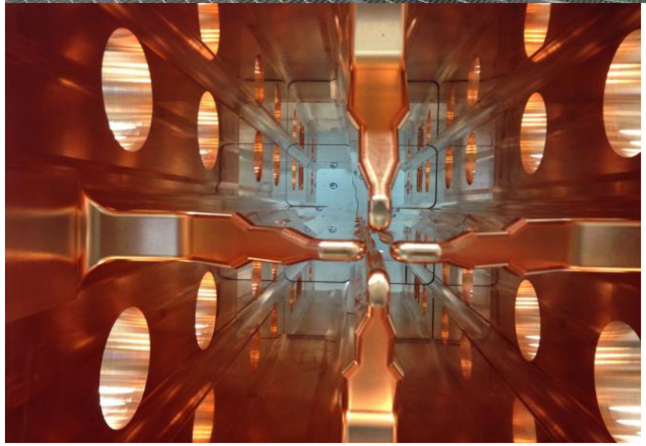


RFQ Inside view: the bulk is in OFE Copper, and the flange in LN316 SS

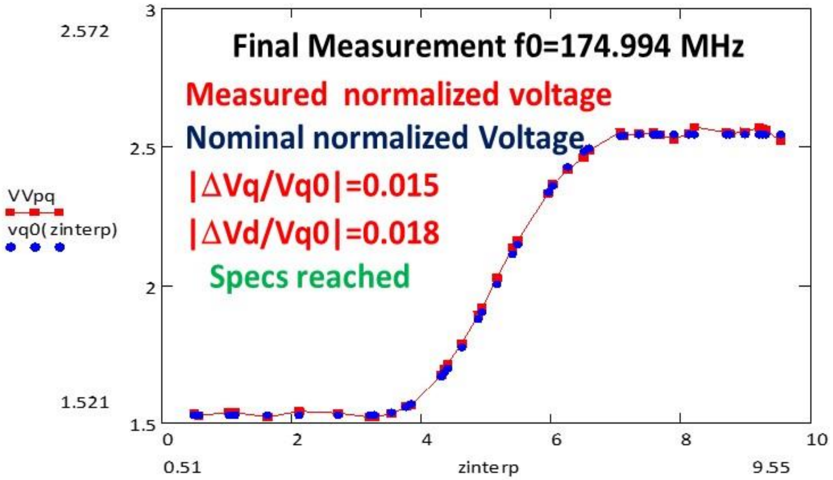
RFQ during High Power Tests @ Saclay (France)

The RFQs developed at LNL: IFMIF





RFQ Inside view: the bulk is in OFE Copper, and the flanges are in LN316 SS



RFQ tuning with dummy tuners: results

	IFMIF
Status	installed in Rokkasho site (Japan)
Particle	d
f [MHz]	175
l [m]	9.8
l/λ	5.7
R_0[mm]	4.13-7.10
r/R_0	0.75
Ib [mA]	125
V[kV]	79-132
W [MeV/u]	5
E.M. segments	1
Mech. modules	18
N. of Couplers	8 (loop type)
Q_0	12000 (25% margin)
RF power [kW] (CW)	1250(25% margin)

The ESS DTL

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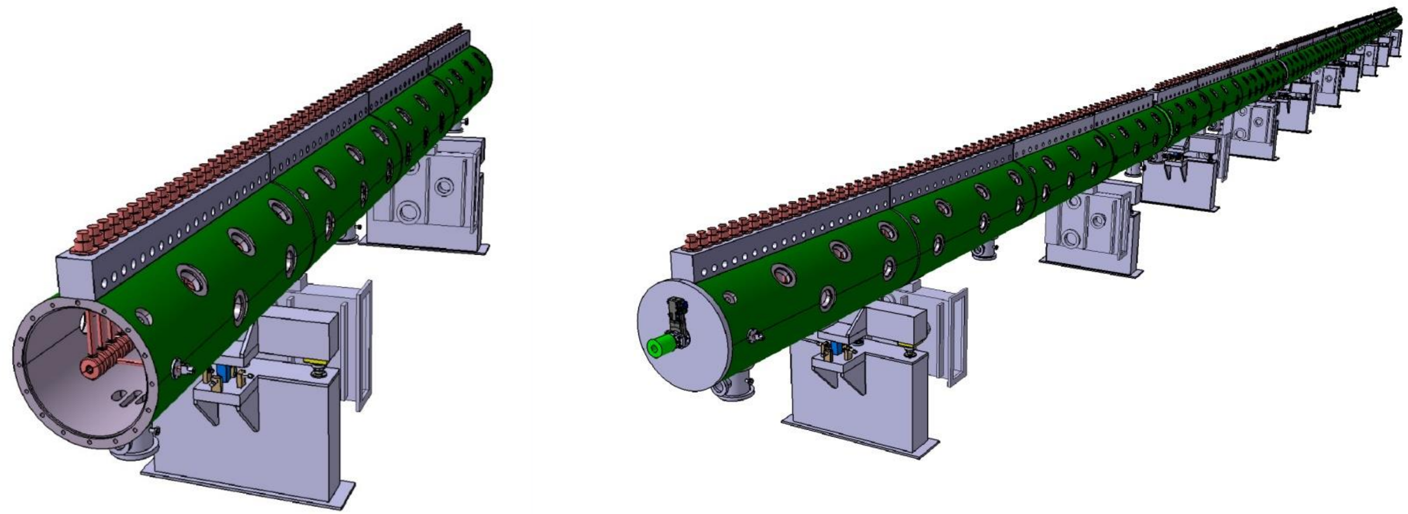
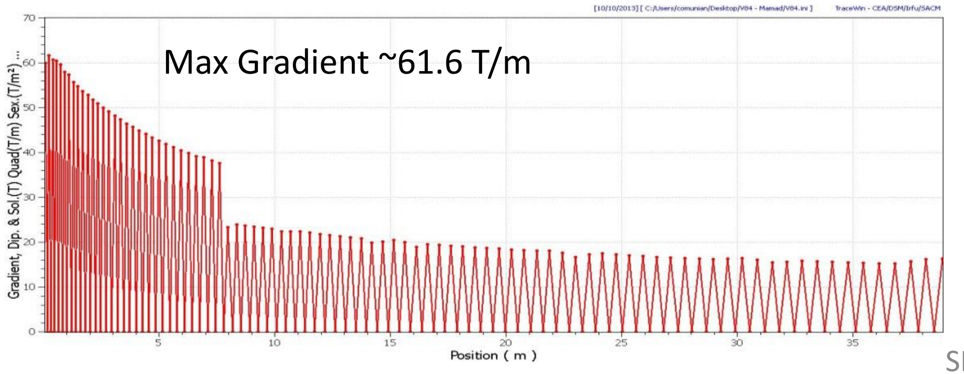
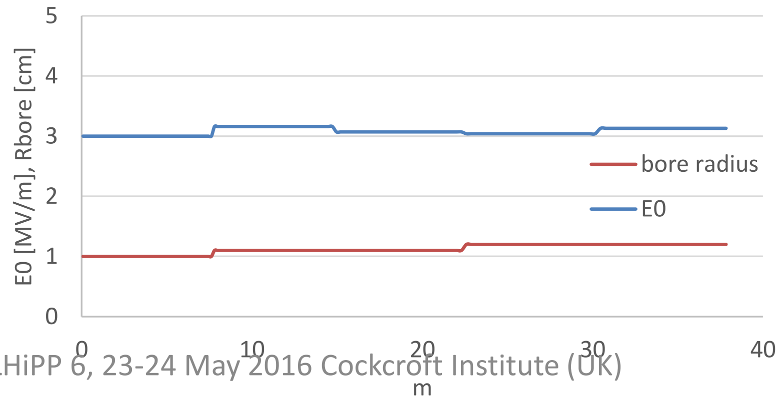
The ESS DTL (Drift Tube Linac) cavity is constituted of 20 modules, assembled in 5 tanks, composed of 4 modules each, for a total length of approximately 40 m.

One 2.9 MW klystron feeds each DTL tank. 30% of this power is put aside for waveguide losses and LLRF regulation. The remaining 2.2 MW enter in the cavity through 2 iris couplers located at 1/4 and 3/4 of the tank length to minimize the induced field perturbation.

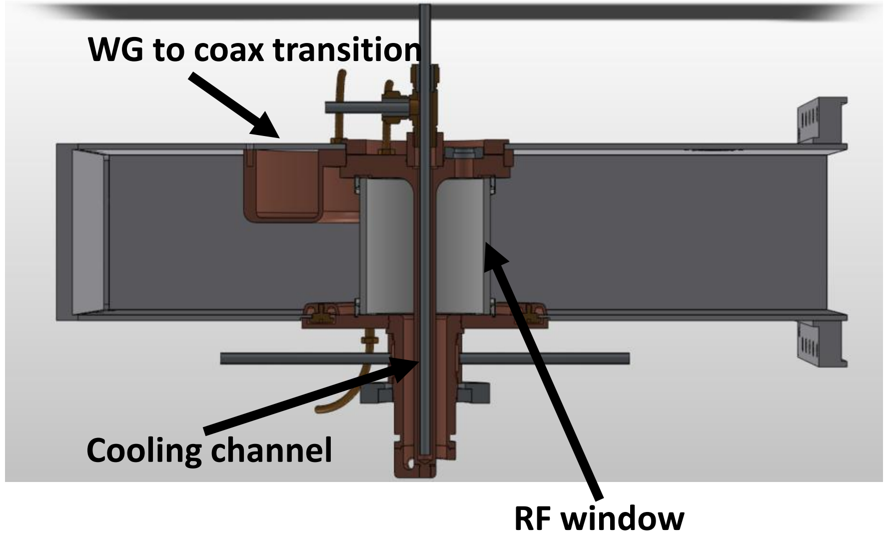






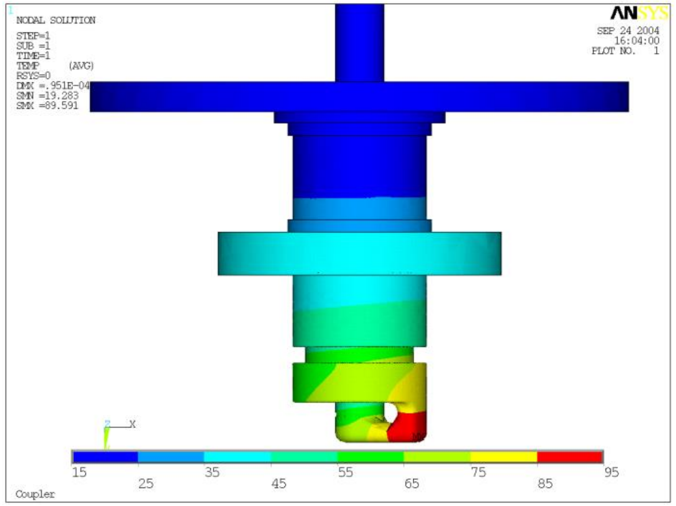
	ESS
Status	Prototyping and tendering
Particle	p+
f [MHz]	352.21
l [m]	8m x 5 tanks
l/λ	9.3
Ib [mA] (2.86 ms pulse,14 Hz rep. rate)	62.5 mA
Eacc [MV/m]	≈3
Focusing	FODO
W [MeV/u]	3.62MeV- 90 MeV, in 5 tanks
E.M. segments	1 x 5 tanks
Mech. modules	4 per tank
N. of Couplers	2 per tank
Q0	42000 (25% margin)
RF power [kW] (peak+Pbeam)	2200 (25% margin on Pcu)

TRASCO RFQ High Power Coupler: design

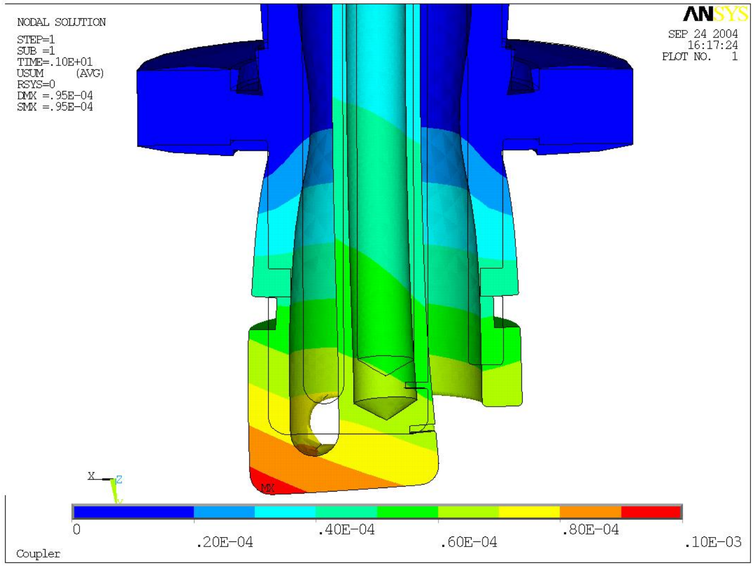


Main challenges:
- CW operation @ 140 kW/coupler power level
- Compact dimensions of the loop to fit 57 mm diameter RFQ port

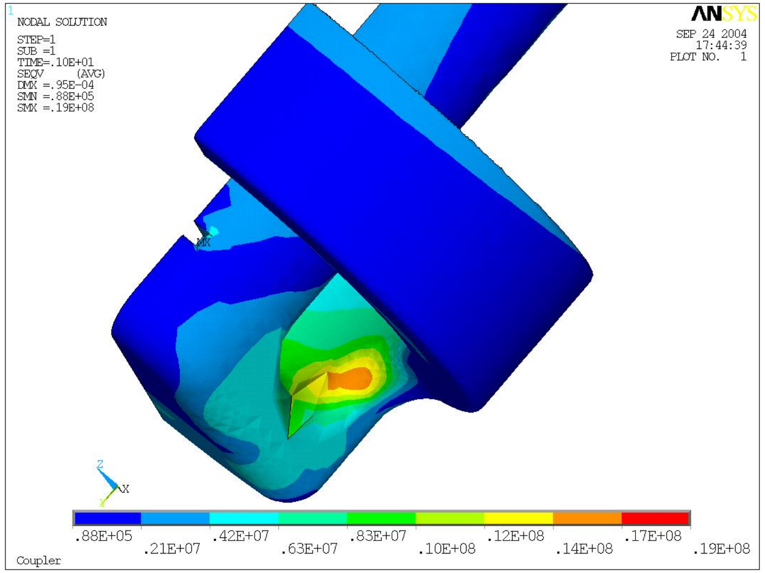
Design chioces:
- Design inspired to LEP NC couplers
- Optimization of loop fileting in order to minimize RF power density (max value 25 W/cm^2)
- Cylindrical Alumina RF windows (LEP kind)
- Spring Cu-Ag RF joint (LA-CUD by Multicontact®)



Temperature distribution (max value 95°C)



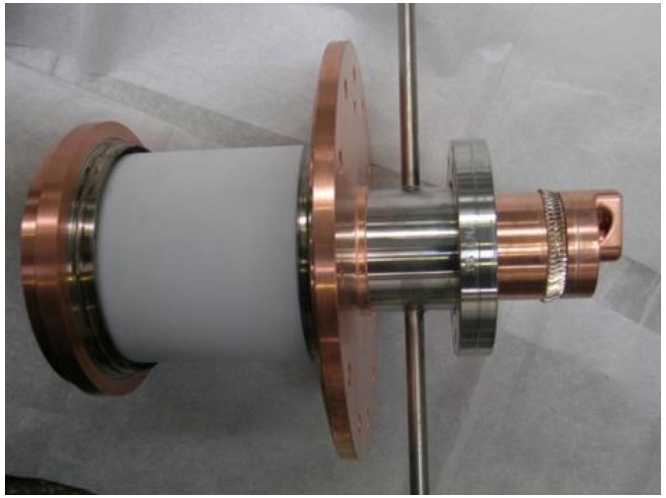
Deformation Map (max value 0.1 mm)



Von Mises Stress (max value 18 MPa)

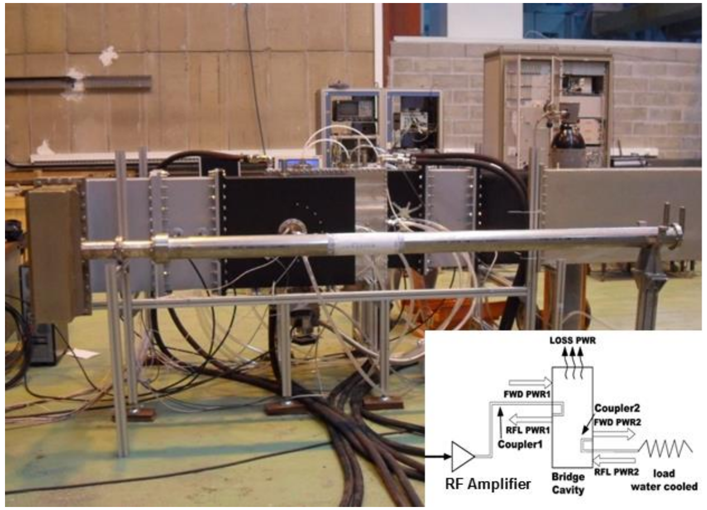
TRASCO RFQ High Power Coupler: construction and test

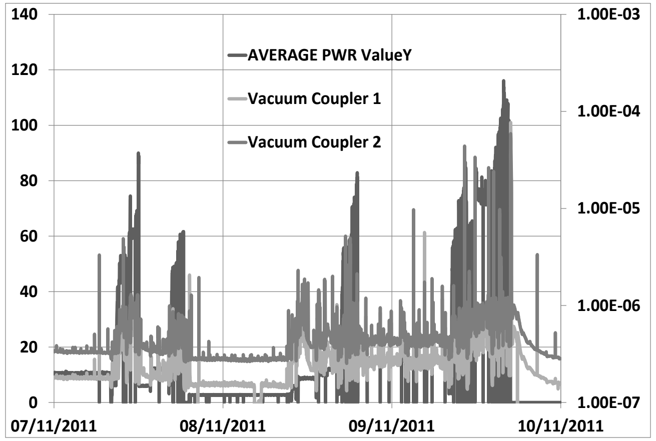
Construction



The inner conductor of the loop is cooled via a coaxial SS tube and the outer conductor with a cooling SS sleeve. In particular, the CU OFE (inner coaxial, outer coaxial and drive loop) and the LN316 (flange and cooling sleeves) sub-assemblies were constructed and cleaned separately and then brazed together in a single brazing step. Prior to the brazing, the dimensions of the loop were determined after a set of measurements performed on the first two modules of the RFQ with on-purpose built aluminium dummy couplers. Finally, the cylindrical RF alumina windows were TIG welded on the coupler body via the Kovar ring.

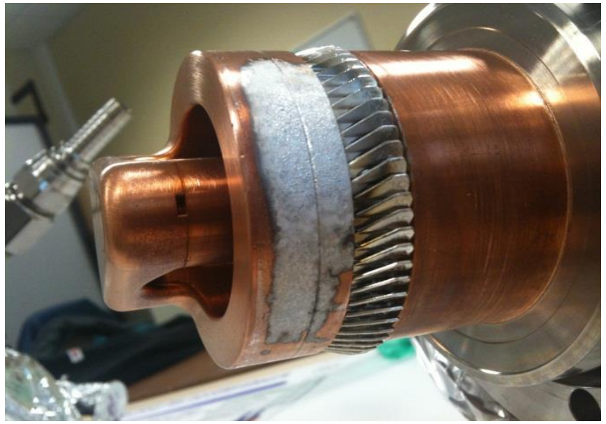
High Power Tests @ CEA-Saclay (May-Nov 2011)





- Two Couplers connected via a Aluminum bridge waveguide cavity (30 % power dissipated in the cavity), one of them connected to the amplifier, the other one to a 100 kW load
- Initial tests performed al LNL on a 10 kW Solid State Amplifier
- In the first step, one of the couplers was conditioned up to 140 kW (requested power for RFQ tests 100 kW) and the second one at 100 kW. In the second step the couplers configuration was reversed and the second couplers was conditioned up to 140 kW
- conditioning rate paced by the vacuum level or multipacting, which were the driving terms for pulse length and/or Rep. Rate. Multipacting level encountered at 400 W, 2kW and in the 90 kW region, as foreseen by calculations (10% error)
- Nominal power reached and kept for some hours on November 2011 (vacuum level in the 10^{-7} mbar scale)

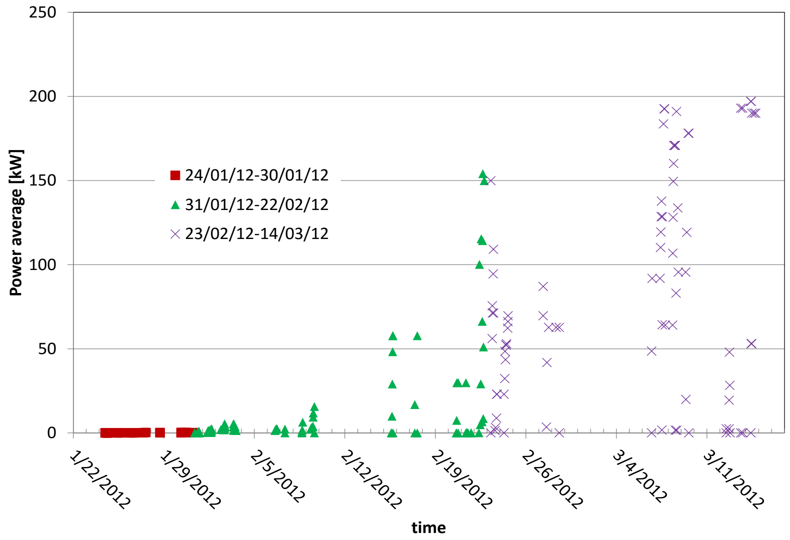
TRASCO RFQ High Power Coupler: checks and High Power conditioning on the RFQ



Coupler checks after RF Conditioning

Strong aluminum sputtering on external surface. A melted O-ring in the interface with Al cavity was responsible for the vacuum tightness very near to the zone where the problem appears. In the same point the RF seal is present. Bad vacuum seal due to the melted NBR O-ring caused some gas to penetrate in the zone of the RF seal. During conditioning, electric field penetrated in a region with poor vacuum, caused by the damaged O-ring, in which free electrons generated by aluminum oxide as well as brazing material in the brazing groove were present. This fact generated sparks at high values of electric field and aluminum sputtering on the copper surface. In order to mitigate the problem, during conditioning of the reversed configuration, the vacuum threshold on the coupler was decreased and the recovery time after vacuum interlock was extended. With this new procedure, the conditioning of the second coupler got fast and smoother. In any case this O-ring interface is not present for RFQ.

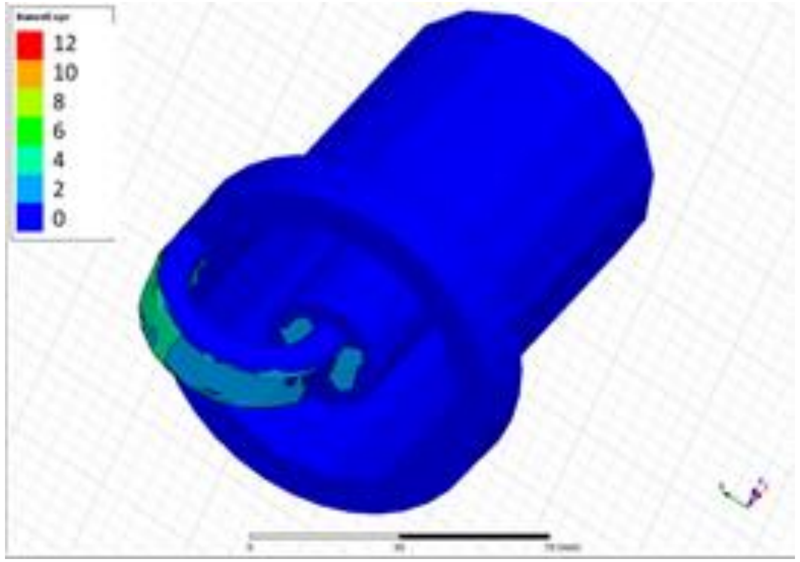
RFQ High Power Conditioning @ CEA-Saclay (France)

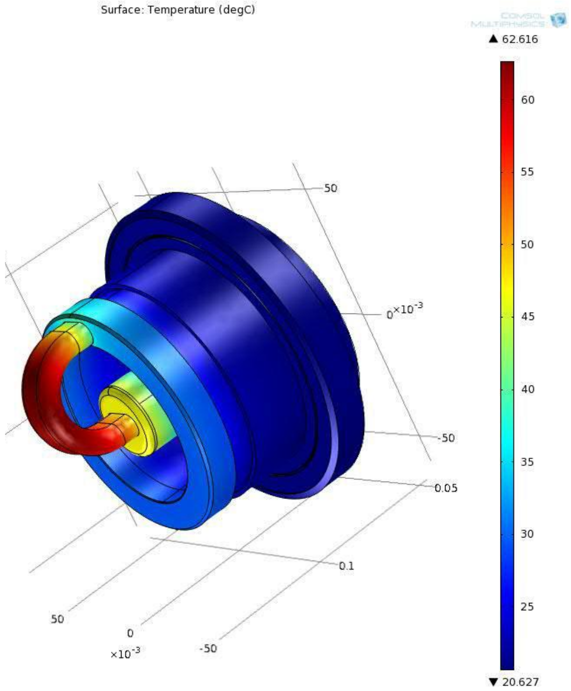


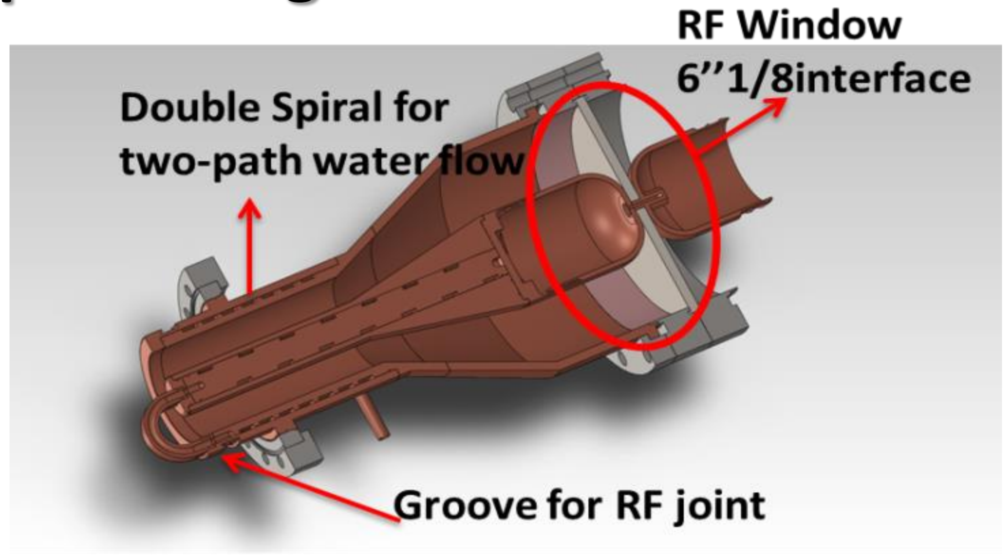
Cavity average power throughout the test

First electromagnetic segment of the TRASCO RFQ (two 1.2 m long OFE copper modules).
CEA 1.3 MW klystron, protected from the reverse power by a 1 MW circulator.
The RF power is led into the RFQ tunnel through full-height WR2300 waveguide and then it is tapered to half-height WR2300 for the final distribution to the RFQ. Just upstream the RFQ, the RF power is split by a magic-TEE: two waveguide arms are coupled into the RFQ through 2 coupling loops, the 4th arm goes to a 100 kW water load The unloaded quality factor Q0=8460 was measured in the RFQ equipped with two aluminum couplers. Once the copper couplers were installed, the sum of the coupling coefficients ($\beta 1+\beta 2$) = 1.036. The nominal voltage (68kV – 192 kW) in CW regime was reached in 07/03, and for 6 days the cavity was conditioned at this field with different duty cycles, always higher than 20 %.Test was considered successfully closed in March 14th, after a continuous running at nominal voltage for 2 h. Temperature monitors did not show anomalous heating in high power steady state operation.

IFMIF RFQ High Power Coupler: design





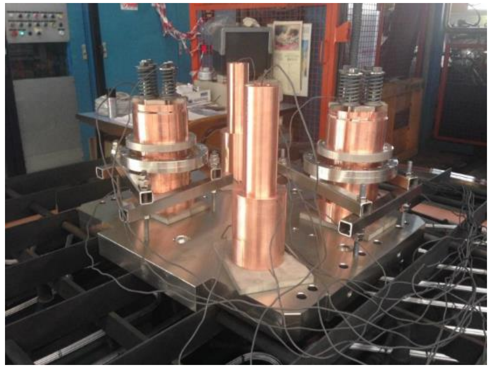


The High Power Coupler is designed to be critically coupled with the 2 meter long RFQ, in conditions of minimum acceptable Q_0 value of Q_{0min}=9000 (ideal Q_0=15000). Moreover, it must be properly cooled in order to manage its RF losses induced by the high power feeding the RFQ (up to 220 kW) and travelling on the RF coaxial line. The design of the loop therefore has a coaxial inner radius R=20 mm, an insertion depth of 29 mm and a thickness of 8 mm. The coupling was verified both with analytical calculations and with HFSS simulations. Should the Quality Factor be higher, a rotating flange can accomplish a proper change of the coupling. The power dissipated in the loop is about 100 W for an input power of 200 kW. The coupler and RF window are two separate devices coupled with a standard 6"1/8 IEC coaxial interface. In particular, the Coupler material is copper OFHC, and the water heat exchange coefficient hc is maintained near 10000 W/m^{2}K on the whole heat exchange surface: water velocity is above 2.5 m/s both in the external and internal spiral and in the loop. The coupler inner connection is used to remove power from the RF window. The RF window is a planar type window, purchased from MEGA Industries (USA). The material used is Alumina 99% with 1.5 to 3 nm TiN coating, with nominal RL of 40 dB and Insertion Loss of less than 0.01 dB.

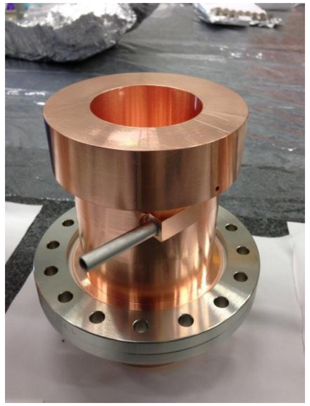
IFMIF RFQ High Power Coupler: construction

The construction procedure foresees three brazing steps. In the first step, the cooling spirals are brazed separately on inner and outer conductors and also the SS flange seats and the water tubes are brazed on the copper bulk. In the second step, the plugs are brazed at both ends of the inner conductor and the outer conductor with cooling spiral is brazed with the tapered coaxial. Finally, in the third step the inner and outer conductors are brazed to the loop, and the assembly is completed. After both the first and the second brazing step, machining is required and brazing defects can be eventually recovered.

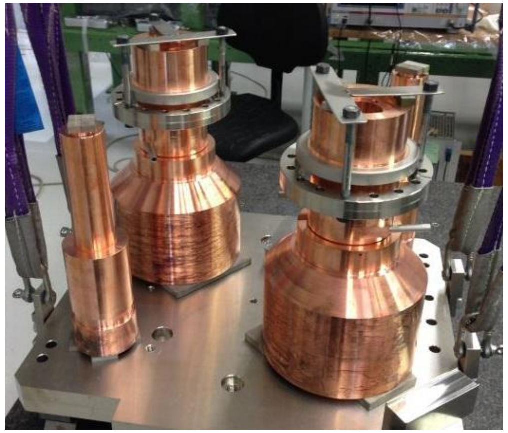
The construction was carried out by CINEL Strumenti Scientifici, Vigonza (PD), while the brazing and thermal treatments were performed at the LNL brazing facility.



1 Pre 1st braze assembly



2 After1st braze



3 After 2nd braze

4 After 3rd braze

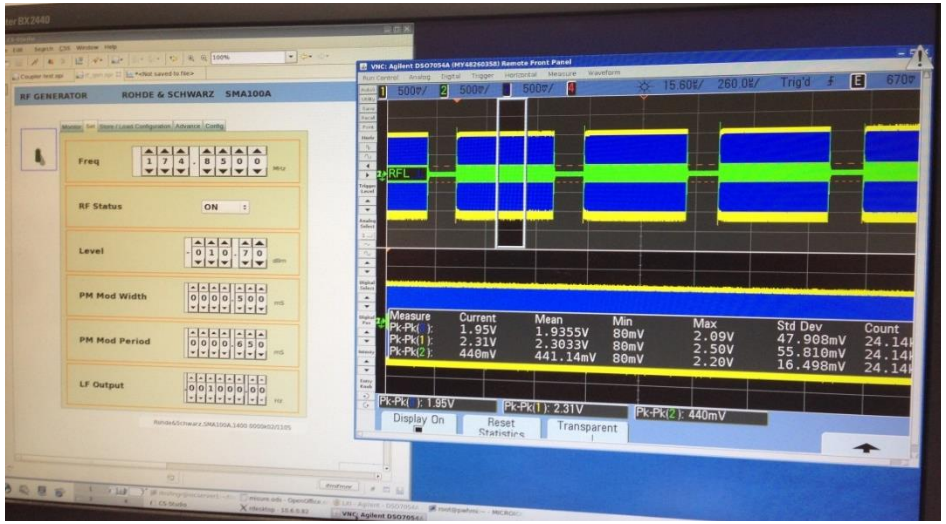
5 Assembly of the Coupler and the RF Window

IFMIF RFQ High Power Coupler: High Power Test Stand

The High Power Test Stand consists of the High Power Amplifier, (with its associated Power Supplies and cooling system), the 6”1/8 to 9”3/16 adapter placed immediately after the amplifier output, the 9”3/16 coaxial waveguide, the 9”3/16 to 6”1/8 adapter, a straight 6”1/8 line, the couplers along with the coupling cavity (with their associated cooling systems) and a High Power water cooled load, capable of withstanding up to 200 kW absorbed power (courtesy of Japanese team). In particular one of the couplers acts as a power feeder, while the other coupler, connected to the load, acts as a receiver. The Test Stand is completed by the Vacuum System (based on TCP256 turbomolecular pump) and by the Diagnostic/Control System.

IFMIF RFQ High Power Coupler: High Power Tests

- Prior to High Power Conditioning, the couplers and the cavity were baked out under vacuum up to the temperature of 95°C, for 2 days.
- on 2014 May 26th, the RF conditioning started.
- As a general rule, the conditioning rate was paced by the vacuum level or multipacting. As the duty cycle was increased, the presence of vacuum instabilities (typically above 10^{-6} mbar) drove the maintenance of the pulse length and rep rate up to re-establishment of the baseline vacuum level (up to $7*10^{-7}$ mbar depending on the average RF power).
- When the vacuum instability was persistent, typically the pulse length was decreased in order to limit the outgassing activity before proceeding with higher power levels.
- Conditioning started with τ=20 ms and T=200 ms with a few kW peak power level, and the peak power level was then progressively increased up to the nominal value. At this point the duty cycle was increased by acting first on the pulse duration, then to the repetition rate.
- When performing an increase of Duty Cycle, the drive signal output power was typically first reduced then little by little increased. During this process, some multipacting levels were encountered in the regions between 40 and 80 kW and 120 and 160 kW. Moreover, due to some overheating, the tuner on the coupling cavity was removed, and the operational frequency increased of about 300 kHz, still within the amplifier bandwidth.
- On June 4th the duty cycle reached 100% with about 150 kW input power, and later on the nominal 200 kW power value at CW was reached and maintained for about 2 hrs. Then in the following days the conditioning continued, although not around the clock, and up to the 13th of June, an integrated time of about 72 hrs at 200 kW power in CW was collected (f=175.22 MHz).
- During the conditioning, the maximum value of the temperature read on each coupler did not exceed 39°C and the temperature read on the aluminium cavity was equal to 33ºC, exactly the same value predicted by thermo-structural calculations..



IFMIF RFQ High Power Coupler: High Power Tests on the RFQ

A unloaded quality factor Q_0=12500 was measured by VNA in the RFQ equipped final coupler, the coupling coefficient β being adjusted by coupler rotation (about 32°) to get the critical coupling with a return loss > 50 dB (Q_L = 6250). This value is confirmed by the loaded cavity filling time τ_L=11.6μs. With that value of Q_0, the nominal voltage of 132 kV corresponds to a cavity power P_{cav} = 173 kW (86 kW/m).

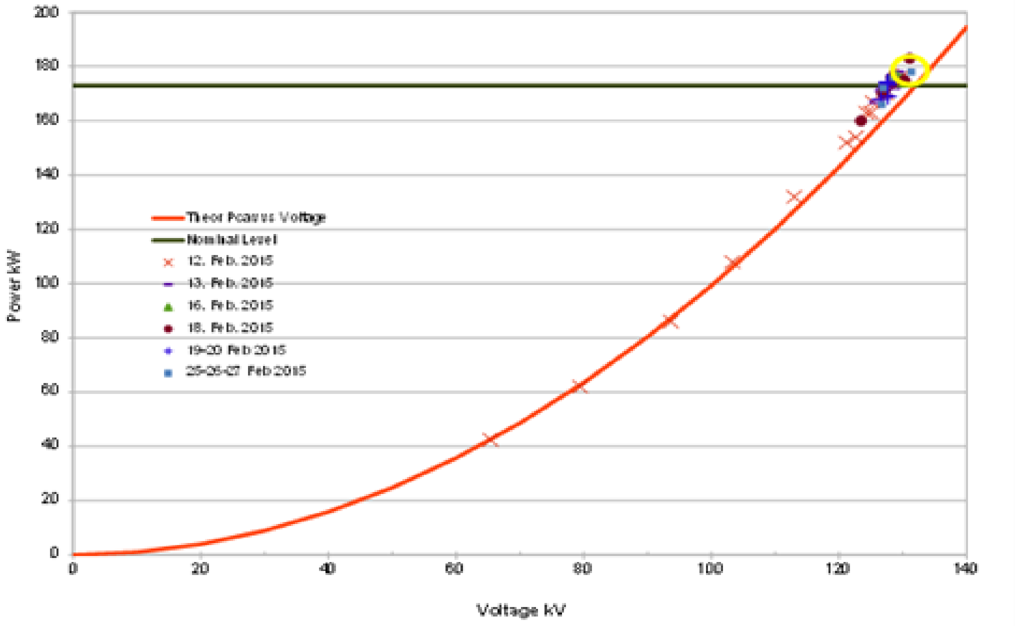
- RFQ cavity: the last 3 modules of the IFMIF RFQ (module 16-17-18) and Prototype 2 as boundary element on the low energy end. Cavity Length = 2.021m (Figure 1)
- RF power system: 175 MHz – 220 kW CW amplifier based on tetrode TH781, coaxial wave-guide line, AFT circulator from CIEMAT (Spain)
- Vacuum system: rough dry pump and turbo pump as pre-vacuum stadium, a cryogenic pump as main pump and a ionic pump as back-up in case of failure or purge of the cryo-pump.
- Cooling system: water skid with 2 independent cooling circuit (warm circuit and cold circuit), to finely tune the resonant frequency by temperature regulation.
- Control system: based on EPICS and PLC technology.

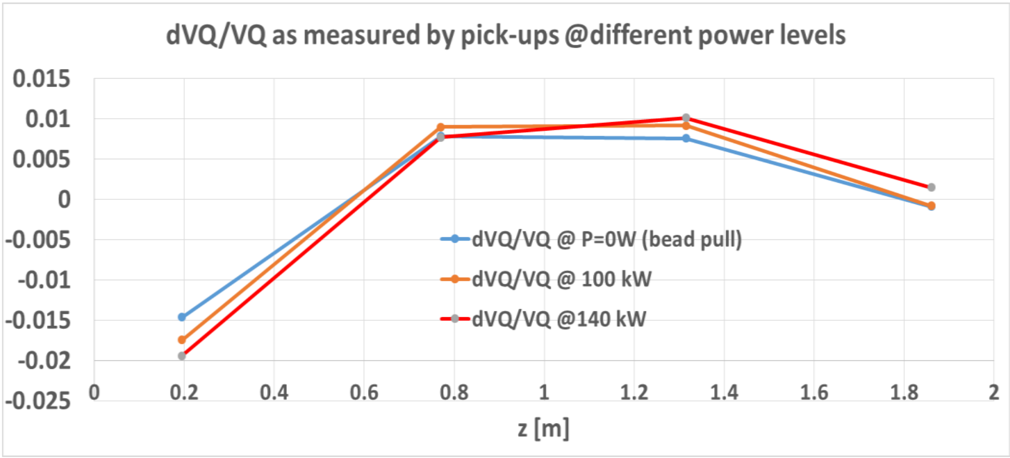
IFMIF RFQ High Power Coupler: High Power Tests on the RFQ (2)

In the conditioning phase the RFQ was let free to be detuned, and the master frequency generator followed the RFQ resonant frequency.

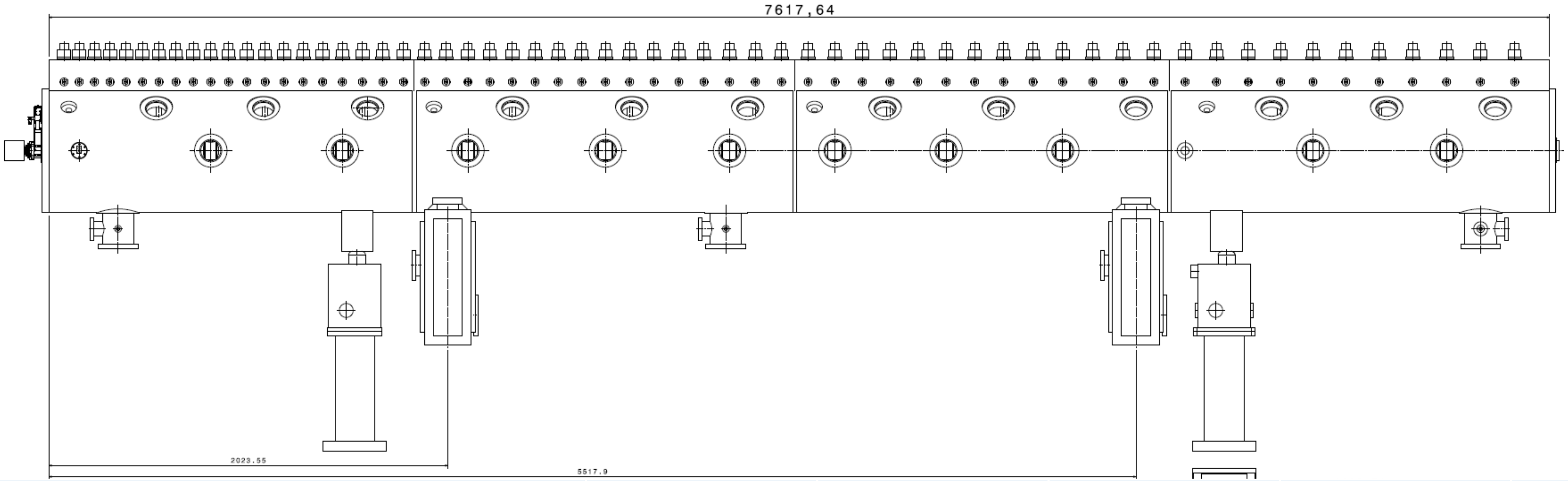
Three main phases of the RFQ conditioning:

- 28/11/14 to 12/12/14, RFQ was conditioned at low duty cycle (<1%), in order to calibrate RF measurements and distinguish multipacting levels.
- 18/12/14 to 09/01/15 (Christmas vacation excluded) was dedicated to cavity conditioning at medium-high duty cycle (Range 10-80%), sometime exceeding nominal peak power (max 192 kW-139 kV at 50% duty cycle).
- 21/01/15 to 27/02/15 (with 2 weeks of stop from 26/01 to 11/02): the RFQ was kept at 100% duty cycle. In this period we also set up the calorimetric measurement of RFQ cavity power, in order to benchmark the RF measurements. At power <100kW, the measured power and voltage corresponds to the theoretical curve, while at higher power level the measurements are above the nominal curve, meaning that a fraction of the power is not converted in accelerating voltage due to electron loading.
- The nominal voltage (132kV – 180 kW) in CW regime was reached on 2015 February 18th, but for less than 2 hours. The test was considered successfully closed in February 27th, after continuous operation at nominal voltage for 5 h.





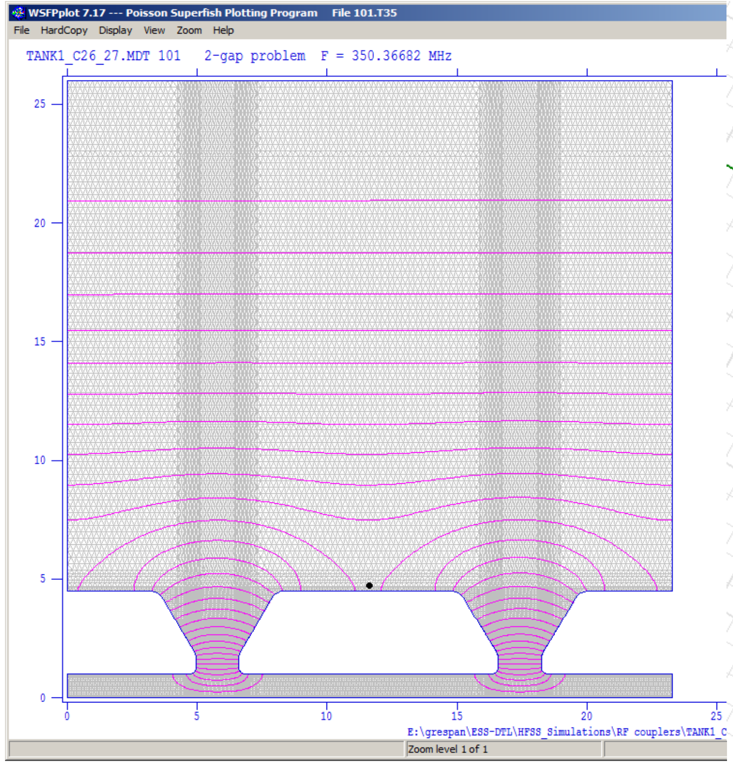
ESS DTL couplers

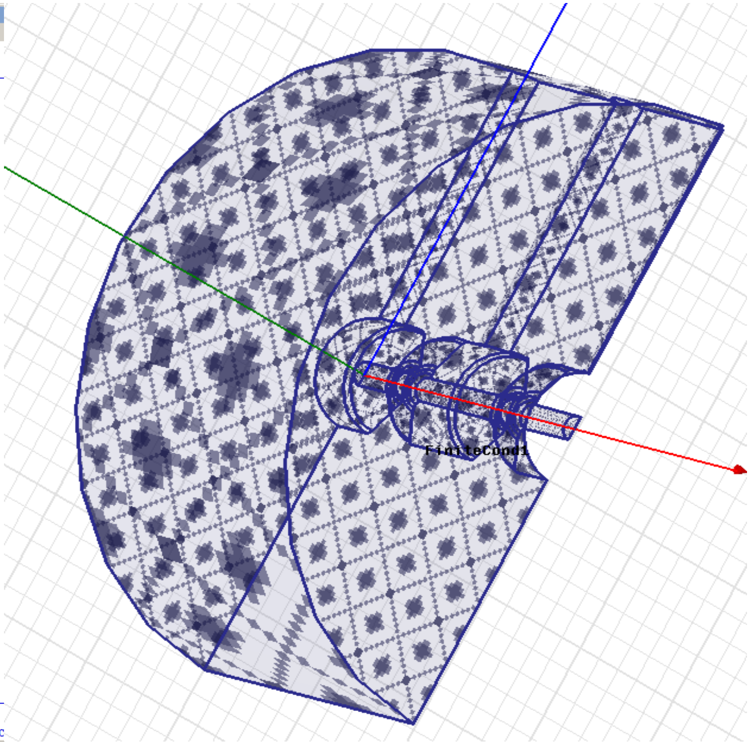


Parameter/Tank	1	2	3	4	5
Peak P_{cu} [MW] (1.25 margin)	1087	1077	1090	1126	1190
Pbeam [MW]	1103	1113	1105	1024	1004
Optimum coupling	2.01	2.03	2.01	1.91	1.84
RF coupler ports	2	2	2	2	2
Q0 (margin 1.25)	42512	44455.6	44345	43895	43416
Optimum detuning [MHz]	352.2123	352.2120	352.2120	352.2118	352.2118
3dB bandwidth (0.5 power)	±12.5 kHz	±12 kHz	±12.0 kHz	±11.7 kHz	±11.5 kHz
1dB bandwidth (0.8 power)	±6.2 kHz	±6 kHz	±6.0 kHz	±5.8 kHz	±5.7 kHz
Filling Time [micros]	12.7	13.2	13.3	13.6	13.8
Required Pgen/(Pcu+Pbeam) in case of detuning=0	1.021	1.016	1.016	1.014	1.014

ESS DTL Couplers: RF design

- Cells 26-27 and 8 are simulated because of their location just below the iris coupler.
- Check the coherence of 2D and 3D simulations (frequency, Q0).
- The power dissipated into the geometry calculated by Superfish, the simulation results are scaled to the real power dissipation (2.2MW → Beam seen as a further resistive load)





	MDT Fish Data	HFSS
	T1_C26_27	T1_C26_27
P tot kW	47.13	
Q0	30166	30591
f0 -MHz	351.764	351.563

P cells 26-27	47.1	kW	MDTFish Pcopper
P tank	2200	kW	Pbeam+Pcopper
rescaled conductivity	106337	S·m	

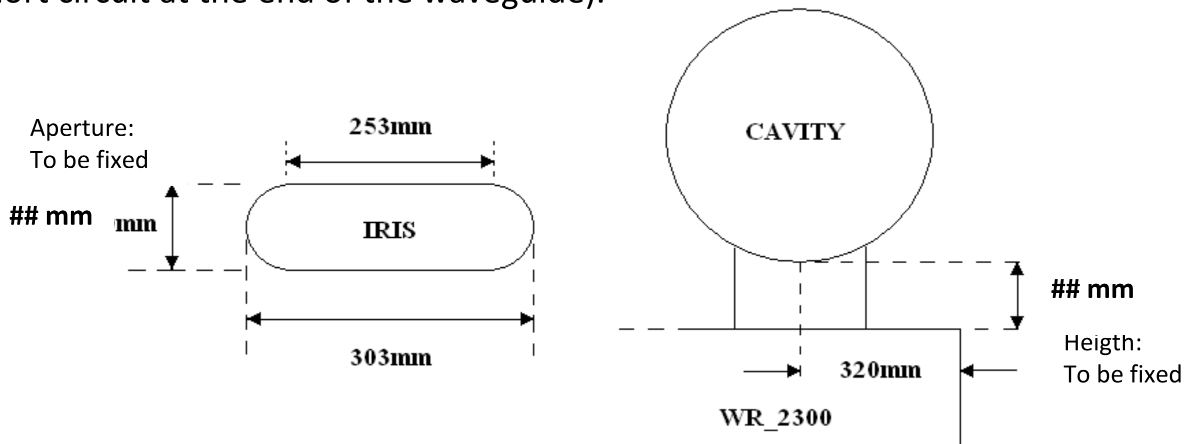
$$\text{Rescaled conductivity} = 5.8e^{7}\left(\frac{2 \times 47.1\ kW}{2200\ kW}\right)^{2}$$

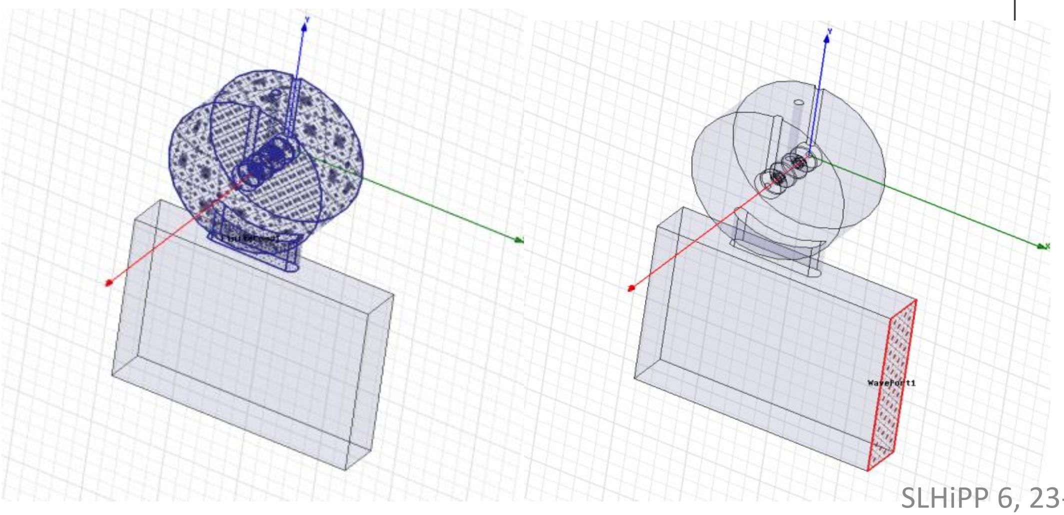
or rescaled beta coefficient

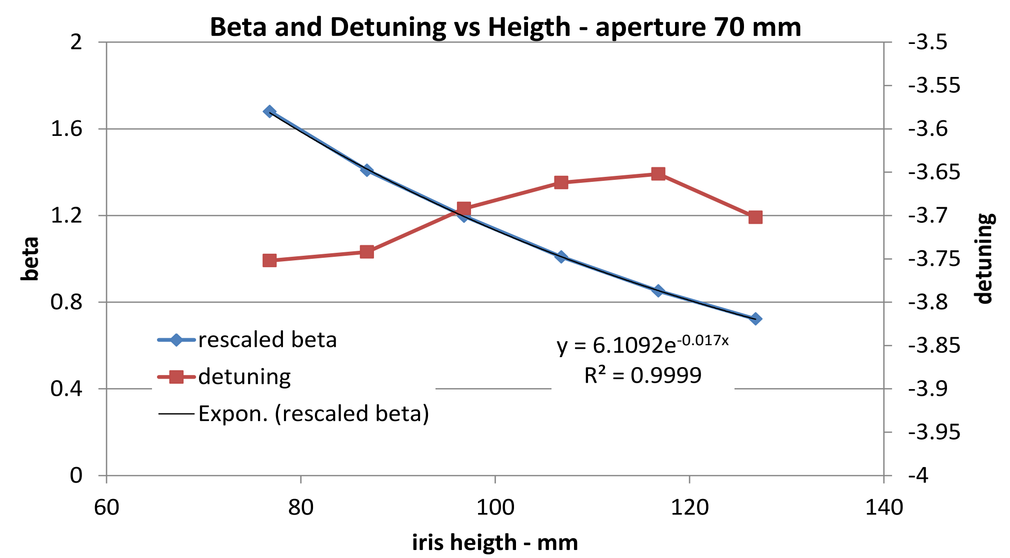
$$\beta_{tank} = 2 \times \beta_{cell26-27}\,\frac{47.1\ kW}{2200\ kW}$$

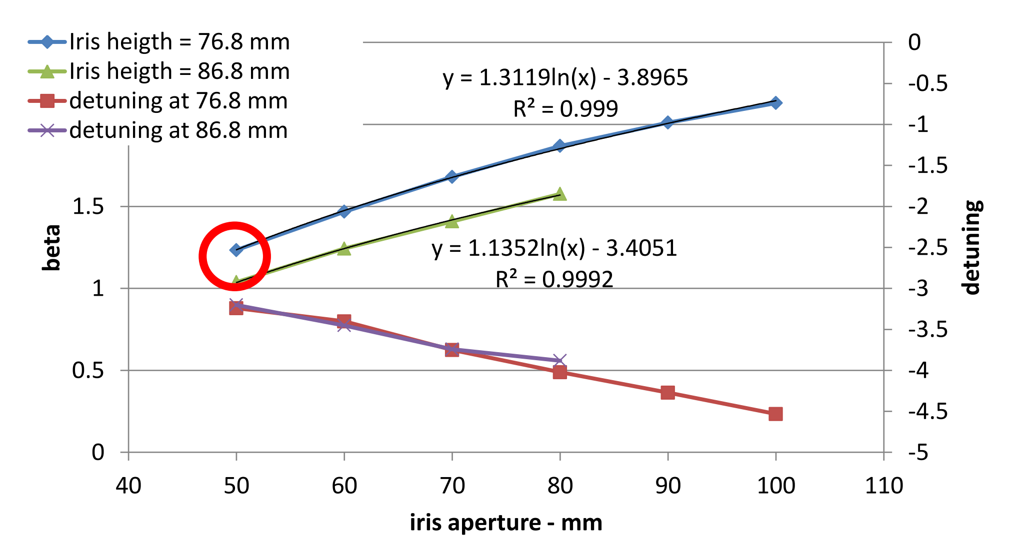
ESS DTL Couplers: RF design

The coupling strength β is optimized in order to critically couple the waveguide and the beam loaded cavity. Different sizes of iris aperture and iris height have been simulated with HFSS in a simplified geometry and then rescaled to the total power of 2.2.MW (Figure 5 and 6). The iris height and aperture allow a coupling strength β=1.2 (20% margin that can be adjusted by shifting the short circuit at the end of the waveguide).









ESS DTL Couplers: nominal dimensions and detuning

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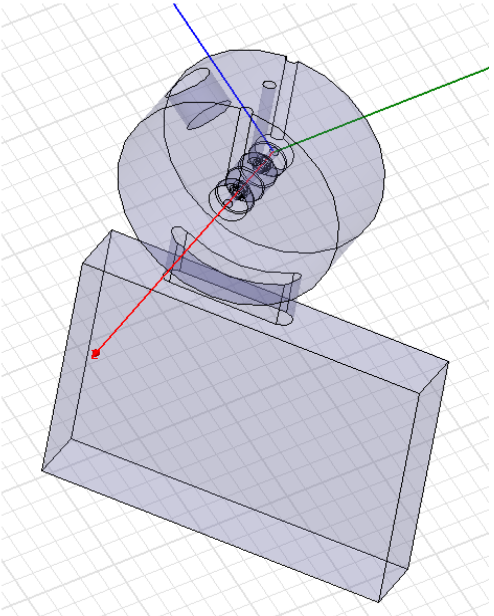
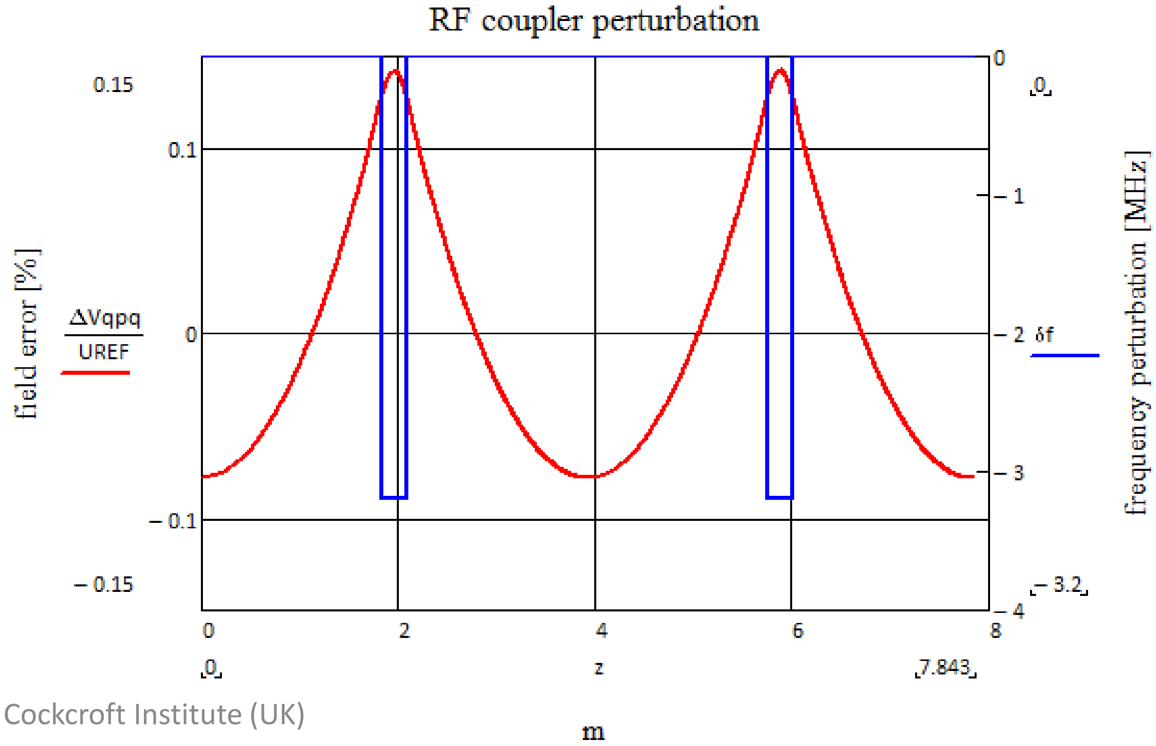
Nominal Iris Geometry						
Iris heigth = 76.8 mm	iris aperture - mm	iris heigth - mm	S11 linmag	freq - MHz	beta	rescaled beta
rescaled beta	50	76.8	0.9329	347.84	28.81	**1.23**
rescaled conductivity	50	76.8	0.1299	348.04	**1.30**	

- Aperture=50 mm
- Heigth = 76.8 mm
- Beta rescaled = 1.25
- Detuning cells 26-27 = -6.5 MHz (local detuning on 23 cm). This value is obtained in two eigenmode simulations, performed with different boundary conditions at the reference plane (perfect magnetic or perfect electric). Frequency is compared with simulation without coupler.
- Transmission line model of the cavity shows a ±10% error on field flatness
- Dedicated tuner located RF at coupler section

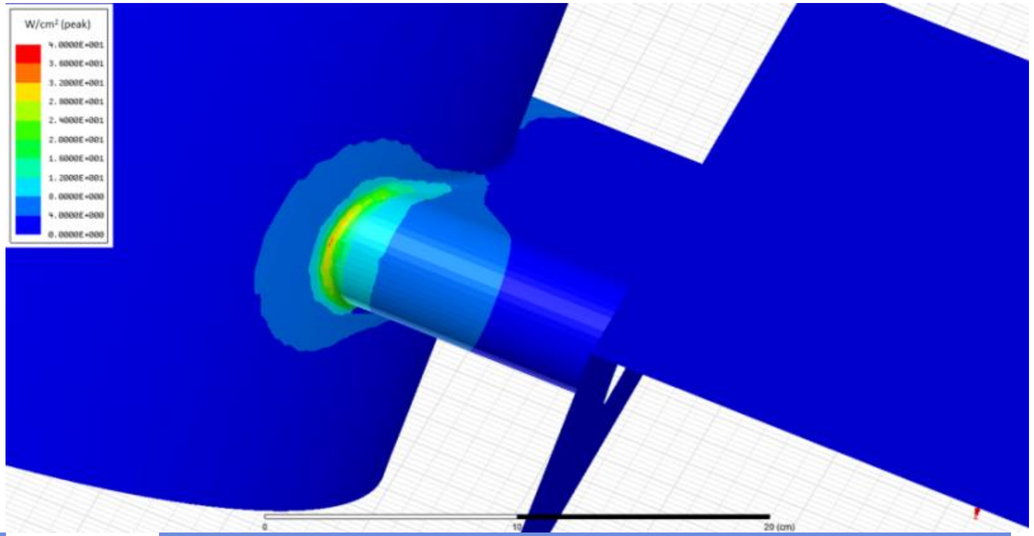


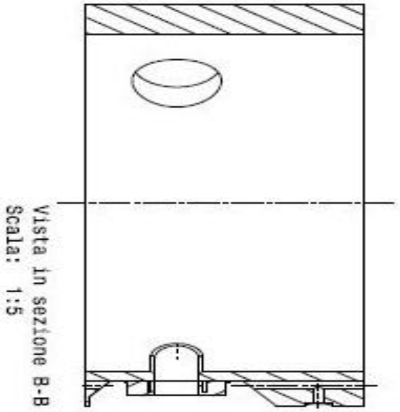


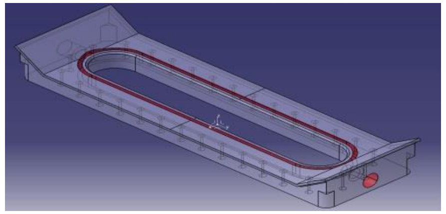
ESS DTL Couplers: mechanical design

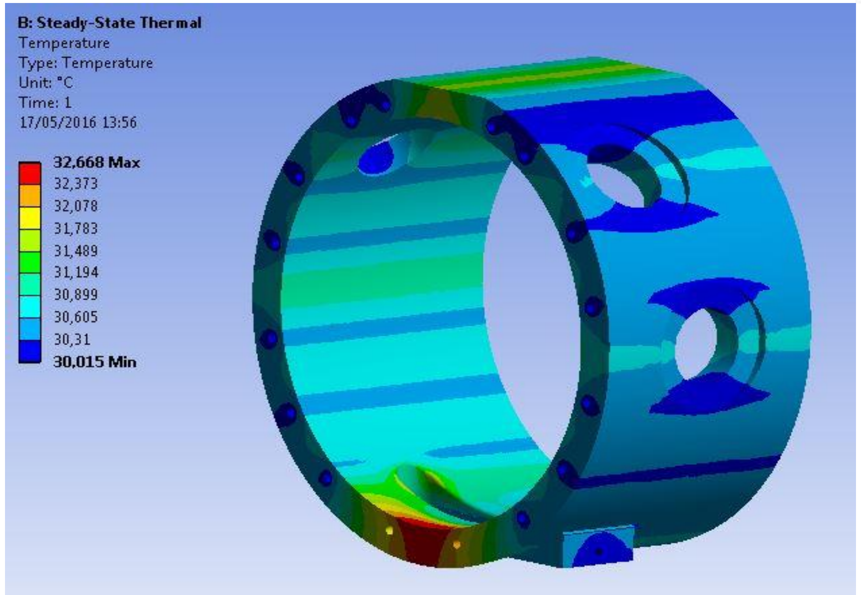
In order to investigate thermal deformation and cooling efficiency the power density is computed, scaling outer wall H field to the nominal value H0. Iris edge is rounded.

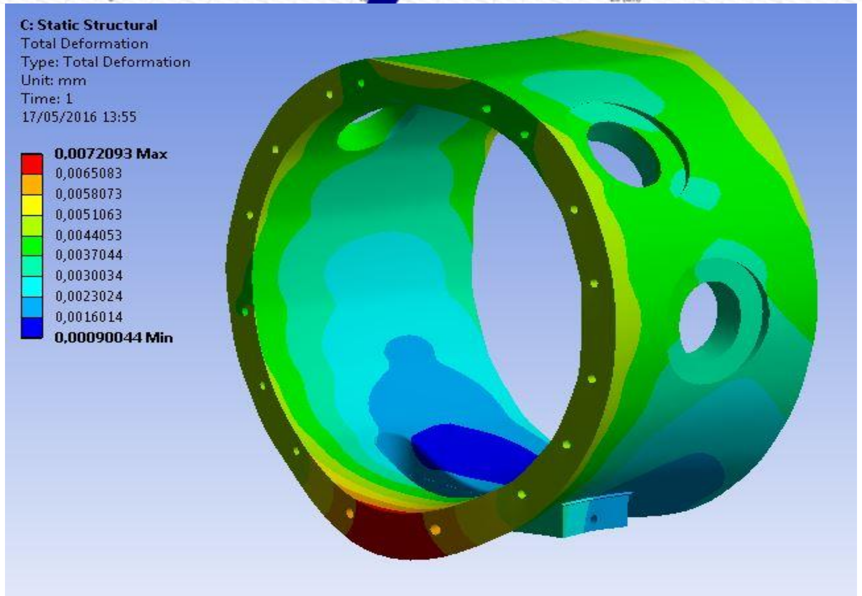
- H0=3.7 kA/m (Superfish)
- H_{max}= 11 kA/m
- Peak power density = 30 W/cm^2
- Rs=0.004896 Ohm
- Max Power density = $(H_{max}{}^2)$x Rs/2 x 5%=1.5 W/cm^2











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RF windows

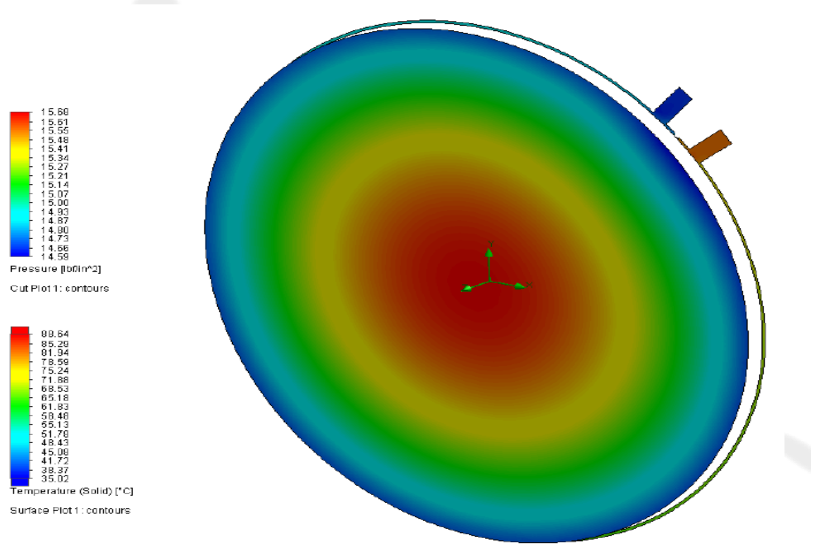
Linac4 RF windows: duty cycle applied was 1 ms pulses at 2 Hz, @1 MW.
Dedicated RF windows design has been requested, to be tested at high power conditions.

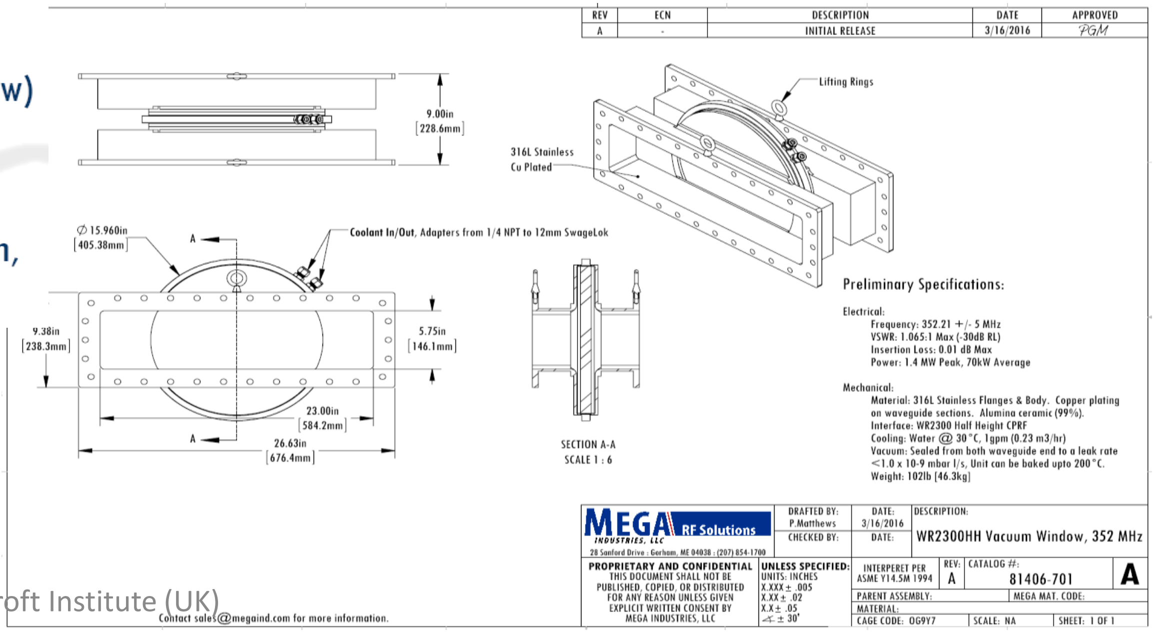
— Electrical:

- Frequency: 352.21 MHz Center, BW: +/- 5 MHz
- VSWR: 1.065:1 Maximum (-30 dB Return Loss)
- Insertion Loss: < 0.01 dB
- Power: 1.4 MW peak, 70 kW ave typical (3 MW & 300 kW Maximum)

— Mechanical:

- Vacuum: full operation at 5 x 10^{-8} Torr or lower
- Leak Rate: < 1 x 10^{-9} mBar-Liter/second
- Pressurization: 2 atm (air side) with vacuum on other (30 psi differential across window)
- Cooling: <1 m^3/hr of 30°C water at 100 psig nominal, static pressure test at 150 psig
- RF Interface: WR2300 ½ Height flat output
- Construction: waveguide-vacuum side Cu plated 316L SS, waveguide-air side Aluminum, window-TiN coated (10 nm) Alumina (>98%)





Multiple couplers: phase and tuning

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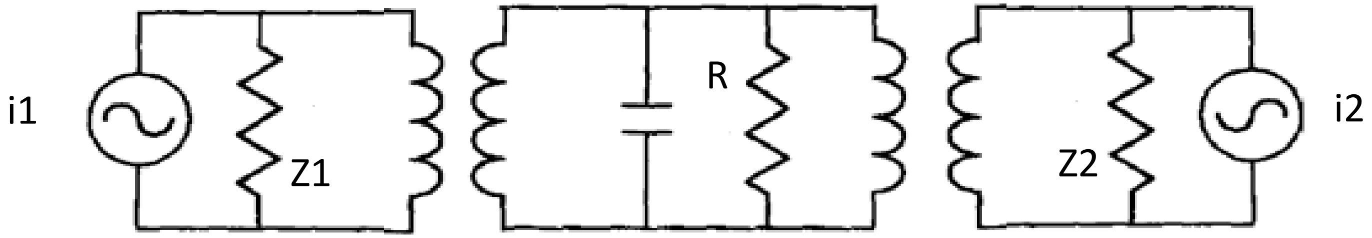
Accelerators driving very intense beam current often require multiple main coupler to withstand the high RF power needed per cavity. It is useful to see the interaction between couplers, using the RLC circuit model and the superposition. To understand the process we solve the problem for 2 couplers, with cavity driven at resonance. A generalization of this result to N coupler ports has been done by H. Safa in MULTIPLE COUPLING AND BEAM LOADING OF A RF CAVITY.

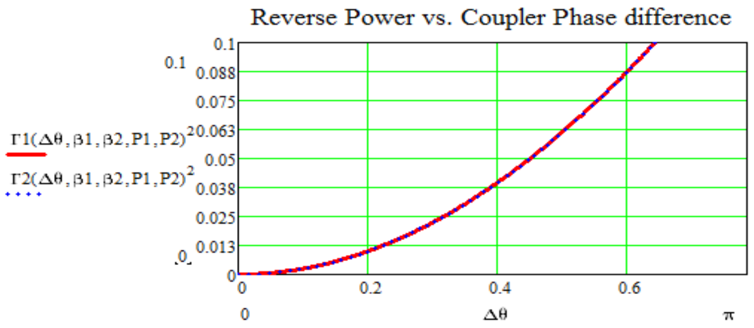


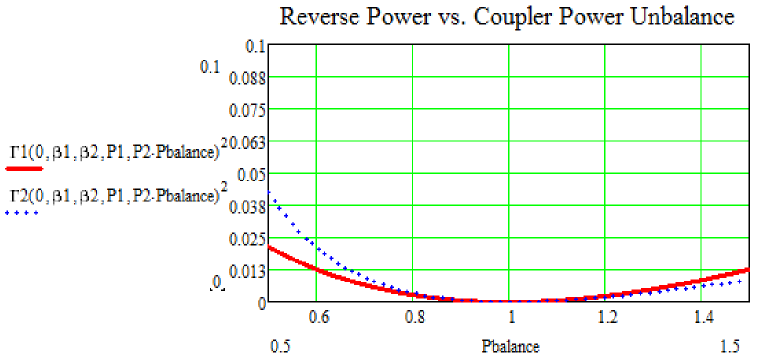
$$\sqrt{\frac{\beta 1 Z1}{R}} = n1:1 \qquad 1:n2 = \sqrt{\frac{\beta 2 Z2}{R}}$$

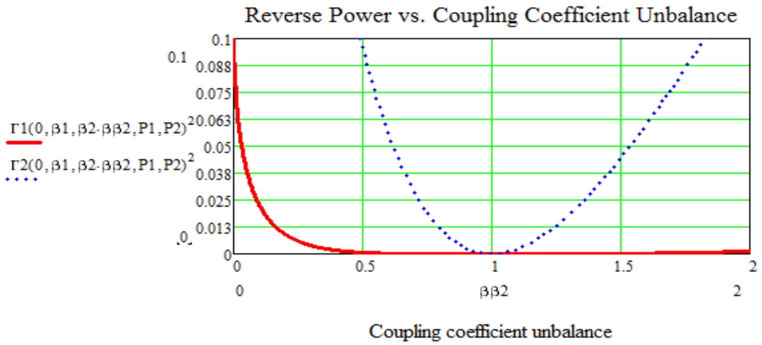
$$\Gamma 1(\Delta\theta, \beta 1, \beta 2, P1, P2) := \left| \frac{\beta 1 - \beta 2 - 1}{\beta 1 + \beta 2 + 1} + \frac{2\sqrt{\beta 1 \cdot \beta 2}}{\beta 1 + \beta 2 + 1} \cdot \sqrt{\frac{Z2}{Z1}} \cdot \frac{\sqrt{P2} \cdot e^{i \cdot \Delta\theta}}{\sqrt{P1}} \right|$$

$$\Gamma 2(\Delta\theta, \beta 1, \beta 2, P1, P2) := \left| \frac{\beta 2 - \beta 1 - 1}{\beta 1 + \beta 2 + 1} + \frac{2\sqrt{\beta 1 \cdot \beta 2}}{\beta 1 + \beta 2 + 1} \cdot \sqrt{\frac{Z1}{Z2}} \cdot \frac{\sqrt{P1}}{\sqrt{P2} \cdot e^{i \cdot \Delta\theta}} \right|$$

- Reflected power depends on the Coupler Power Unbalance, Coupler Phase Unbalance and Coupling coefficient unbalance.
- RF Coupler tolerances (phase Δθ, coupling β, power splitting) should take into account power margin
- Accuracy around β=1 → 1mm on short length = 1% on β







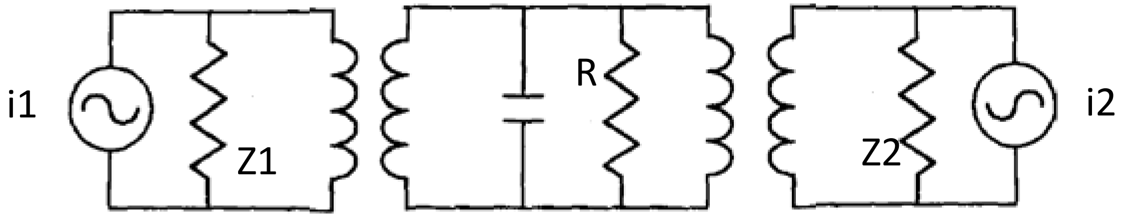
Conclusions

- To date INFN-LNL developed and tested up to full power in CW (>100 kW) four loop type couplers for RFQ's: 2 couplers at 352.2 MHz and 2 couplers at 175 MHz
- Two different kinds of RF windows technologies were employed successfully (cylindrical welded and planar)
- All the design steps (RF and thermomechanical in particular) were addressed in all their aspects and the test performed proved the correctness of the design procedures adpopted so far.
- Two Aluminum cavities were designed and developed with the purpose of testing the couplers prior to their high power cycling in the RFQ, in order to "debug" them. This approach proved well-conceived, since no major issue came out from the couplers during RFQ high power tests.
- Next steps: Full power testing of all remaining couplers for the IFMIF-RFQ, using the same technology developed for the two already tested
- Design finalization and development of the ESS DTL couplers: RF and thermo-mechanical defined
- 2 WR2300 RF windows delivery in autumn 2016 for full power RF test
- Interfaces defined with ESS RF group in term of reflected power, amplitude unbalance, phase difference between two couplers

Back-up slides

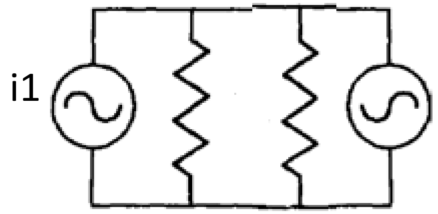
Multiple couplers

It could be useful to see the interaction between couplers, using the RLC circuit model and the superposition. To understand the process we solve the problem for 2 couplers, with cavity driven at resonance.



$\sqrt{\frac{\beta 1 Z1}{R}} = n1:1$ $\quad 1:n2 = \sqrt{\frac{\beta 2 Z2}{R}}$

System transformed into the generator 1



$i2\frac{n2}{n1}$

Z1 $\quad Z = \frac{\beta 1 Z1}{\beta 2 + 1}$

The reflected current at generator 1 is defined as the current reflected back through Z1 when i2=0 plus the current flowing through Z1 when i1=0: $i_1^- = i_{11}^-\big|_{i2=0} + i_{12}^-\big|_{i1=0}$

As the forward current is defined as the maximum current available to the load Z: $i_1^{FWD} = i_1/2$

the current reflected back to generator 1 is defined as $i_{11}^- = i_1/2 - i_z = \frac{\beta_1 - \beta_2 - 1}{\beta_1 + \beta_2 + 1} i_1/2$

The current from generator 2 to Z1 is $i_{12}^- = i_2 \frac{n_2}{n_1}\left(\frac{1}{Z1} + \frac{1}{Z}\right)^{-1} = i_2 \sqrt{\frac{Z_2}{Z_1}} \frac{\sqrt{\beta_1 \beta_2}}{\beta_1 + \beta_2 + 1}$

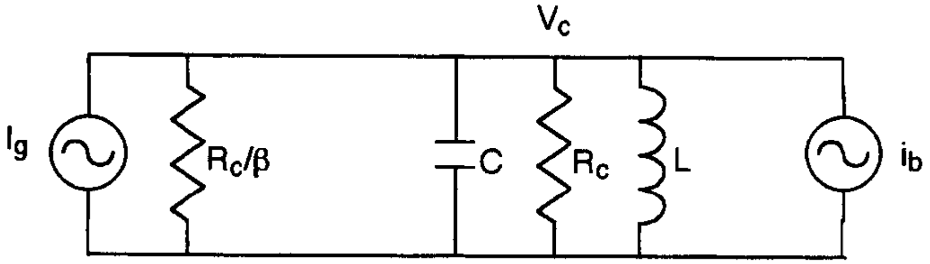
The total reflection at generator 1 is then $\Gamma_1 = \frac{i_{11}^- + i_{12}^-}{i_1/2} = \frac{\beta_1 - \beta_2 - 1 + 2\sqrt{\beta_1 \beta_2}\sqrt{P_2/P_1}}{\beta_1 + \beta_2 + 1}$

Beam Loading

How the RF generator sees the multi-cell accelerating cavity and beam?
In particular, which is the effective synchronous phase between Vc and ib?

$\widetilde{i_b}$=2I **beam current** component at RF frequency

$\Delta W = q|\widetilde{V_c}|cos\phi = q\sum_i V_i T_i cos\phi_i$ **energy gain**



$P_b = \frac{1}{2}|\widetilde{V_c}||\widetilde{i_b}|cos\phi = I\sum_i V_i T_i cos\phi_i$ **beam active power**

$Q_b = \frac{1}{2}|\widetilde{V_c}||\widetilde{i_b}|sin\phi = I\sum_i V_i T_i sin\phi_i$ **beam reactive power**

Effective sync.phase

$$\phi = atan\frac{\sum_i V_i T_i sin\phi_i}{\sum_i V_i T_i cos\phi_i}$$

Effective accelerating voltage

$$|\widetilde{V_c}| = \frac{\sum_i V_i T_i cos\phi_i}{cos\phi}$$

Effective shunt impedance

$$r_s = \frac{|\widetilde{V_c}|^2}{P_{Cu}}$$

Optimum coupling

$$\beta_0 = 1 + \frac{P_b}{P_{Cu}}$$

Optimum detuning

$$2Q_L\frac{f_{RF} - f_0}{f_0} = \frac{\beta_0 - 1}{\beta_0 + 1}tan\phi$$