From Theory to ESS accelerator

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INTRO AND OUTLOOK

- Accelerators are using the electromagnetic force to control the particles
  - This could be acceleration, deceleration, steering, focusing, …
- Some of these accelerators increase the beam energy to relativistic energies

- A beam (bunch) is an ensemble of charge particles that interact with each other through Coulomb force (space-charge)

- Using ESS linac (LINear ACcelerator) as an example, some of the main components of the accelerators will be looked at.

MAXWELL EQUATIONS

- Differential form of Maxwell equations (SI):

Gauss's law	$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0}$
No magnetic monopoles	$\nabla \cdot \mathbf{B} = 0$
Faraday's law (of induction)	$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$
Ampere's circuital law*	$\nabla \times \mathbf{B} = \mu_0 \left(\mathbf{J} + \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} \right)$

LORENTZ FORCE

$$\frac{d\mathbf{p}}{dt} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

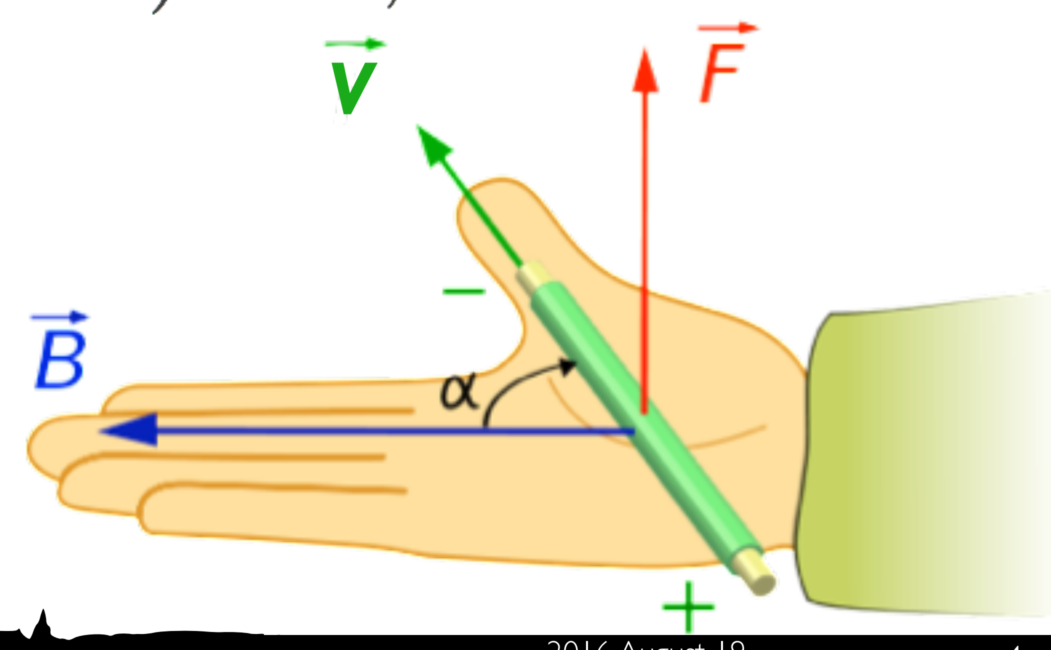
The electric field will increase the kinetic energy of the charged particles,

$$\Delta W = d\mathbf{x} \cdot q\mathbf{E}.$$

The magnetic field cannot affect the energy,

$$d\mathbf{x} \cdot (\mathbf{v} \times \mathbf{B}) = dt\ (\mathbf{v} \cdot \mathbf{v} \times \mathbf{B}) = 0,$$

but, it can change the moving particles trajectory.



Can you show the ratio of electric (10 MV/m) to magnetic (1 T) force for a particle travelling at c?

GAUSS'S LAW

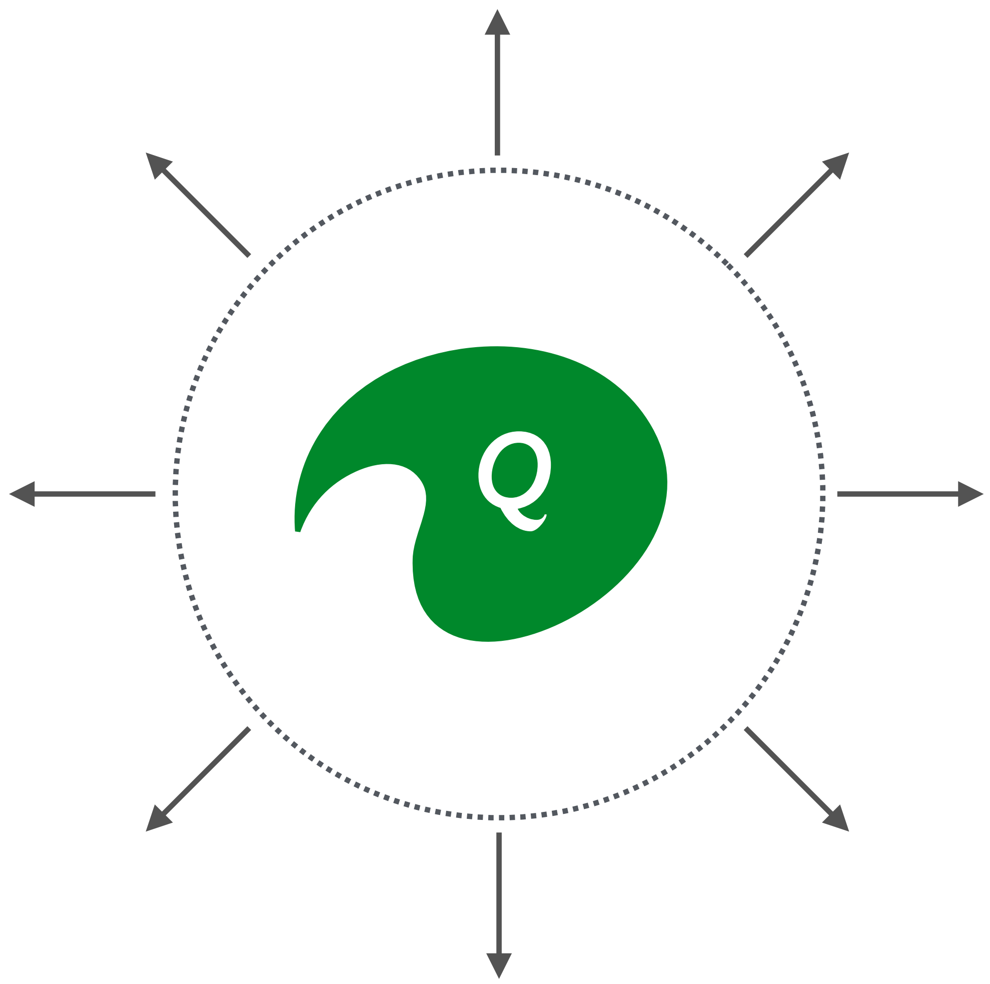
- Using Gauss's law one can calculate the electric field of *any* charge distribution,

$$\oint \mathbf{E}\cdot d\mathbf{A} = \frac{1}{\epsilon_0}\int \rho dv = \frac{Q}{\epsilon_0}$$

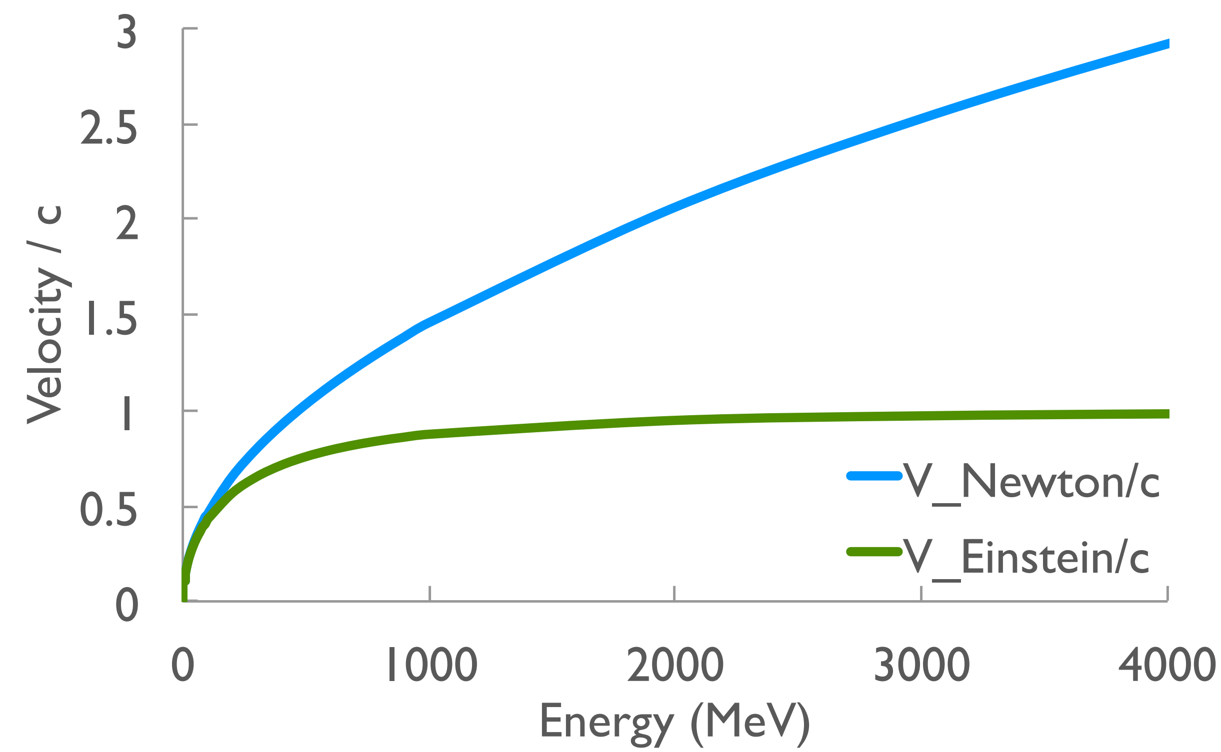
or

in 2D: $$\mathbf{E} = \frac{Q}{2\pi\epsilon_0 r}\hat{\mathbf{r}}$$

in 3D: $$\mathbf{E} = \frac{Q}{4\pi\epsilon_0 r^2}\hat{\mathbf{r}}$$



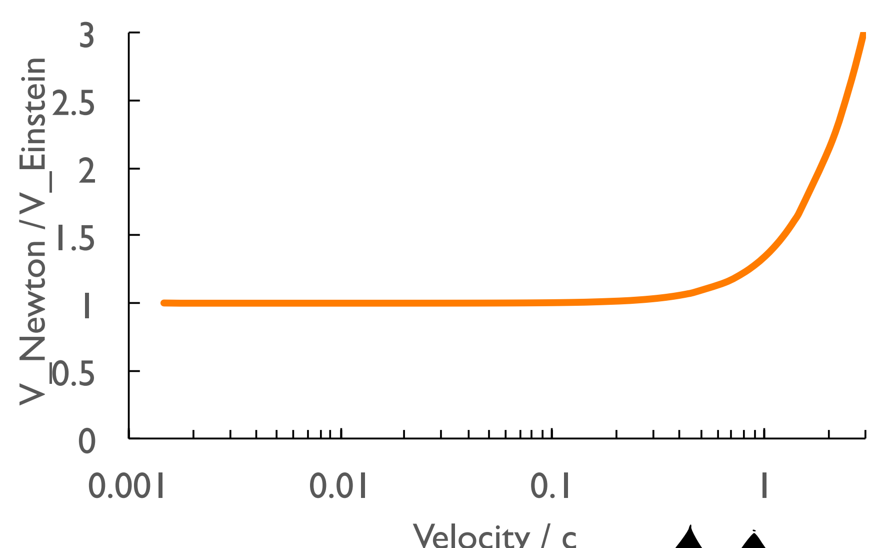
SPECIAL RELATIVITY



$$E = \frac{1}{2} m v^2$$

$$E = \sqrt{(pc)^2 + (m_0 c^2)^2}$$

- In majority of the accelerators particles are too fast to be treated non-relativistically.



SOME OF SPECIAL RELATIVITY EQS.

Relative velocity:
$$\beta = \frac{v}{c}$$

Lorentz factor:
$$\gamma = \frac{1}{\sqrt{1-\beta^2}} = 1 + \frac{E_k}{m_0}$$

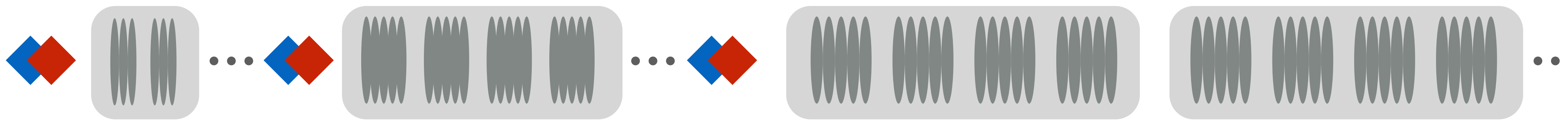
Transformation of the electromagnetic fields:

$$\mathbf{E}'_{\parallel} = \mathbf{E}_{\parallel}$$
$$\mathbf{E}'_{\perp} = \gamma\left(\mathbf{E}_{\perp} + \mathbf{v} \times \mathbf{B}\right)$$
$$\mathbf{B}'_{\parallel} = \mathbf{B}_{\parallel}$$
$$\mathbf{B}'_{\perp} = \gamma\left(\mathbf{B}_{\perp} - \frac{\mathbf{v}}{\mathbf{c}^2} \times \mathbf{B}\right)$$

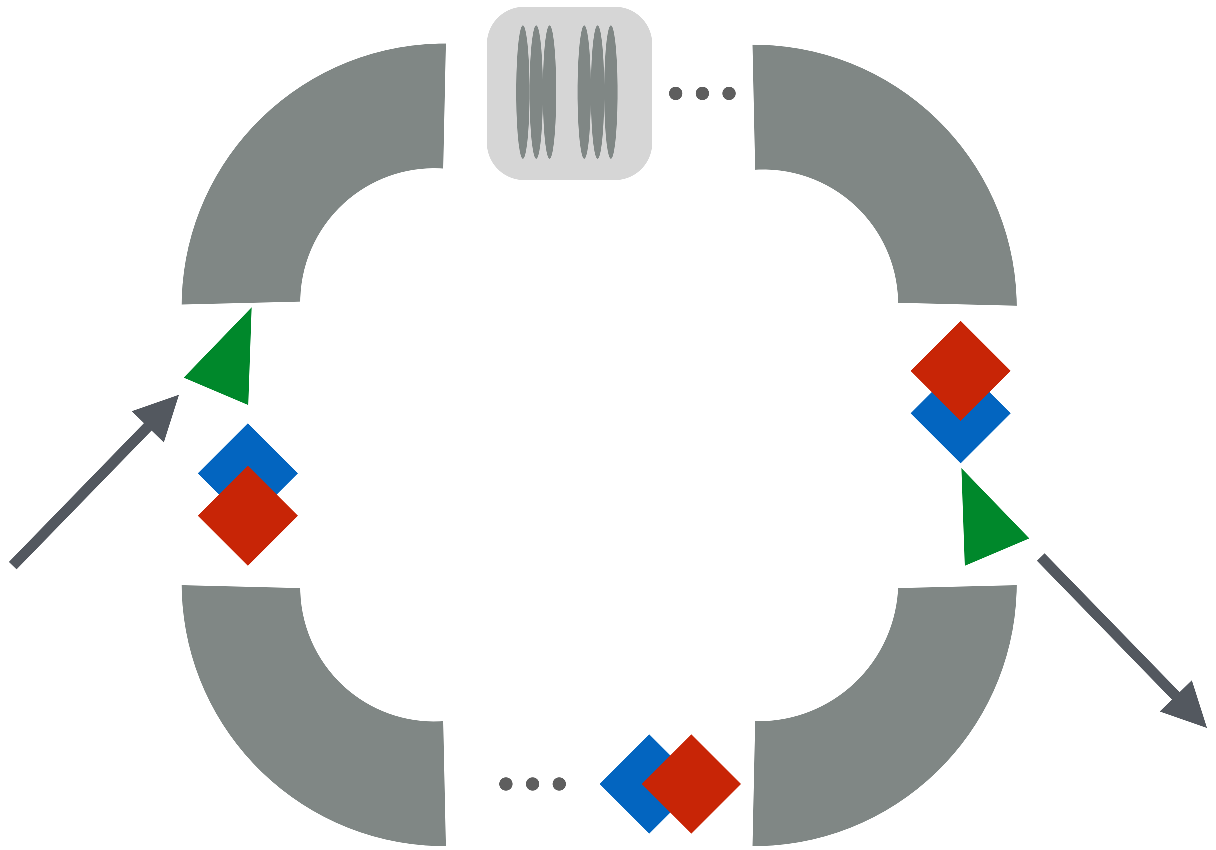
Can you calculate the force between two relativistic charged particles?

SYNCHROTRON VS. LINAC I

- Linear accelerators:



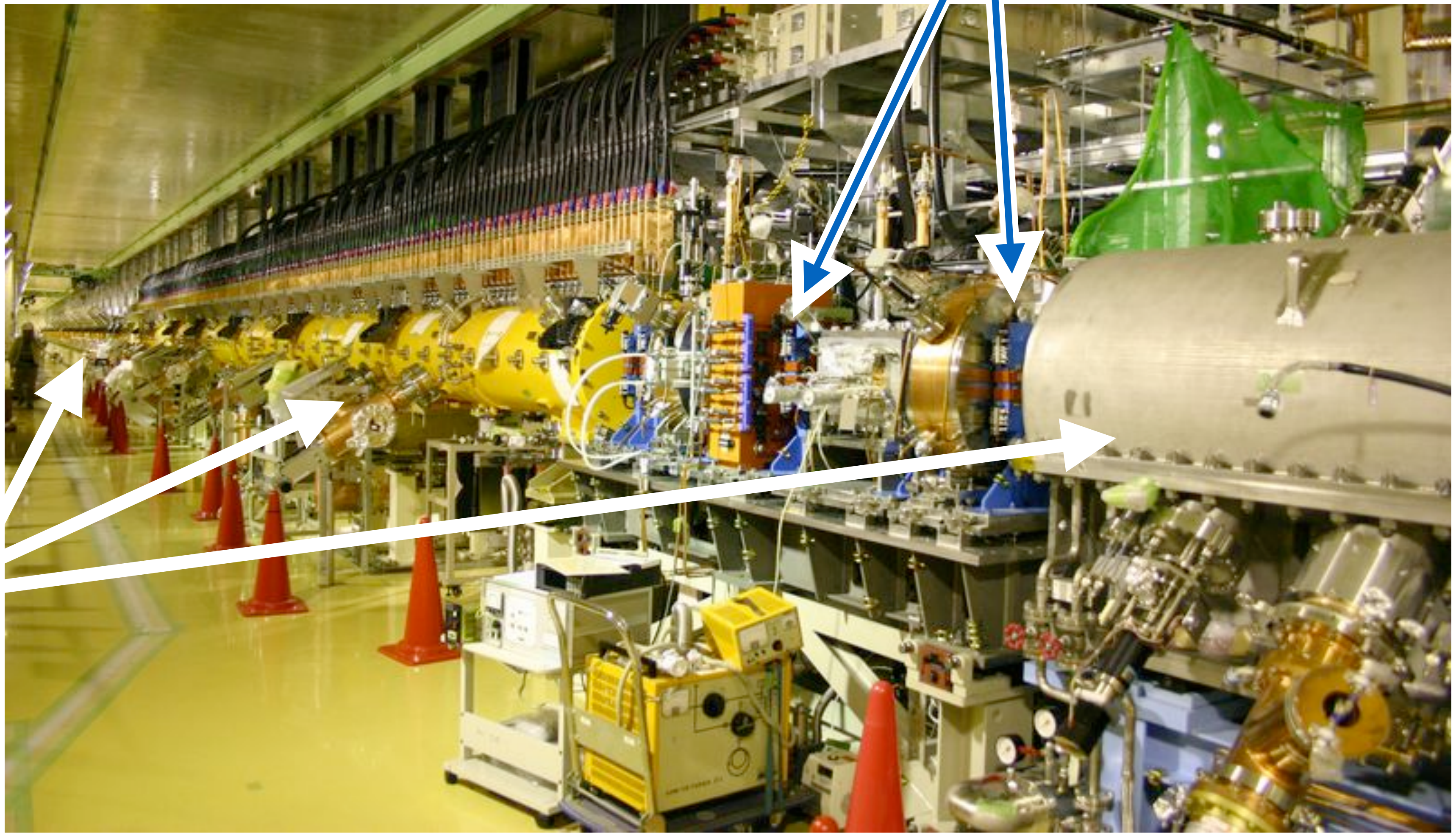
- Circular accelerators:



SYNCHROTRON VS. LINAC II

- **Linear accelerators:**

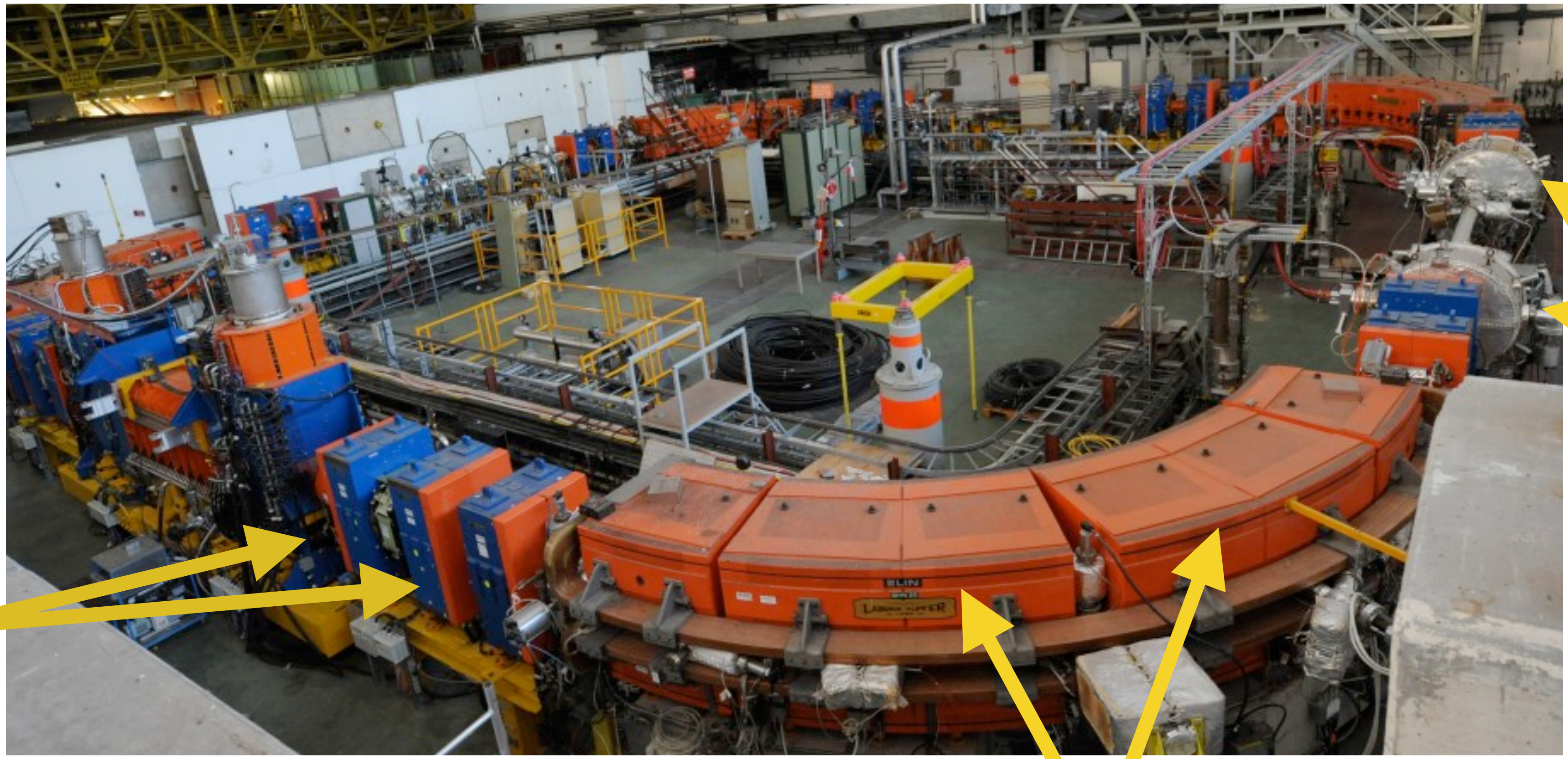
Focusing elements



Accelerating elements

SYNCHROTRON VS. LINAC III

- *Not-so*-Circular accelerators:



Focusing elements

Accelerating elements

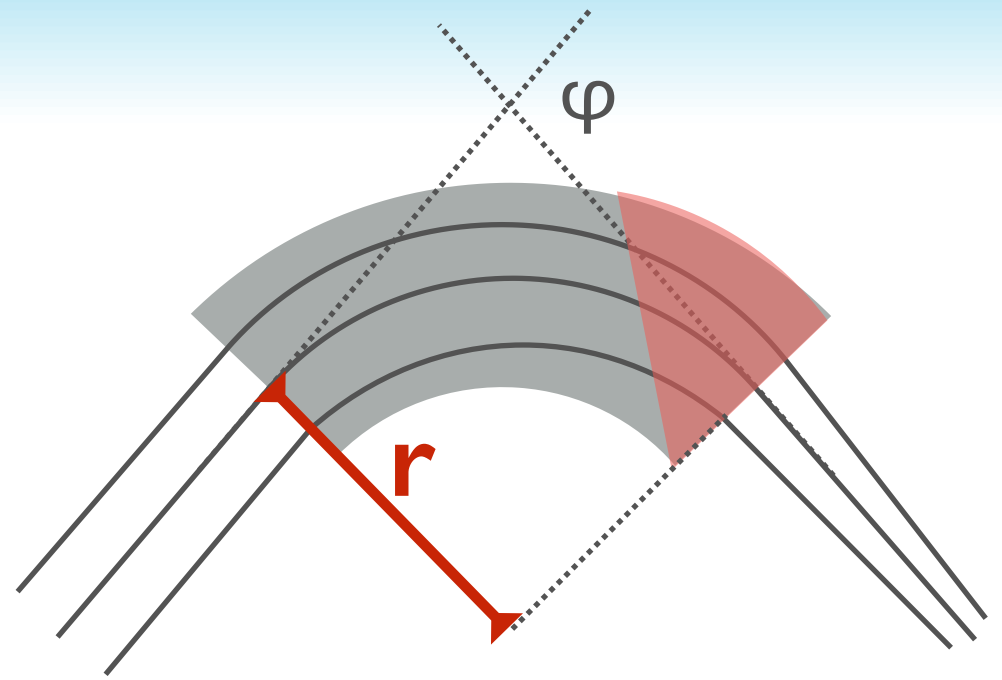
Bending elements

SYNCHROTRON VS. LINAC IV

- Linear accelerators:
  - Are mainly filled with cavities (and focusing elements)
  - Do not have an ultimate energy
  - Are single pass (errors do not accumulate)
  - Can provide any pulse length (train of bunches)

- Circular accelerators (synchrotrons):
  - Are mainly filled with dipoles (and focusing elements)
  - Are limited in their ultimate energy *(do you know why?)*
  - Beam is accelerated for several thousands to millions of turns (errors accumulate)
  - Pulse length is limited to the circumference of the ring

BENDING THE BEAM



- Under a uniform magnetic field B, the beam of charged particles is bent by φ, which could be calculated:

$$F = qvB,$$

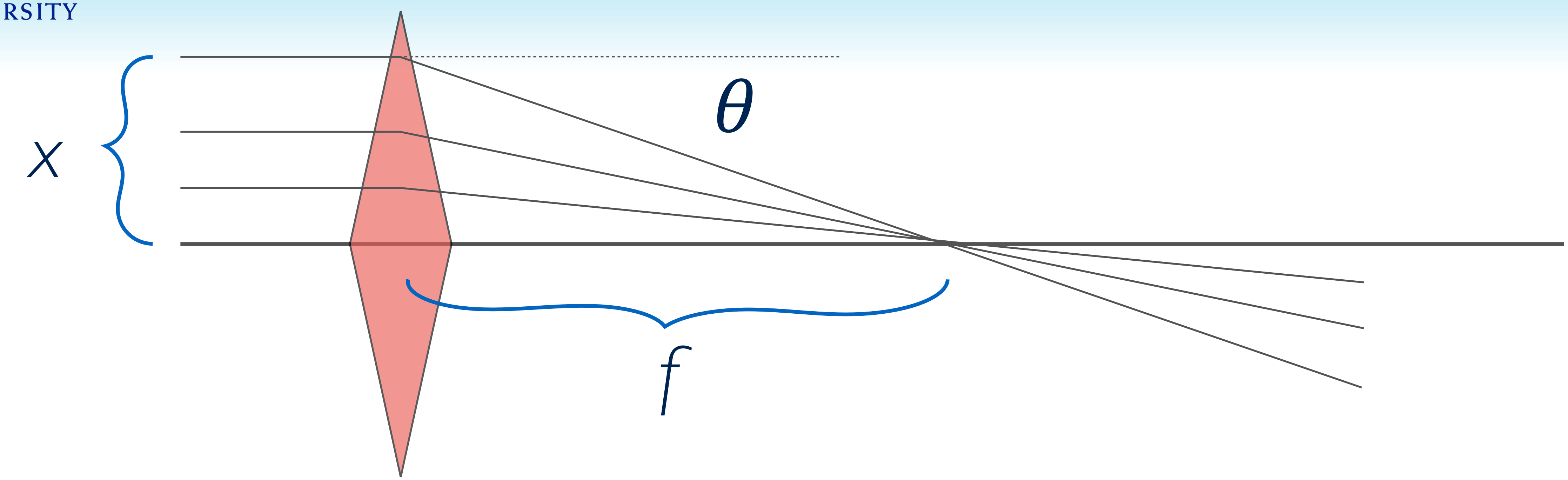
$$F = mv^2/r,$$

$$r = mv/qB$$

the product rB is called beam rigidity.

Can you show: $$\frac{1}{r[m]} = 0.2998\frac{q[e].B[T]}{p[GeV/c]}$$

TRANSVERSE FOCUSING



- A force which is linear with *x* will converge all the incoming parallel, fixed energy, charged particles at a distance *f*,

$$\theta = -x/f$$

$$F_x = -a_1 x \quad (1)$$

$$F_x = -qv_z B_y (2)$$

$$(1),(2): B_y = a_1 x / qv_z = gx$$

similarly

$$B_x = a_1 y / qv_z = gy$$

where g is called the quadrupole gradient

TRANSVERSE FOCUSING I

- One can calculate the equipotential lines of these fields

$$\mathbf{B} = -\nabla\phi_m \quad \therefore \quad \phi_m = gxy$$

so, the equipotentials are hyperbolae with $gxy = const.$

- Now we integrate the Ampere's law over A *(green area)*

$$\int_A \nabla \times \mathbf{H} \cdot d\mathbf{s} = \int_A \mathbf{J} \cdot d\mathbf{s}$$

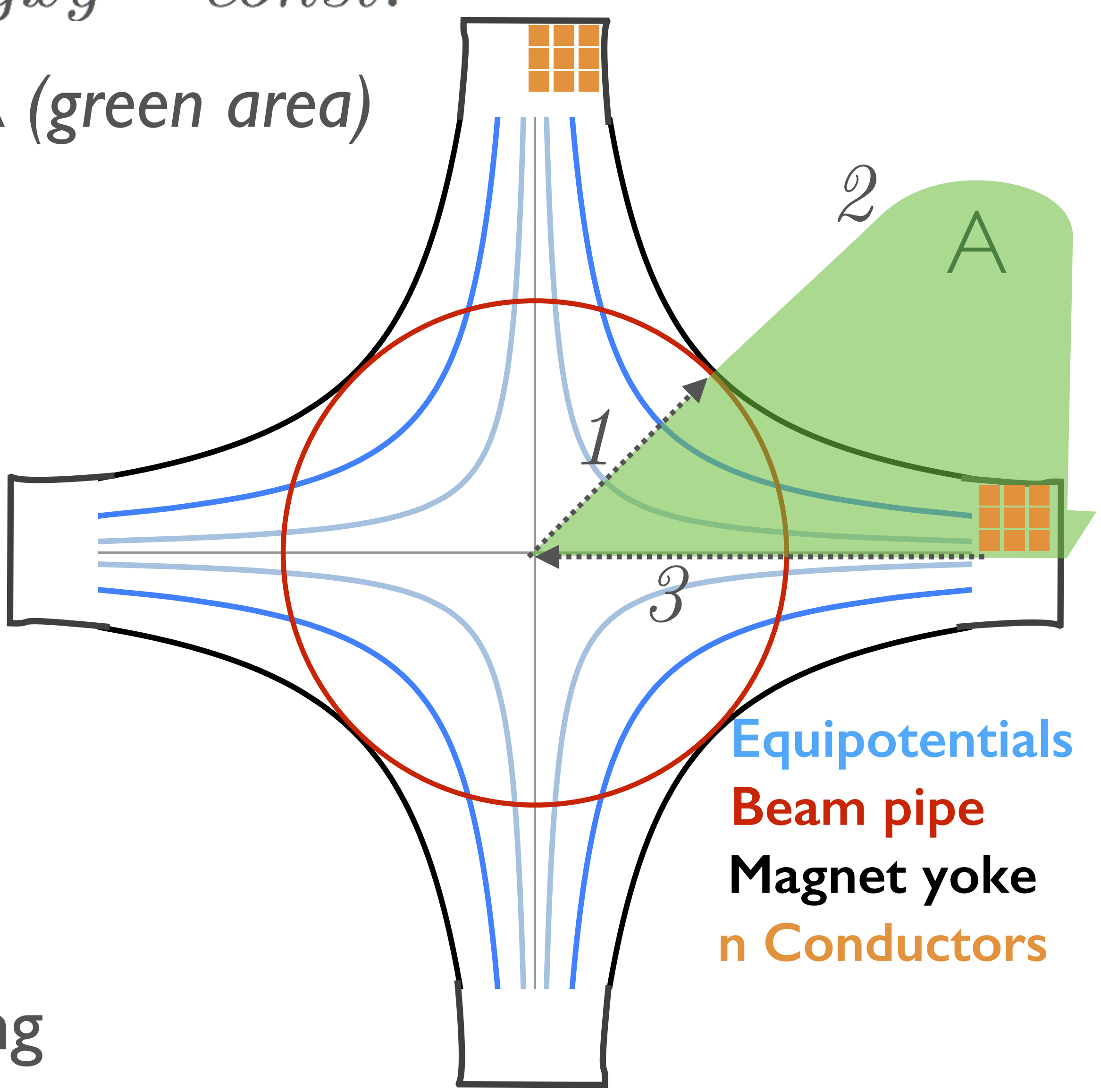
$$\oint_S \mathbf{H} \cdot d\mathbf{l} = nI$$

$$\int_1 \mathbf{H} \cdot d\mathbf{l} + \int_2 \mathbf{H} \cdot d\mathbf{l} + \int_3 \mathbf{H} \cdot d\mathbf{l} = nI$$

the 2nd & 3rd terms $(\mu >> 0)_2$ & $(\mathrm{H} \perp \mathrm{l})_3$ are ~zero, and $(\mathrm{H}=gr/\mu_0)_1$, therefore:

$$g = \frac{2\mu_0 nI}{r_0^2}$$

with $r^2 = x^2 + y^2$ and r_0 the minimum opening of hyperbola

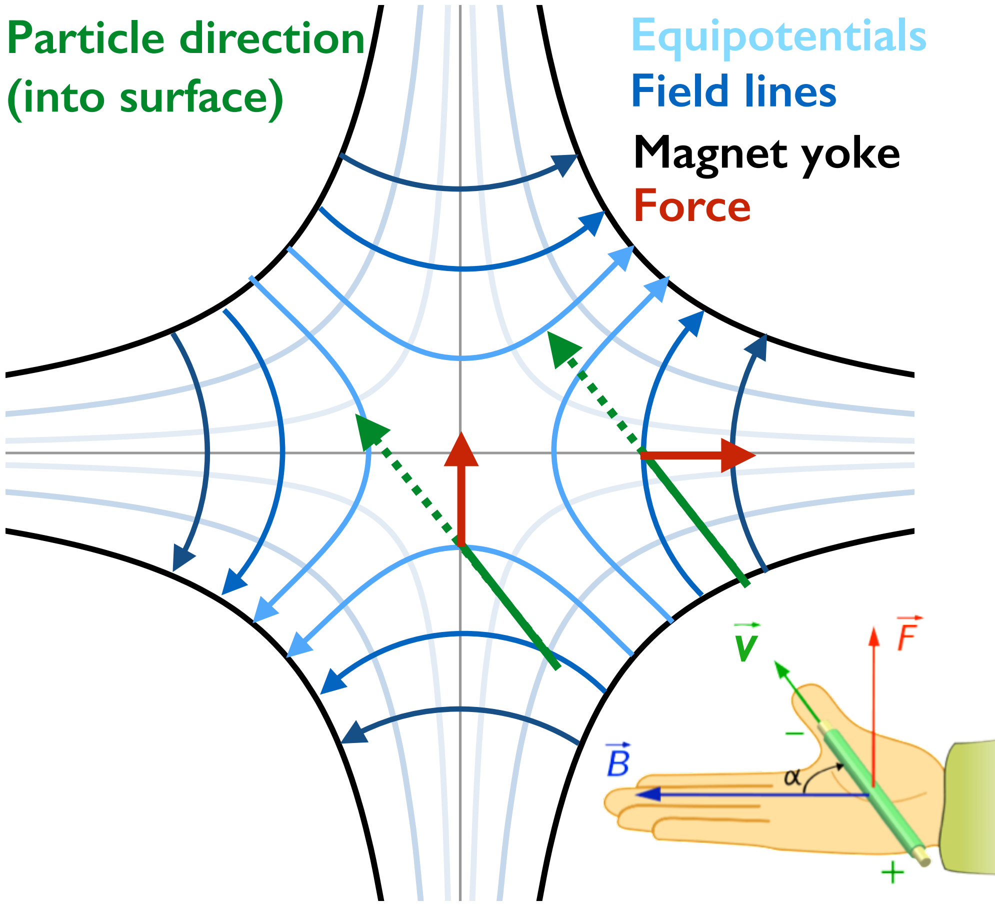


QUADRUPOLE FOCUSING II

- Plotting the field lines, one can check the force on the beam of charged particles.

- It is clear that the beam gets an inward kick in y-axis, while the kick in x-axis is outward, i.e.,

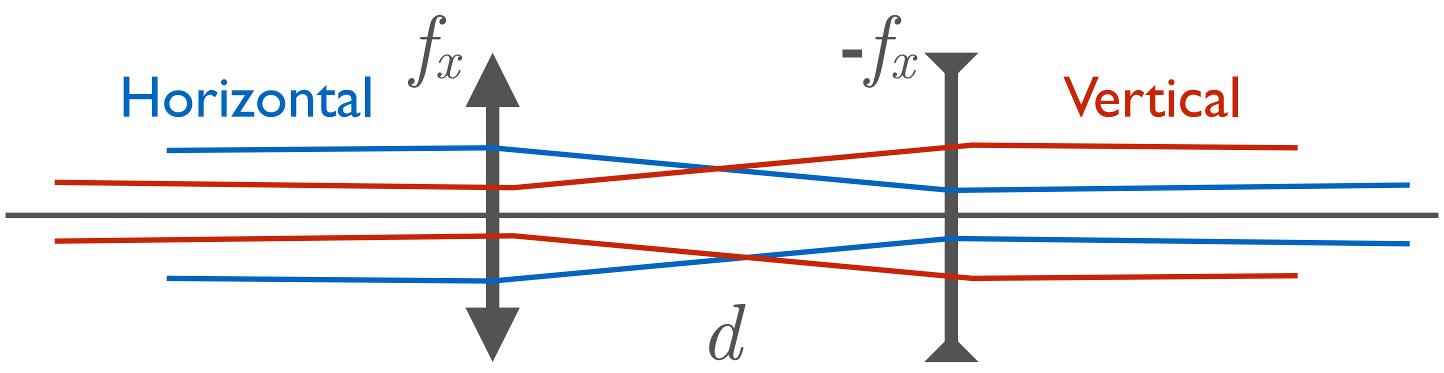
  focuses the beam in y (positive focal point), and defocuses in x (negative focal point).



QUADRUPOLE FOCUSING III

- The focal point of a system of lenses with focal lengths f_1 and f_2, separated by d:

$$1/f = 1/f_1 + 1/f_2 - d/f_1 \cdot f_2$$



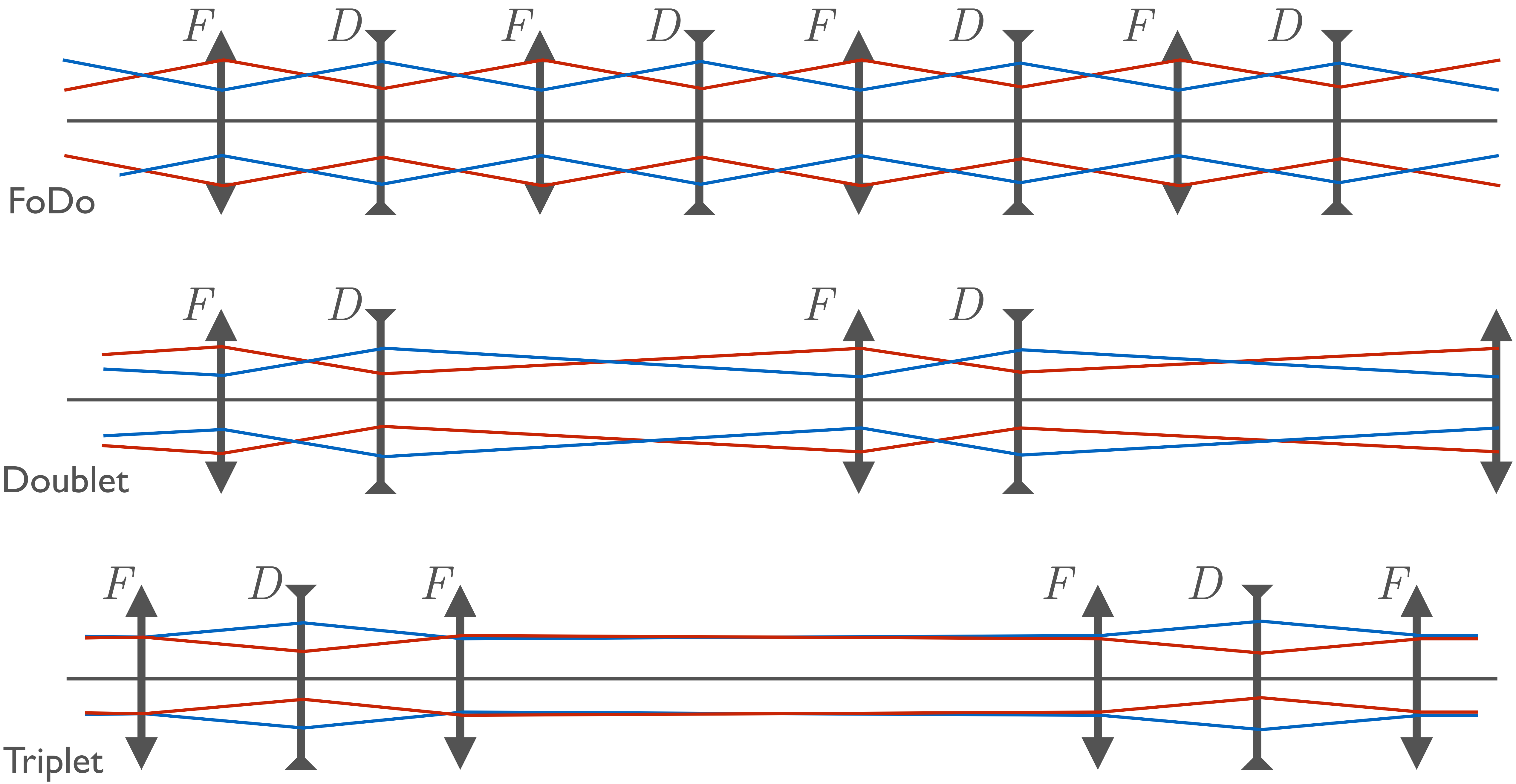
with $f_1 = f_x$ and $f_2 = -f_x$:

$$1/f = d/f_x^2 \quad \therefore \quad f = f_x^2/d$$

- Stability of the whole system puts another constraint on f_x vs. d.

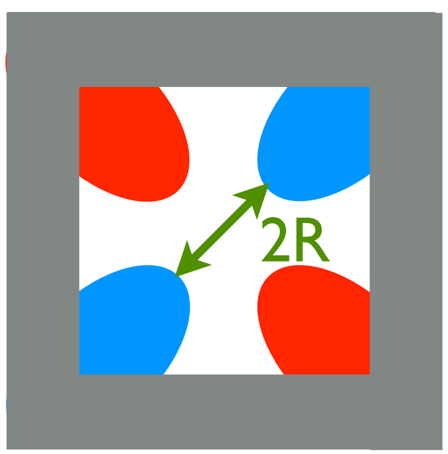
FOCUSING CHANNEL EXAMPLES

- A focusing channel is (usually) a periodic focusing structure:



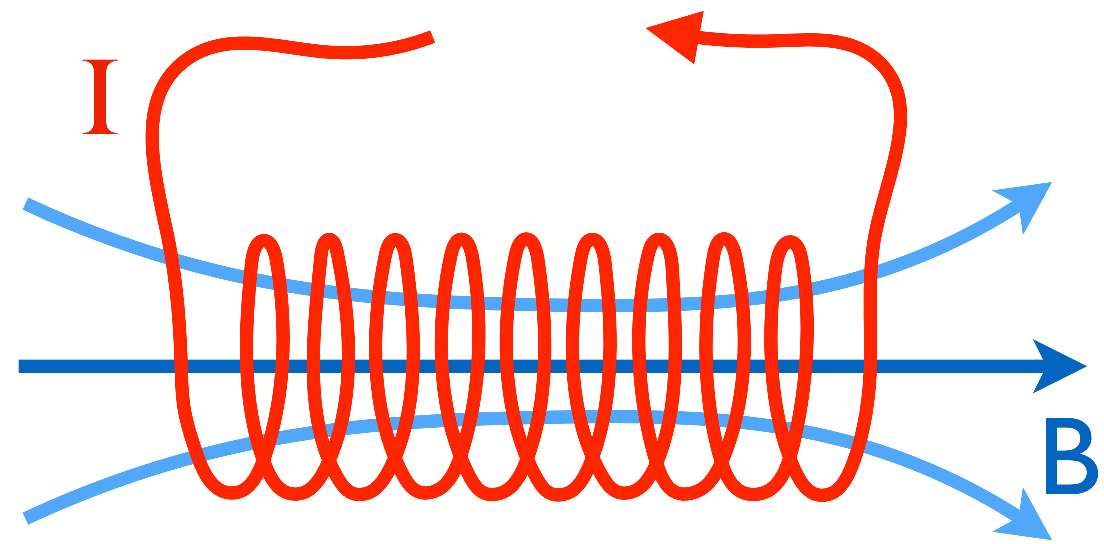
MAGNETIC QUAD. / SOLENOID

Magnetic Quadrupole



$$f_{MQ} = \frac{p}{q}\frac{R^2}{2\mu_0 n I}\frac{1}{L_Q}$$

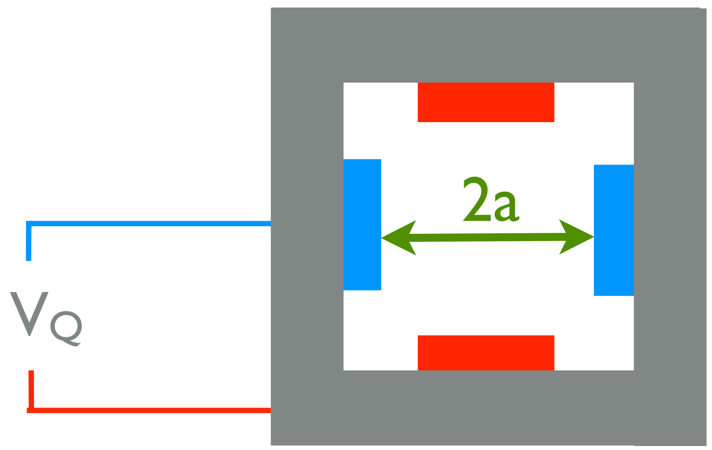
Solenoid



$$f_{sol} = \frac{4p^2}{q^2 B^2}\frac{1}{L_{sol}}$$

ELECTRIC QUAD. / EINZEL LENS

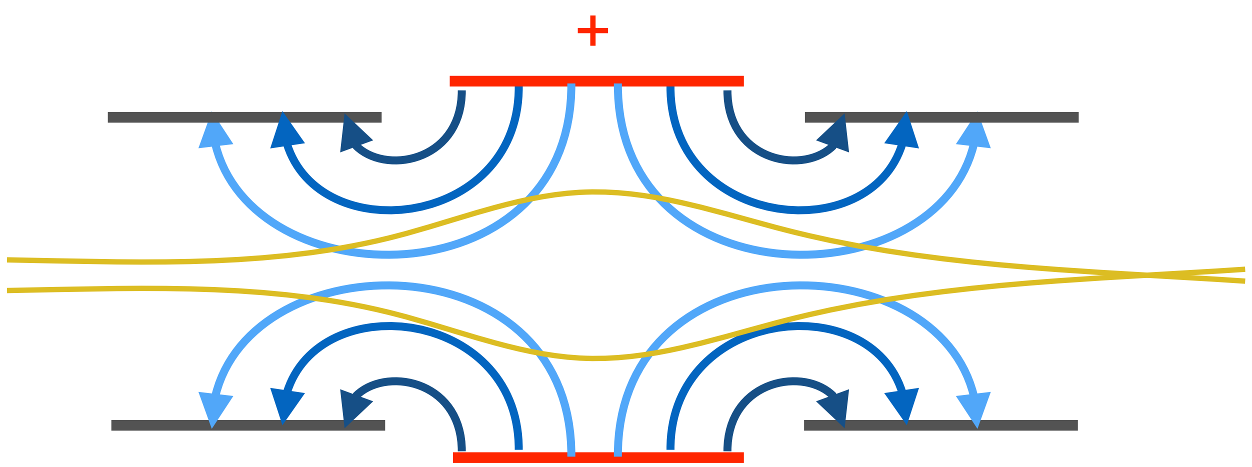
Electric Quadrupole



$$f_{EQ} = \frac{pc}{q}\frac{a^2}{2V_Q}\frac{1}{L_Q}$$

It works best at low energies

Einzel lens



$$f_{Einzel} = \frac{pc}{q}\frac{1}{E_r}\frac{1}{L_{Einzel}}$$

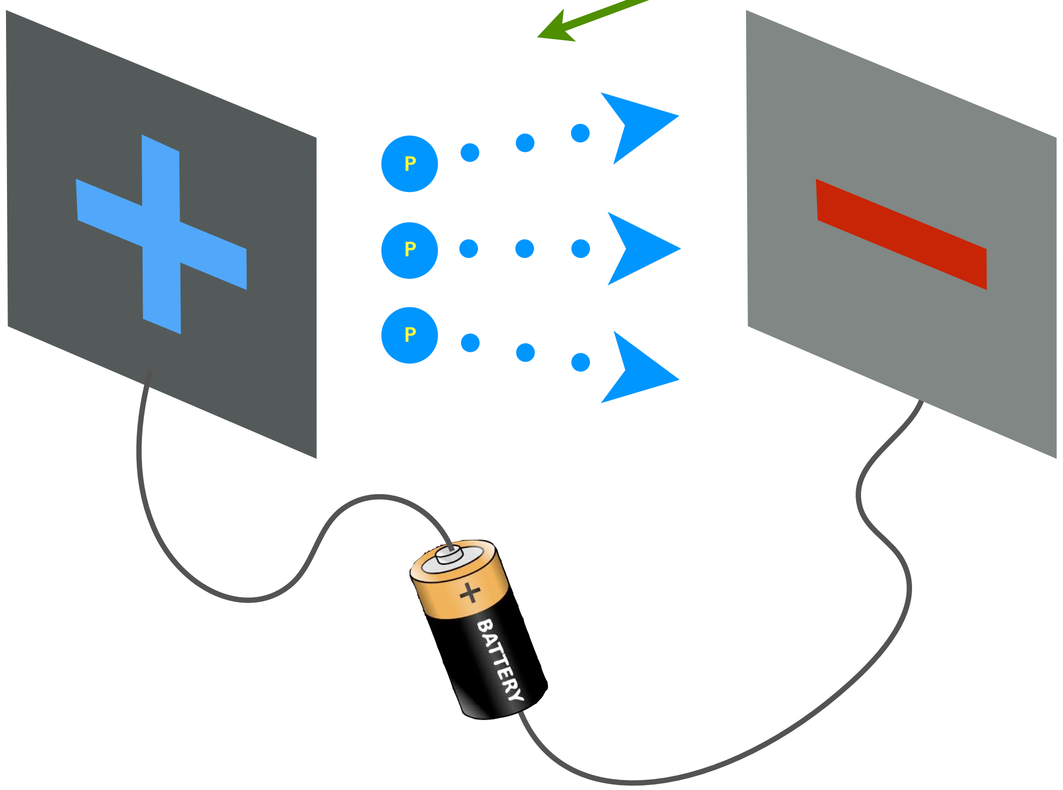
It focuses irrespective of charge

DC ACCELERATION

- Using a DC voltage one can make a DC accelerator.
  - Produce a very precise beam energy
  - Produce DC beams
  - Usually low current
  - Max energy limited (~20 MeV)
- Examples are:
  - Van de Graaff generator
  - Cockcroft-Walton generator
  - Tandem accelerators

Can you show the relation between J and eV?

1.5 eV



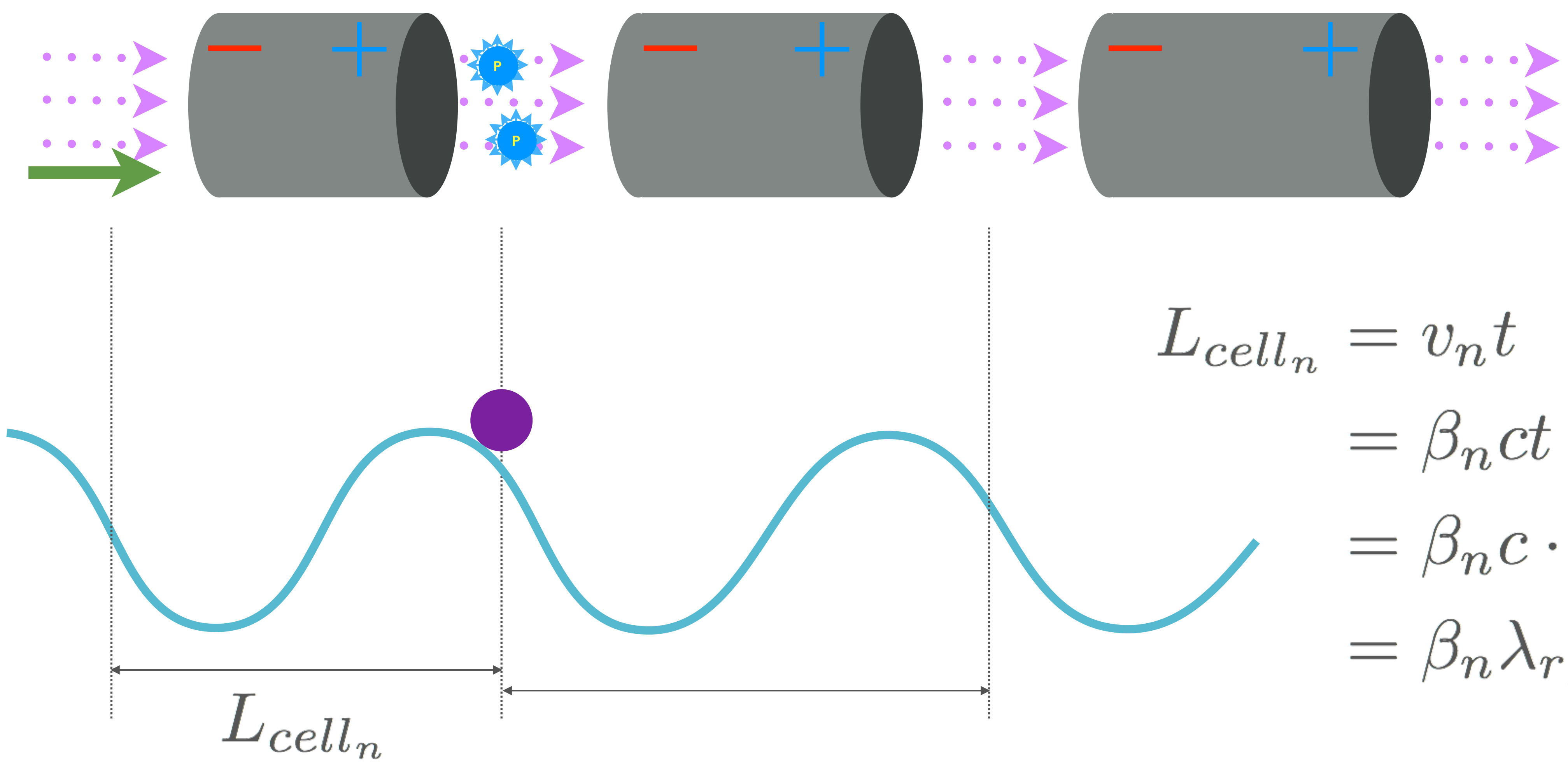
This protons are moving 2 Swedish miles per second

m (mili)	0.001
k (kilo)	1000
M (Mega)	1,000,000
G (Giga)	1,000,000,000

1 eV = 1.602E-19 J, 1 Swedish mile = 10 km

RF ACCELERATION

- As DC accelerators are limited in their ultimate energy, to reach higher energies another acceleration principle is needed



$$\begin{aligned} L_{cell_n} &= v_n t \\ &= \beta_n c t \\ &= \beta_n c \cdot {}^{1}/{f_{rf}} \\ &= \beta_n \lambda_{rf} \end{aligned}$$

ESS LINAC AT A GLANCE

352.21 MHz | 704.42 MHz

Source → LEBT → RFQ → MEBT → DTL → Spokes → Medium β → High β → HEBT & Contingency → Target

Lengths: LEBT 2.4 m, RFQ 4.6 m, MEBT 3.8 m, DTL 39 m, Spokes 56 m, Medium β 77 m, High β 179 m

Energies: 75 keV, 3.6 MeV, 90 MeV, 216 MeV, 571 MeV, 2000 MeV

Proton source (also called Ion source) produces a continues beam of protons at 14 Hz

Low Energy Beam Transport, LEBT, adjusts and matches the beam out of source to the next structure

Radio Frequency Quadrupole, RFQ, Focuses, Accelerates and bunches the beam

Medium Energy Beam Transport, MEBT, adjusts and matches the beam out of RFQ to DTL

Drift Tube Linac, DTL, is the longest normal conducting part of ESS linac

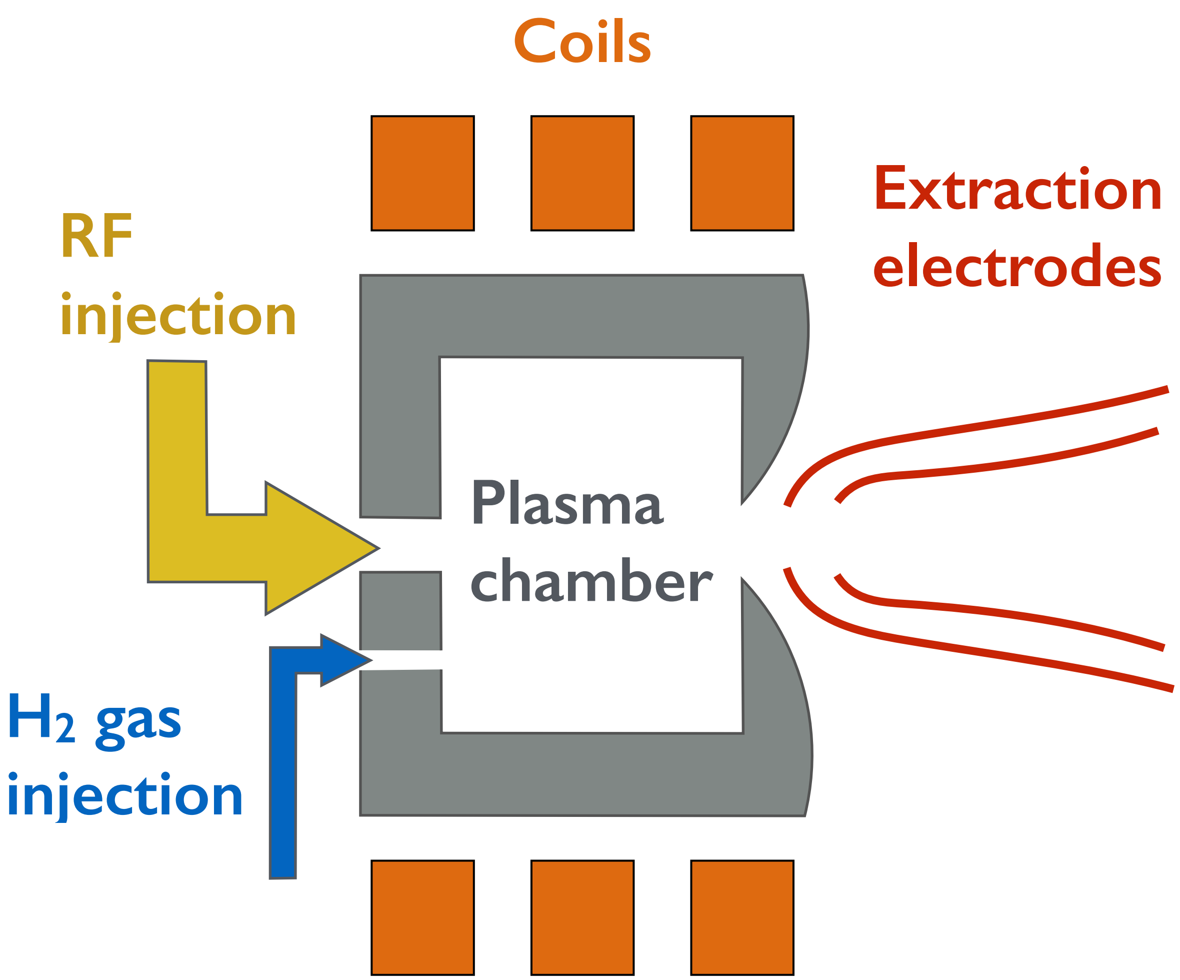
Spoke section is the first superconducting part of ESS linac

Medium and High beta elliptical cavities increase the beam energy to 2 GeV

High Energy Beam Transport, HEBT, adjusts and matches the beam to the target requirements

ION SOURCE

- An Electron Cyclotron Resonance, ECR, ion source uses the resonance between the magnetic and electric field to create ionized beams of particles.



$$F = qvB$$

$$F = \frac{mv^2}{r}$$

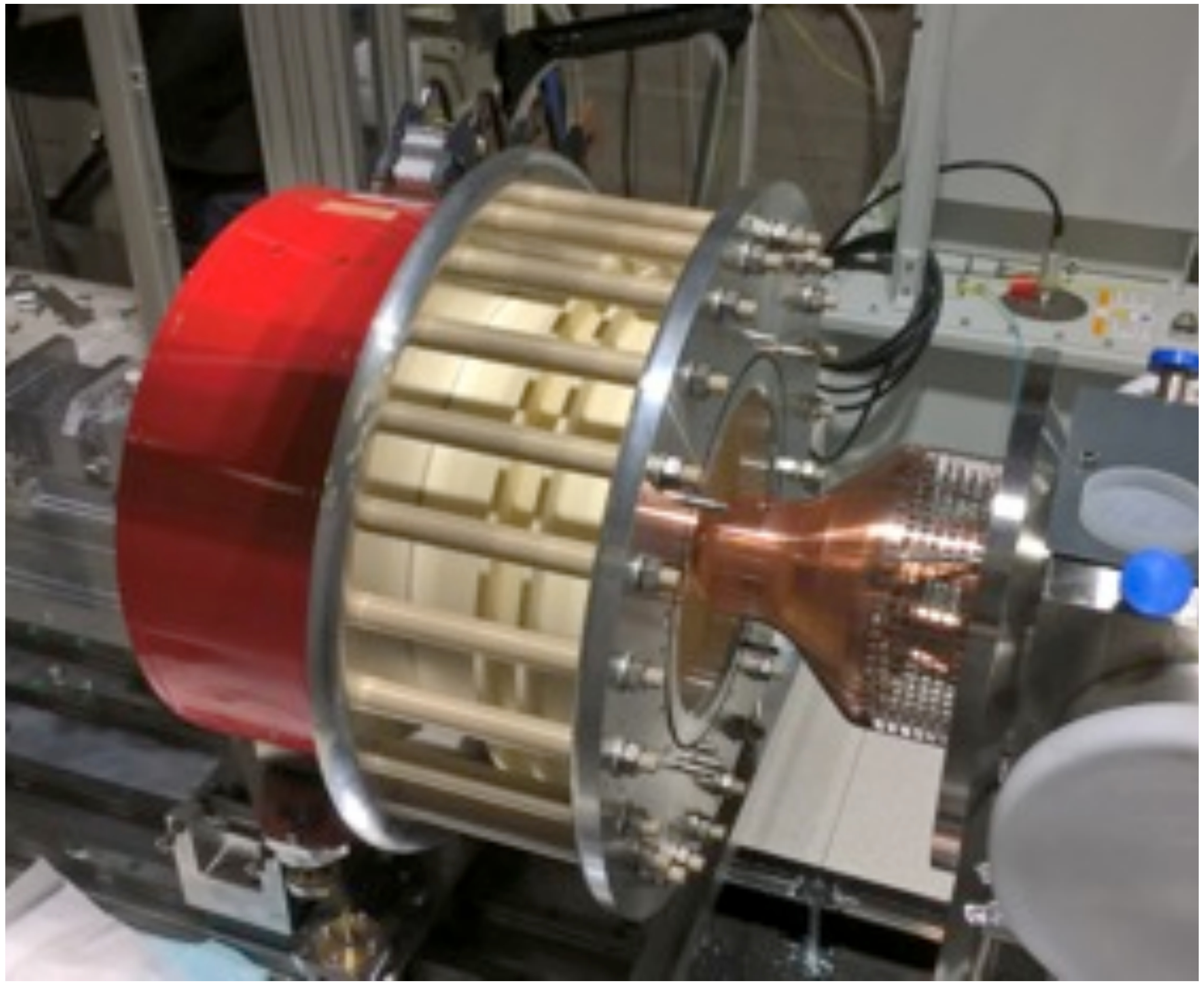
$$\frac{mv^2}{r} = qvB$$

$$\frac{mv}{r} = qB$$

$$\omega = \frac{qB}{m}$$

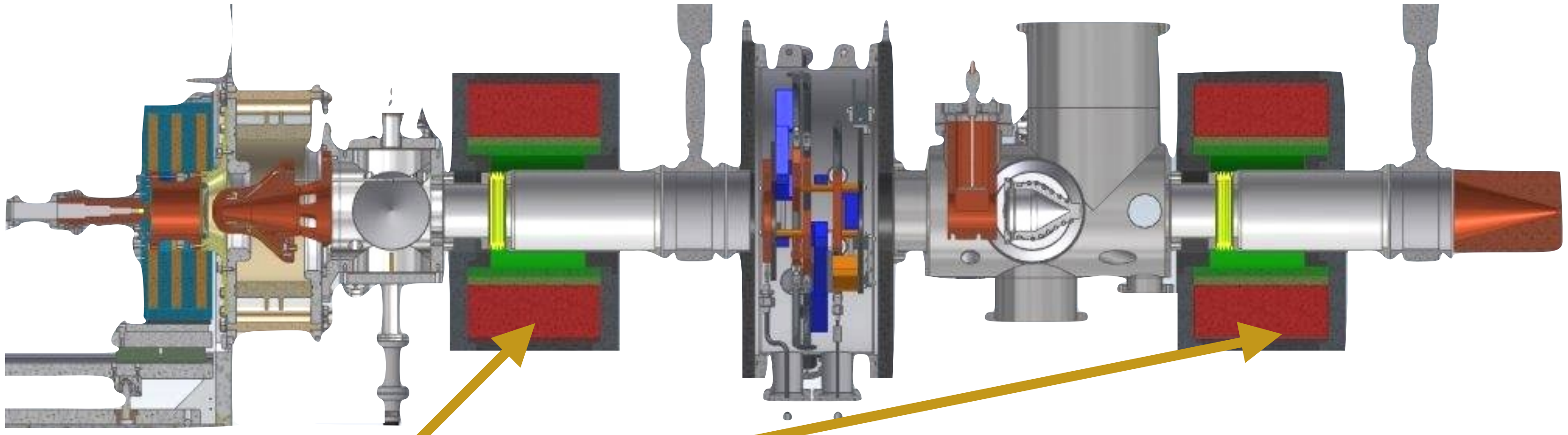
ESS ION SOURCE

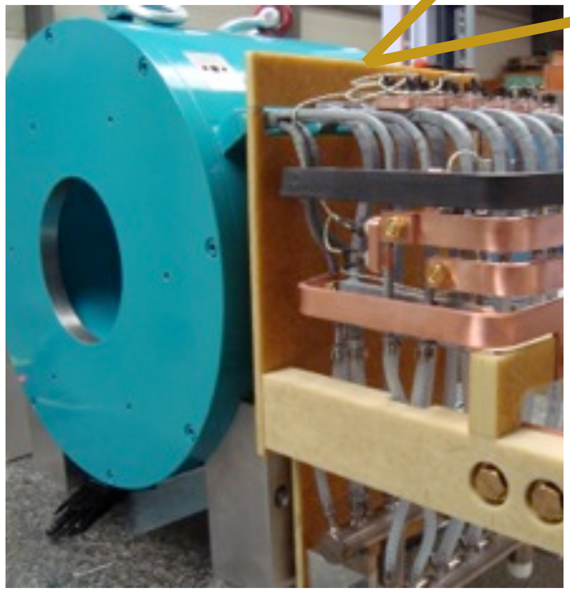
- A Microwave Discharge Ion Source generates a proton beam pulse of up to 3 ms with an energy of 75 keV and an intensity exceeding 80 mA at the source exit.

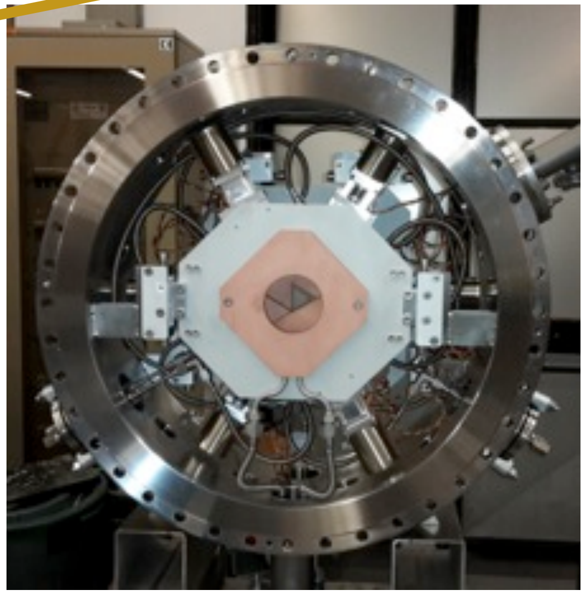


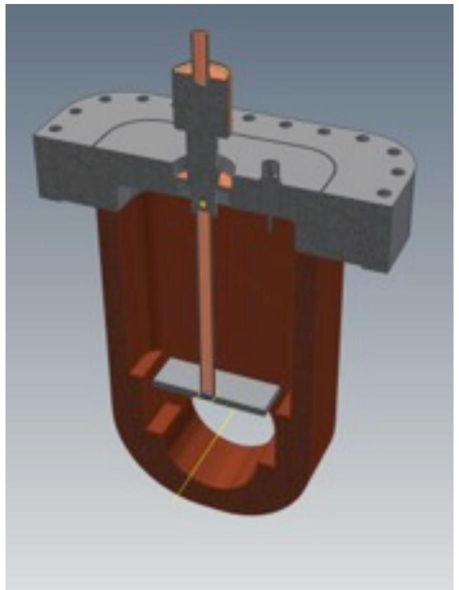
LOW ENERGY BEAM TRANSPORT

- The diverging beam out of ion source should be transported, measured and adjusted to the next structure.
- Two magnetic solenoids provide the required transverse focusing.

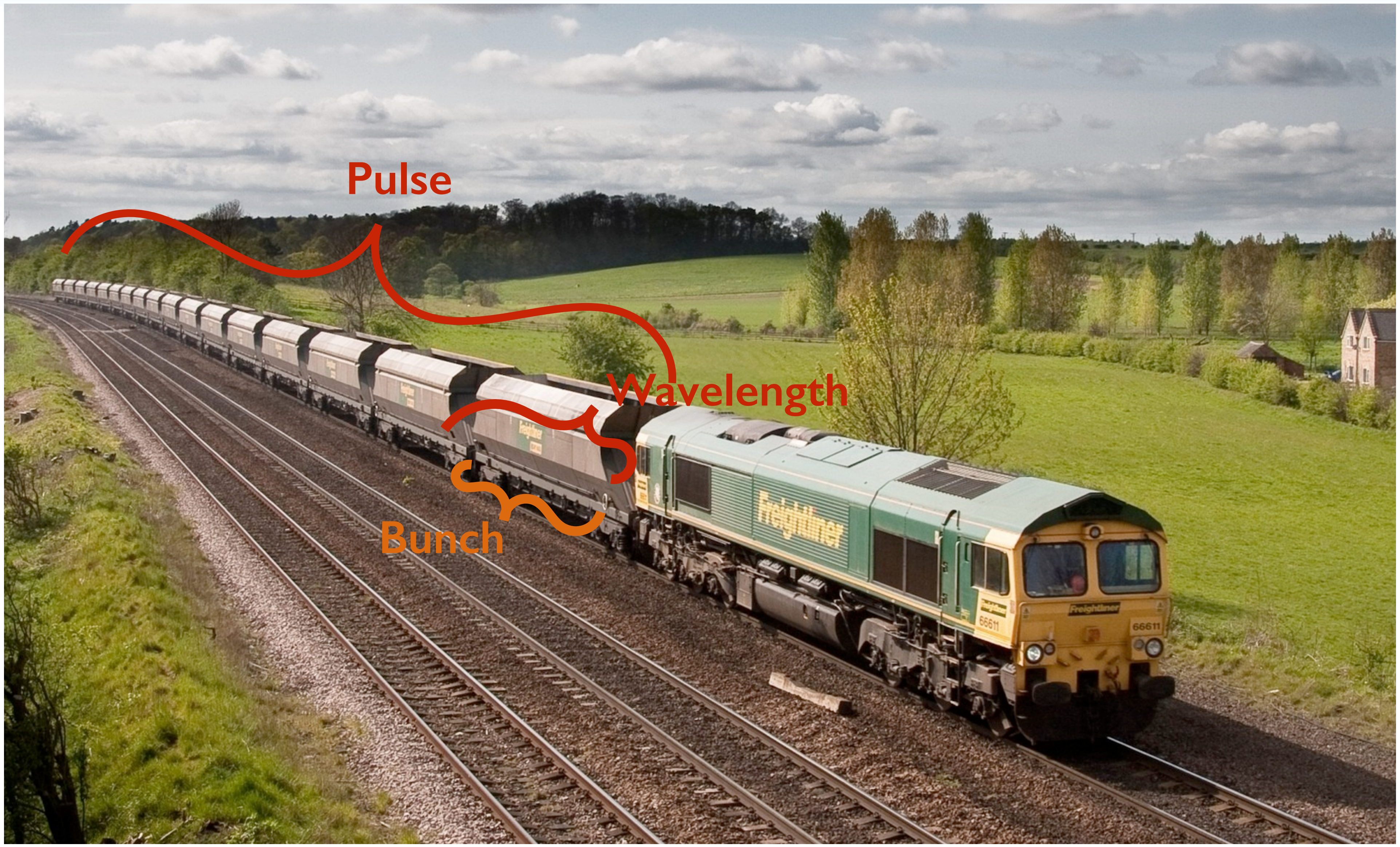



Solenoid


Iris


Chopper

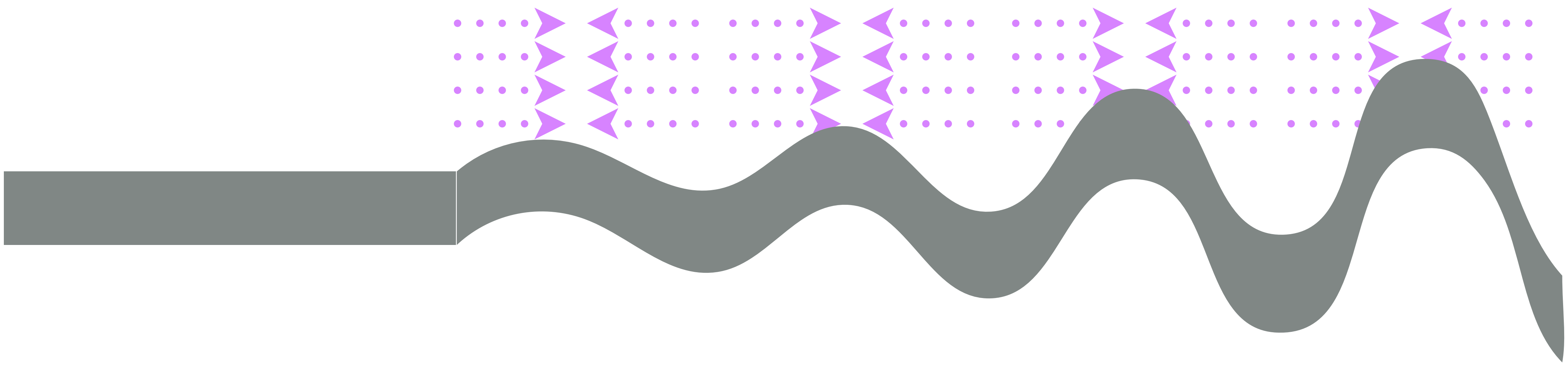
PULSE VS. BUNCH



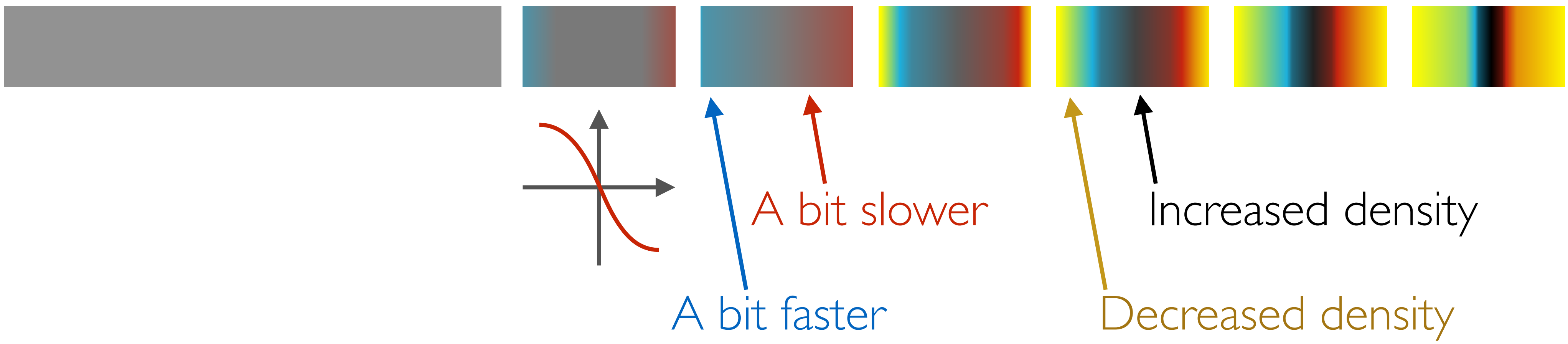
- ESS case:
  - 14 pulses per second, each 2.86 ms long (at 0.95 c this is ~815 km)
  - ~1E6 Bunches per pulse

RF ACCELERATION OF DC BEAM

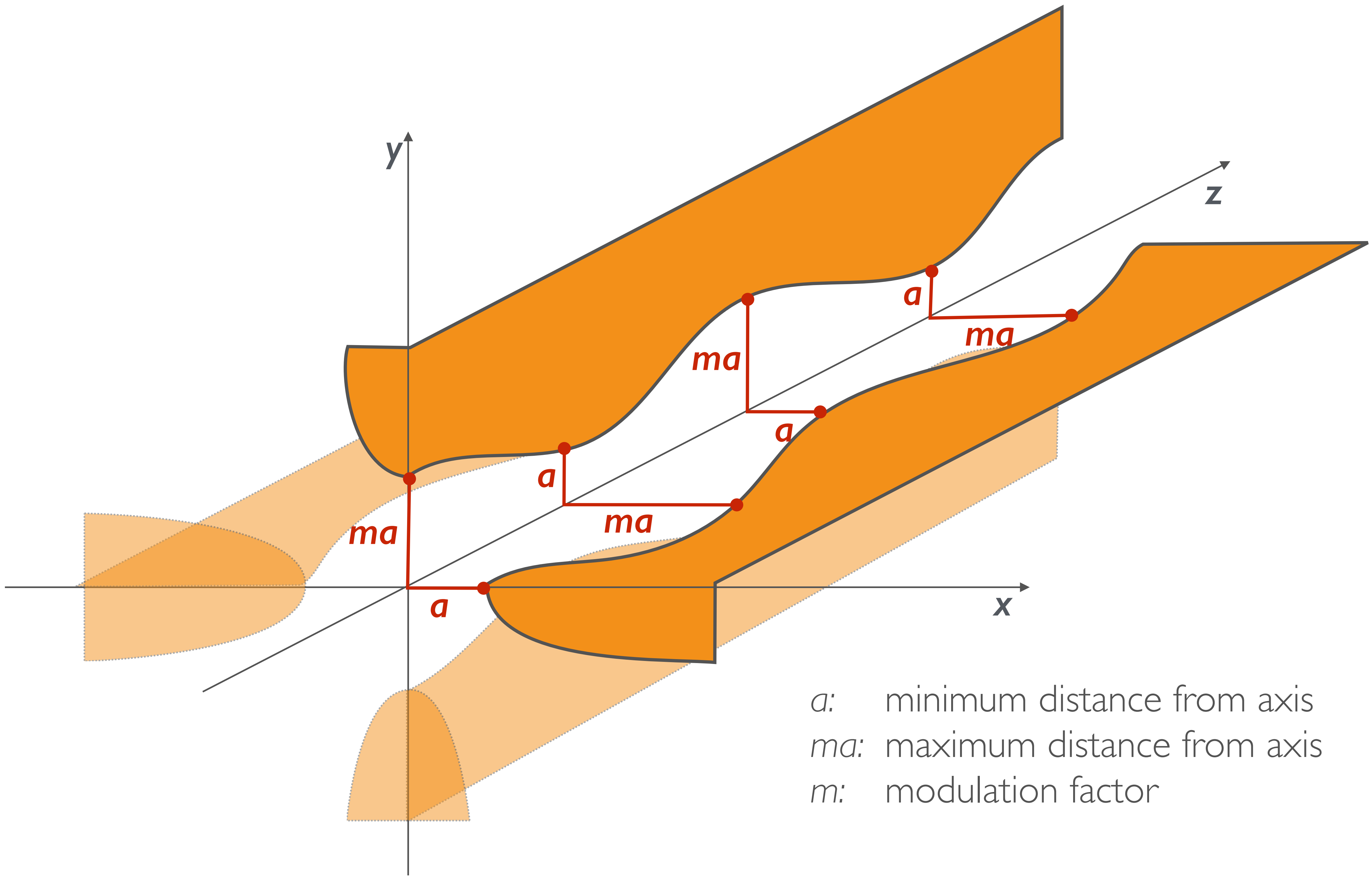
- The full pulse length of a DC beam could not be efficiently accelerated using an RF field as part of beam (half of it) losses energy in the decelerating field and a significant part does not get enough acceleration



- Classic bunching relies on velocity modulation

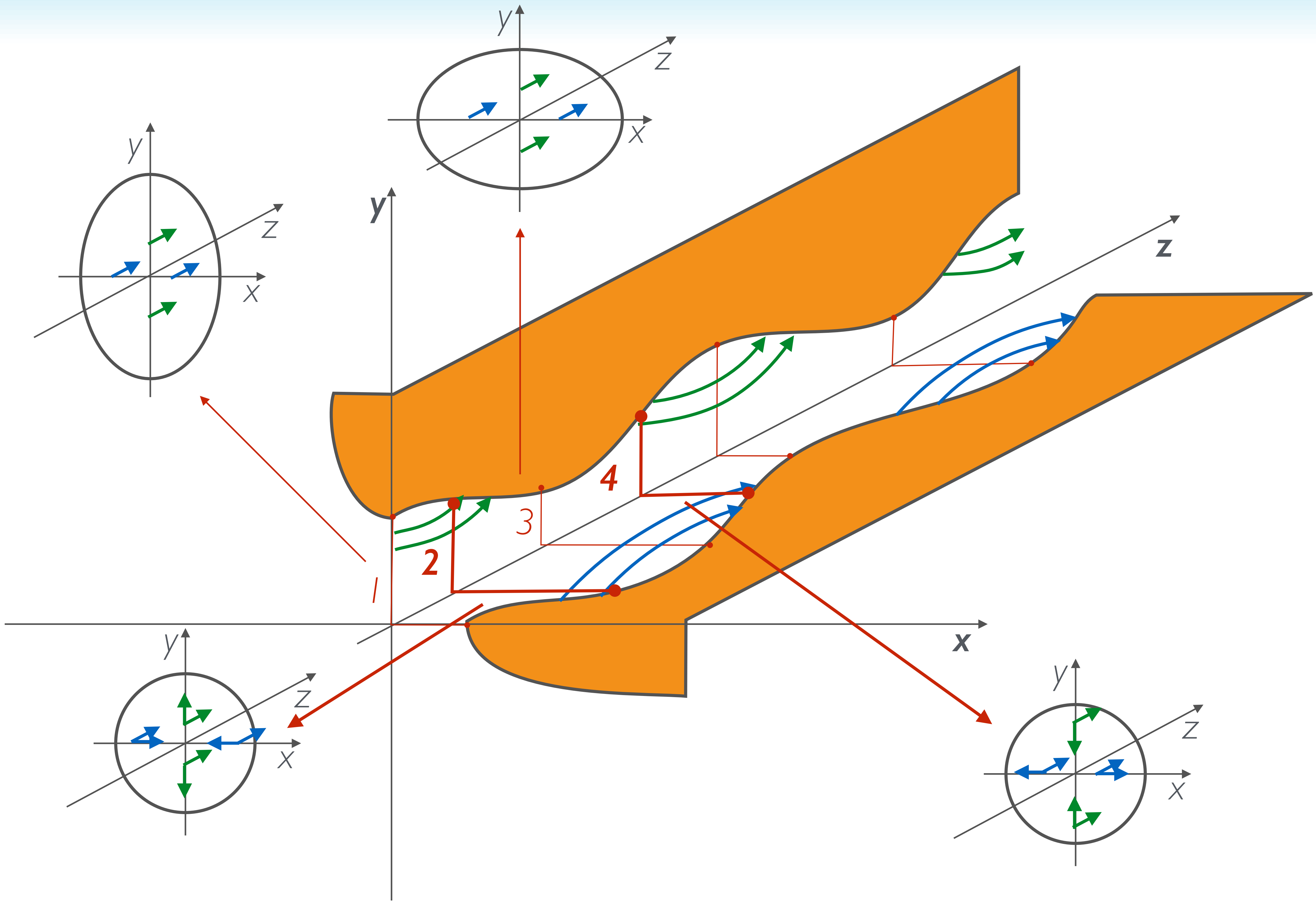


RADIO FREQ. QUAD. (RFQ) I



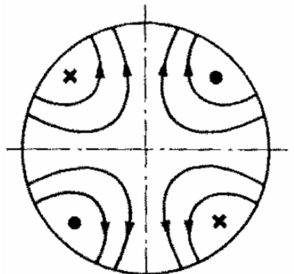
a: minimum distance from axis
ma: maximum distance from axis
m: modulation factor

RADIO FREQ. QUAD. (RFQ) II

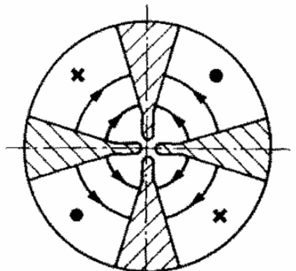


RADIO FREQ. QUAD. (RFQ) III

- Modern hadron accelerators use RFQs as their first accelerating and bunching structure.
  - RFQs can have bunching efficiency of >90%, while the classic methods are limited to ~50-60%.
  - RFQs can provide beams of high quality and high current
- There are two main types of RFQs, four-rods and four-vanes
- Both acceleration and bunching is achieved by RF field
- RFQs work in the TE_{21} mode (in TE mode, the electric field is perpendicular to beam direction of propagation.



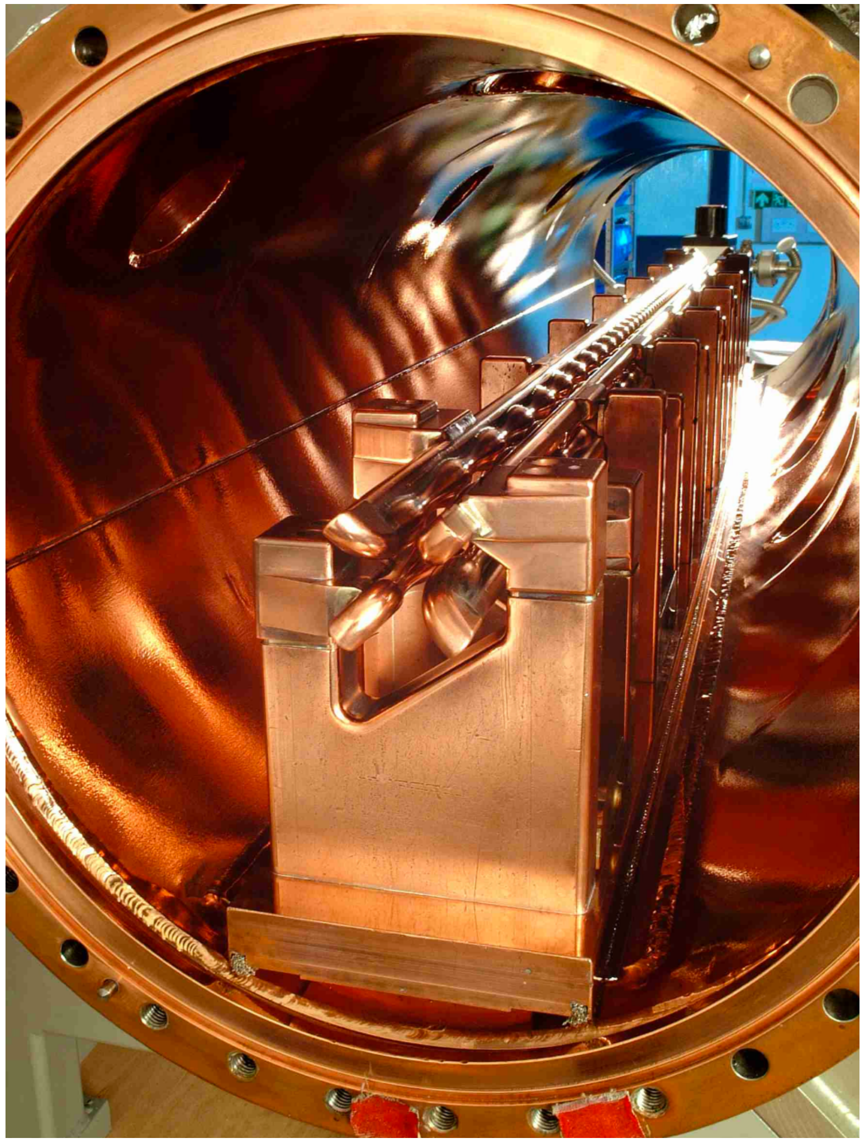
empty cavity



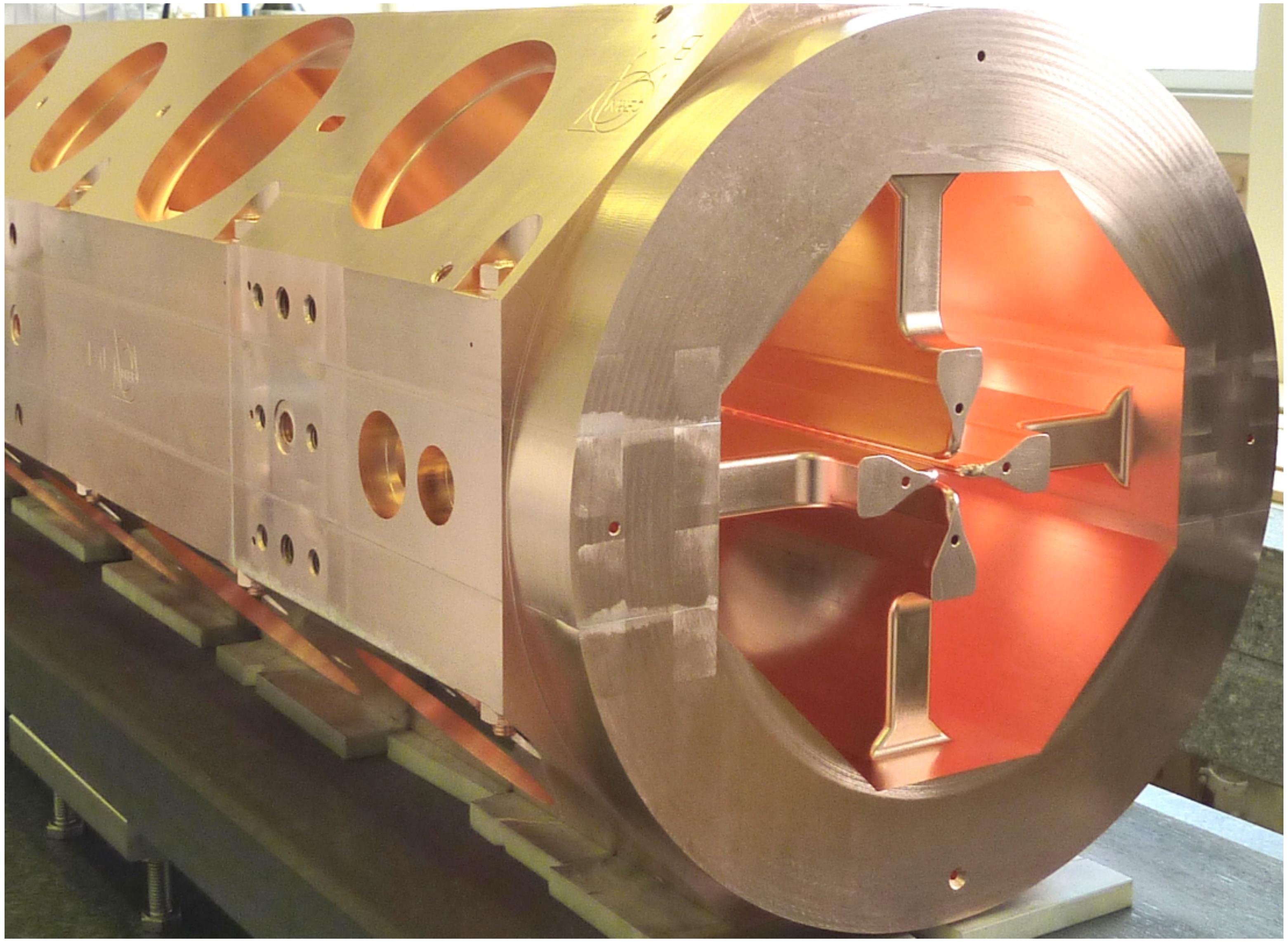
Cavity with vanes

- Longitudinal modulation of the electrodes (vanes or rods) creates a field in the the direction of propagation.

FOUR-ROD RFQ

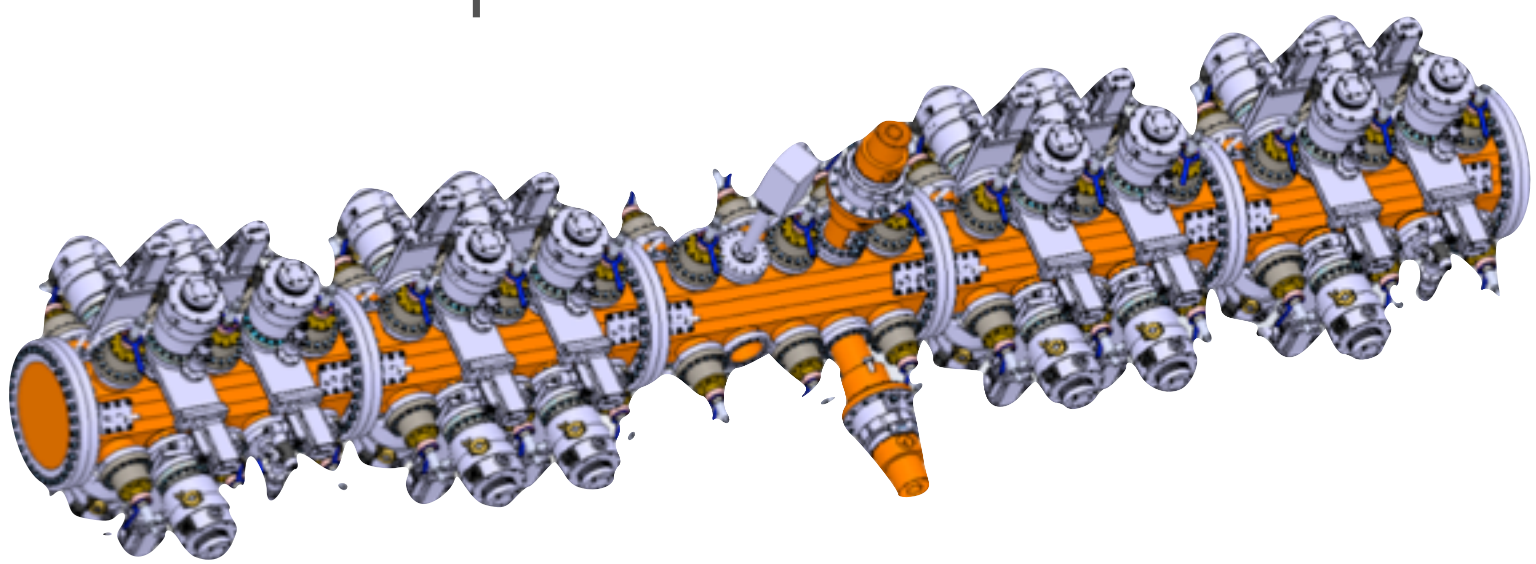


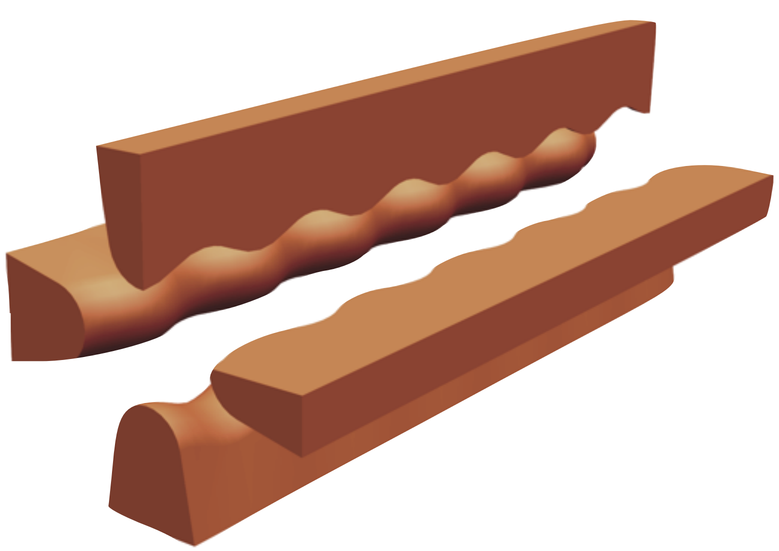
FOUR-VANE RFQ



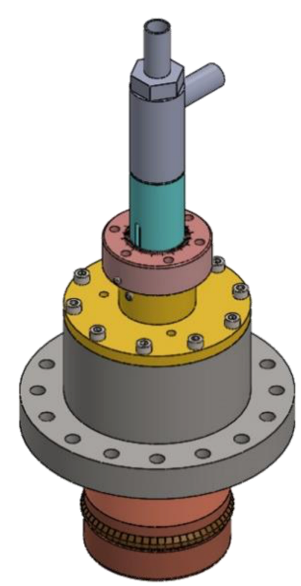
ESS RFQ

- Accelerates the beam from 75 keV to 3.62 MeV.
- It has 60 tuners and 2 RF couplers.

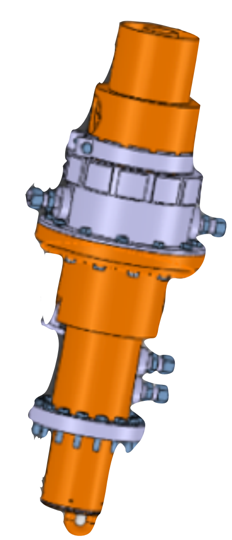




4 vane RFQ



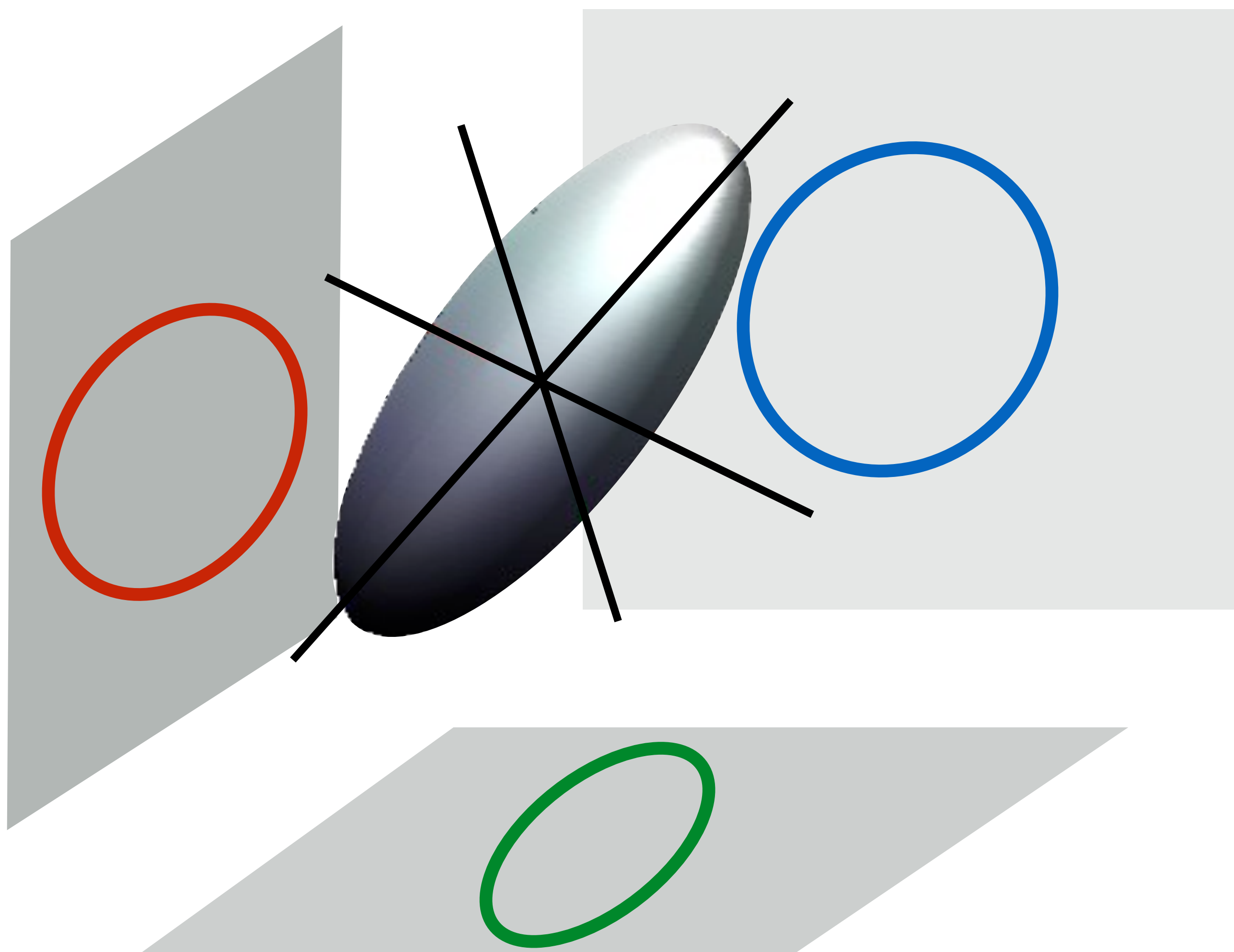
Tuners



RF coupler

A BUNCH OF PARTICLES

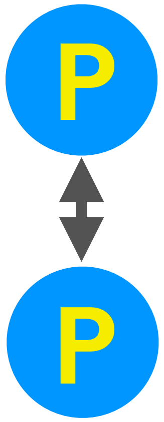
- Beam at the exit of the RFQ is an ellipsoid in the 6D phase-space (3D space shown here).



- ESS case:
  - There are ~1E9 protons per bunch, how about their repulsion?

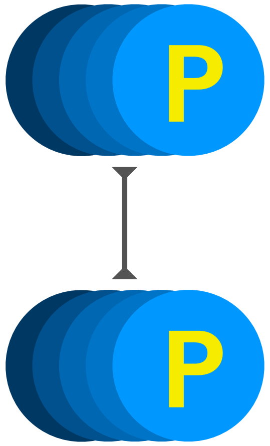
SPACE CHARGE I

2D point charges



- $F_E = q^2/2\pi\epsilon_0 . r$

- $F_B = -\mu_0 I^2/2\pi r = -(1/c^2\epsilon_0)(q\mathbf{v})^2/2\pi r = -\mathbf{v}^2/c^2 .\ F_E$



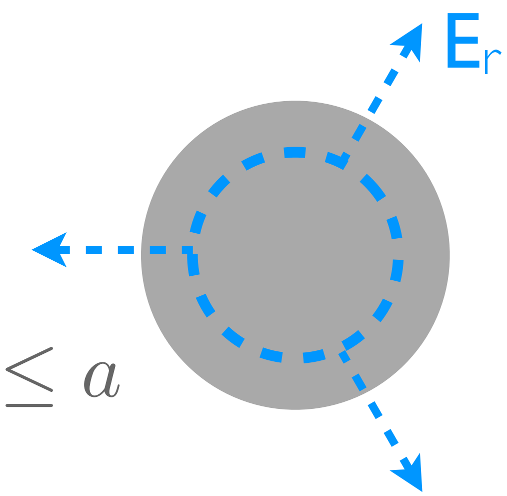
- $F_{tot} = (1-\mathbf{v}^2/c^2).\ F_E$

SPACE CHARGE III

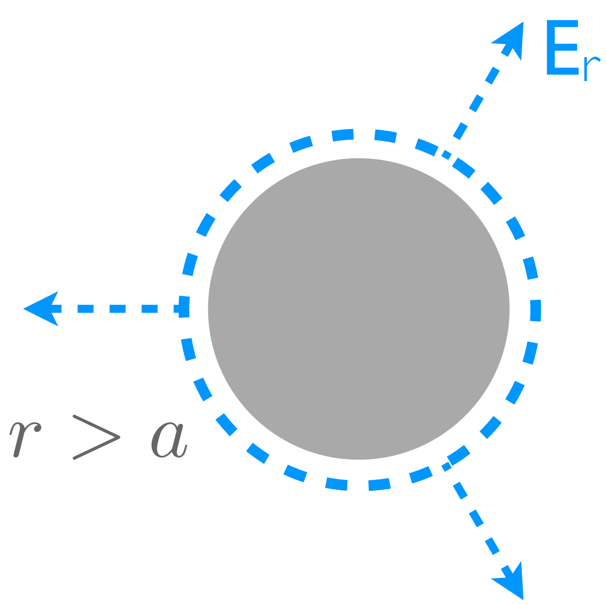
- Unless the beam has a very special distribution, the space charge forces are non-linear.
- A non-linear force will increase the beam emittance.
- It also has a defocusing effect which could be approximated be a series of defocusing lenses.
- In rings the space charge will also affect the tune of the ring
- As the forces are much stronger in lower energies the beam should be confined in all the three planes, horizontal, vertical and longitudinal.

$$2\pi r \mathbf{E} = \frac{Q}{\epsilon_0}$$

$$\mathbf{E} = \frac{Nqr}{2\pi\epsilon_0 a^2} \quad r \leq a$$



$$\mathbf{E} = \frac{Nq}{2\pi\epsilon_0 r} \quad r > a$$

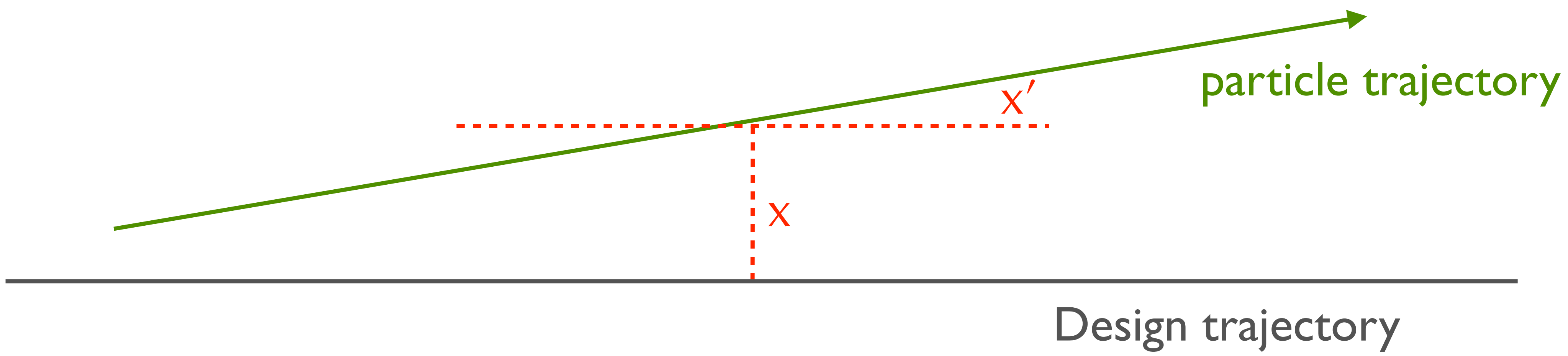


HILL'S EQUATION

- Equation of motion for paraxial particles in periodic systems:

  $x''(s) + k_x(s) = 0,$
  $y''(s) + k_y(s) = 0,$

  where $k_x(s)$ and $k_y(s)$ are periodic focusing functions,
  $k(s+L_p) = k(s)$, and s is the distance along the design trajectory.



G.W. Hill, "On the part of the Motion of the Lunar Perigee Which is a function of the mean motions of the sun and moon", Acta Math., 8, 1-36 (1877)

EMITTANCE

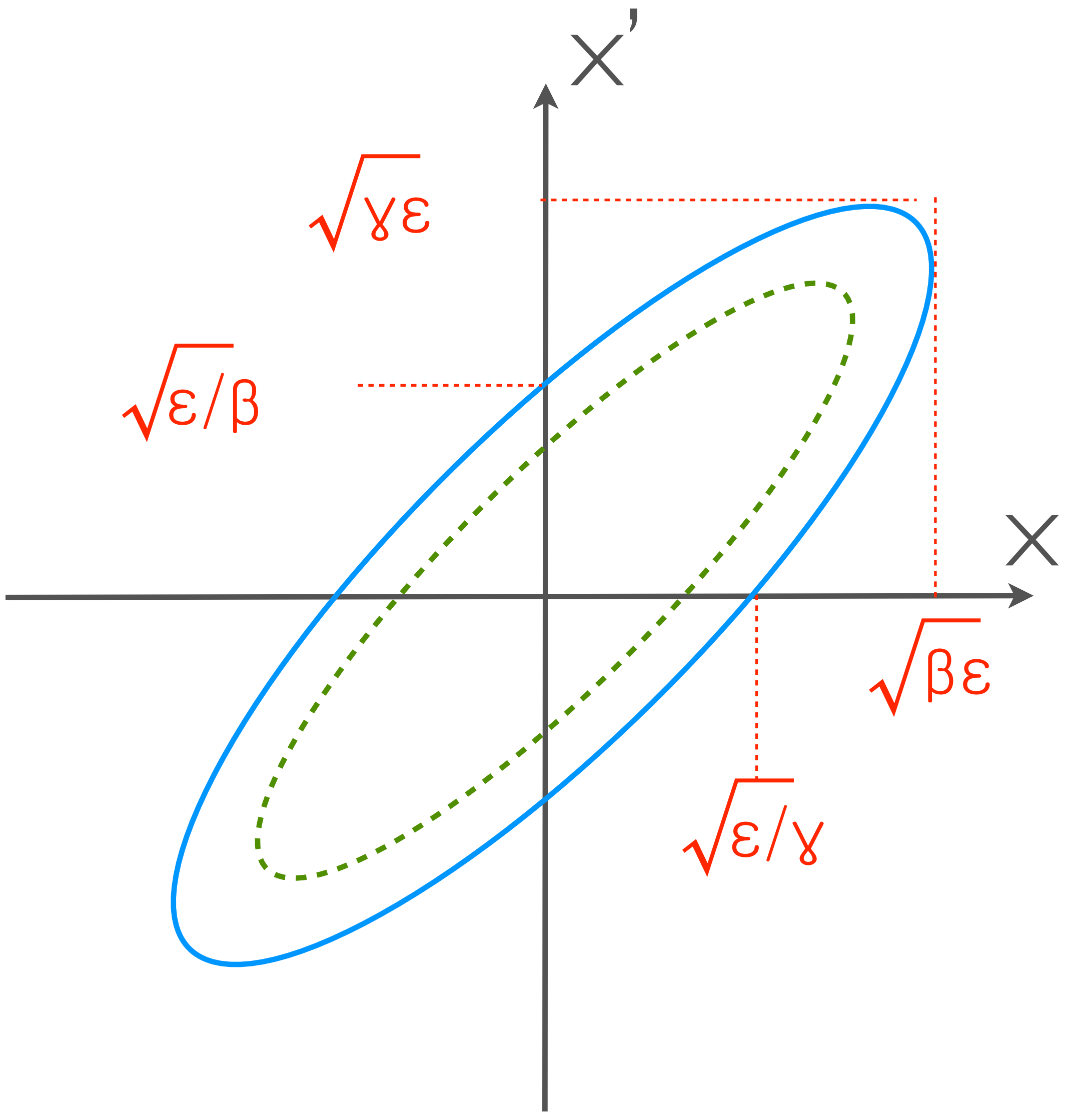
- Solving the Hill's equation one gets:

$$x = A\,\omega(s)\,\cos(\sigma(s)+\phi)$$

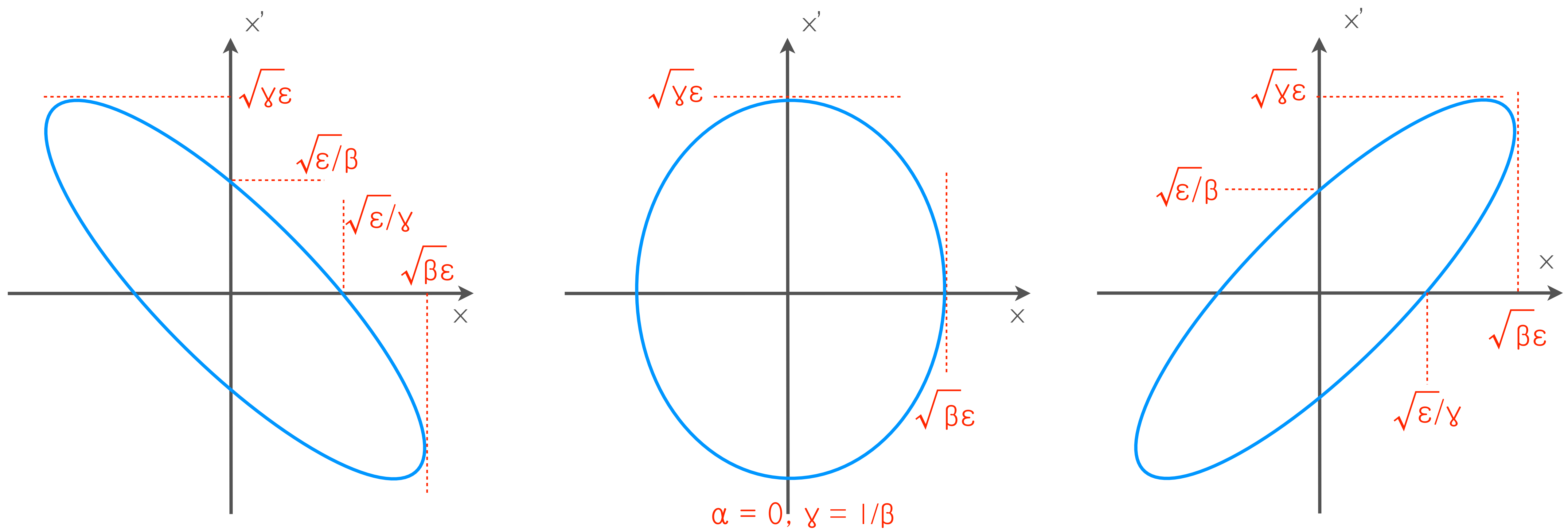
and a little manipulation gives:

$$\gamma x^2 + 2\alpha x x' + \beta x'^2 = A^2$$

which is the equation for an ellipse in xx' phase space. Area of the largest ellipse is denoted by $\pi\varepsilon$ which encloses all particles.



COURANT SNYDER



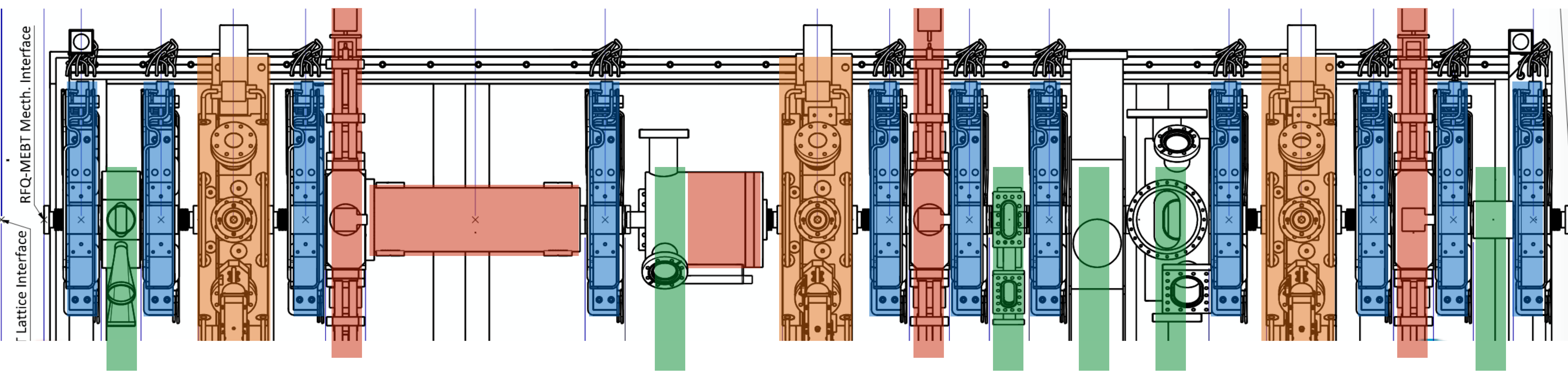
In the absence of nonlinear forces, the ellipse area, the **emittance,** is a constant of motion, Liouville's theorem.

A. A. Vlasov, "The vibrational properties of an electron gas", Sov. Phys. Usp. 93, 444-470 (1967), [translation from Zh. Eskp. Teor. Fiz. 8, 291 (1938)]

MEBT

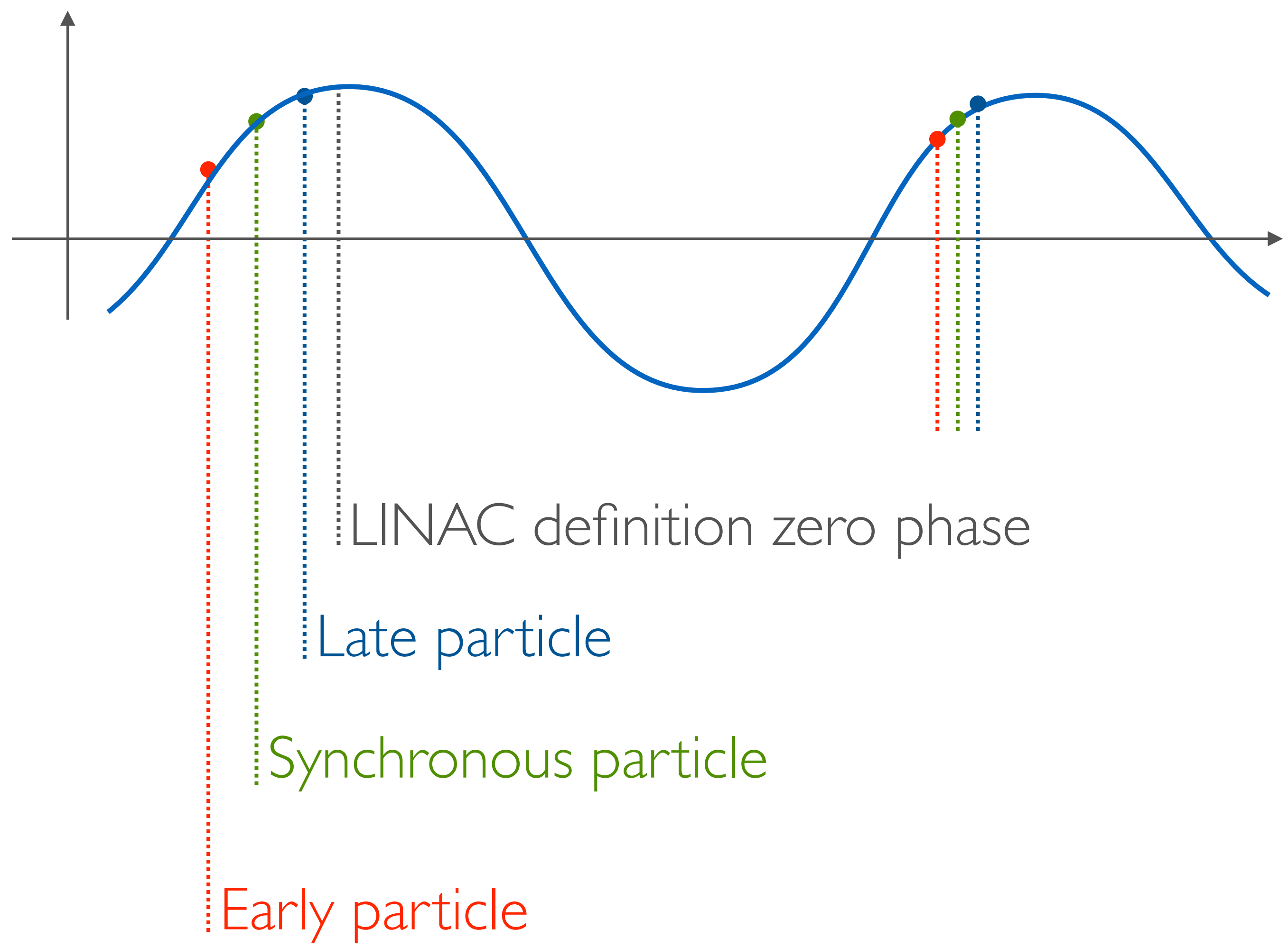
- Match RFQ output beam to the DTL (Three planes)
- Characterise the beam (Three planes)
- Clean the head of pulse using a fast chopper
- Clean the transverse halo using scrapers

QUADs/Correctors, Cavities, Measurement Devices, Cleaning Devices



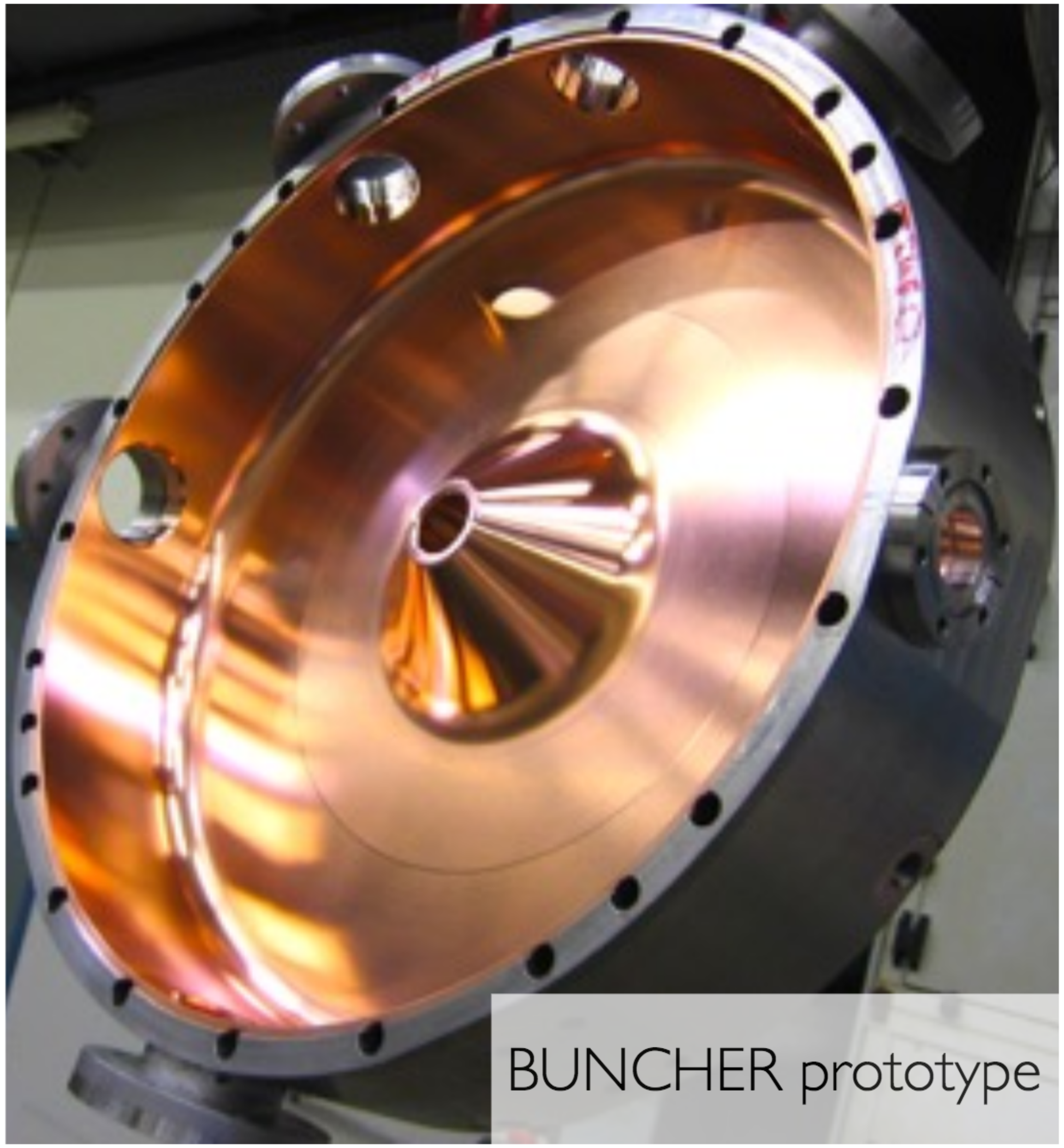
SYNCHRONOUS PHASE

- The highest acceleration is on the voltage peak.
- To keep the beam bunched, acceleration must be done at a lower phase



MEBT BUNCHER

- A buncher cavity is used to focus the beam in the longitudinal direction (direction of propagation).
- ESS MEBT has three buncher cavities
  - E0TL ~150 kV
  - Power coupler limit ~22.5 kW
- These bunchers do not increase the beam energy.



MEBT DIAGNOSTICS

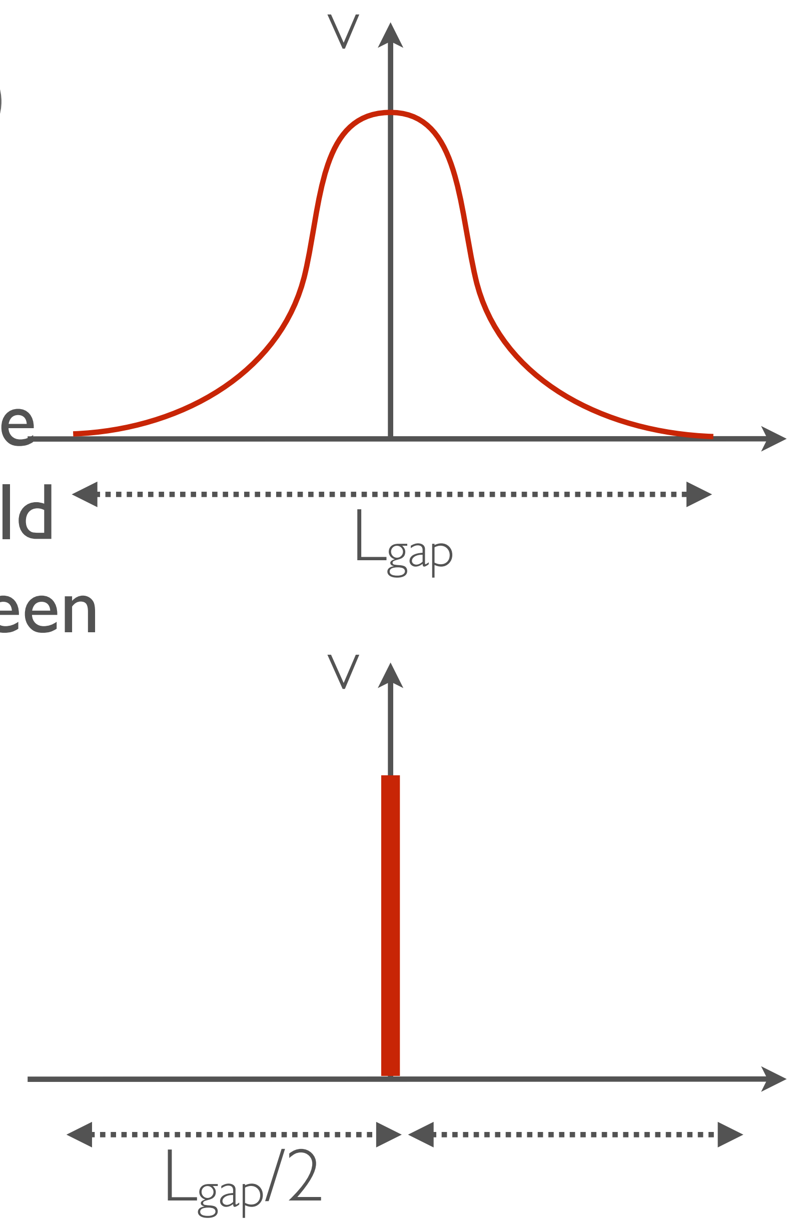
- ESS linac is equipped with beam instruments to measure the beam properties:
  - Beam Current Monitors (BCM or BCT) for measuring the beam current
  - Beam Position Monitors (BPM) to measure the beam's position and time
  - Emittance Measurement Unit (EMU) which measures the beam emittance and its orientation
  - Beam Profile Monitors, these measure only the beam profile in transverse
  - Beam Shape Monitors (BSM) to measure the longitudinal span of the beam

DRIFT-KICK-DRIFT

- The RF acceleration in a gap with finite length could be approximated by a drift (with half the gap length), a sudden increase in energy, and a second drift.

$$E_z(r,z) = \sum_0^{\infty} A_m J_0(a_m r)\cos(2\pi m z/L_{gap})$$

$$E_z(r,z,t) = E_z(r,z)\cdot\cos(\omega t+\phi)$$

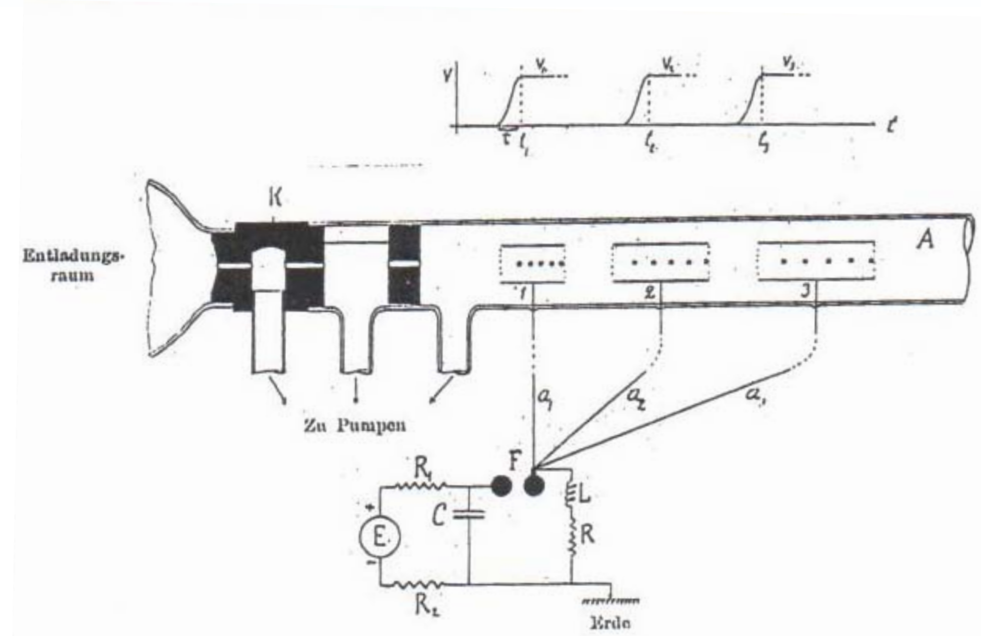


- However, while a particle with finite velocity traverses the cavity, the field changes and the effective voltage seen by the beam is reduced by a factor called Transit Time Factor.

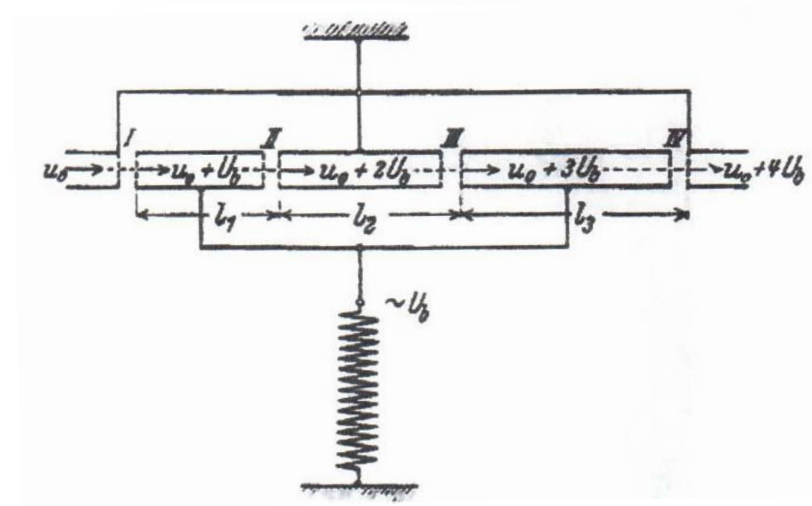
$$\Delta W = qE_zTL_{gap}cos(\phi)$$

DRIFT TUBE LINAC I

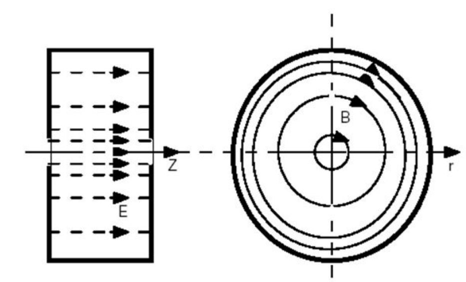
- In 1924 Gustav Ising (Swedish) published a concept where voltage waves (from a spark discharge) to a set of drift tubes would accelerate a beam of particles, but he was unable to make it work!



- In 1927 Rolf Wideröe (Norwegian) read about Ising's concept and replaced the spark discharge with an ac oscillator. He only used one drift tube in between two accelerating gaps and it worked in 1928.

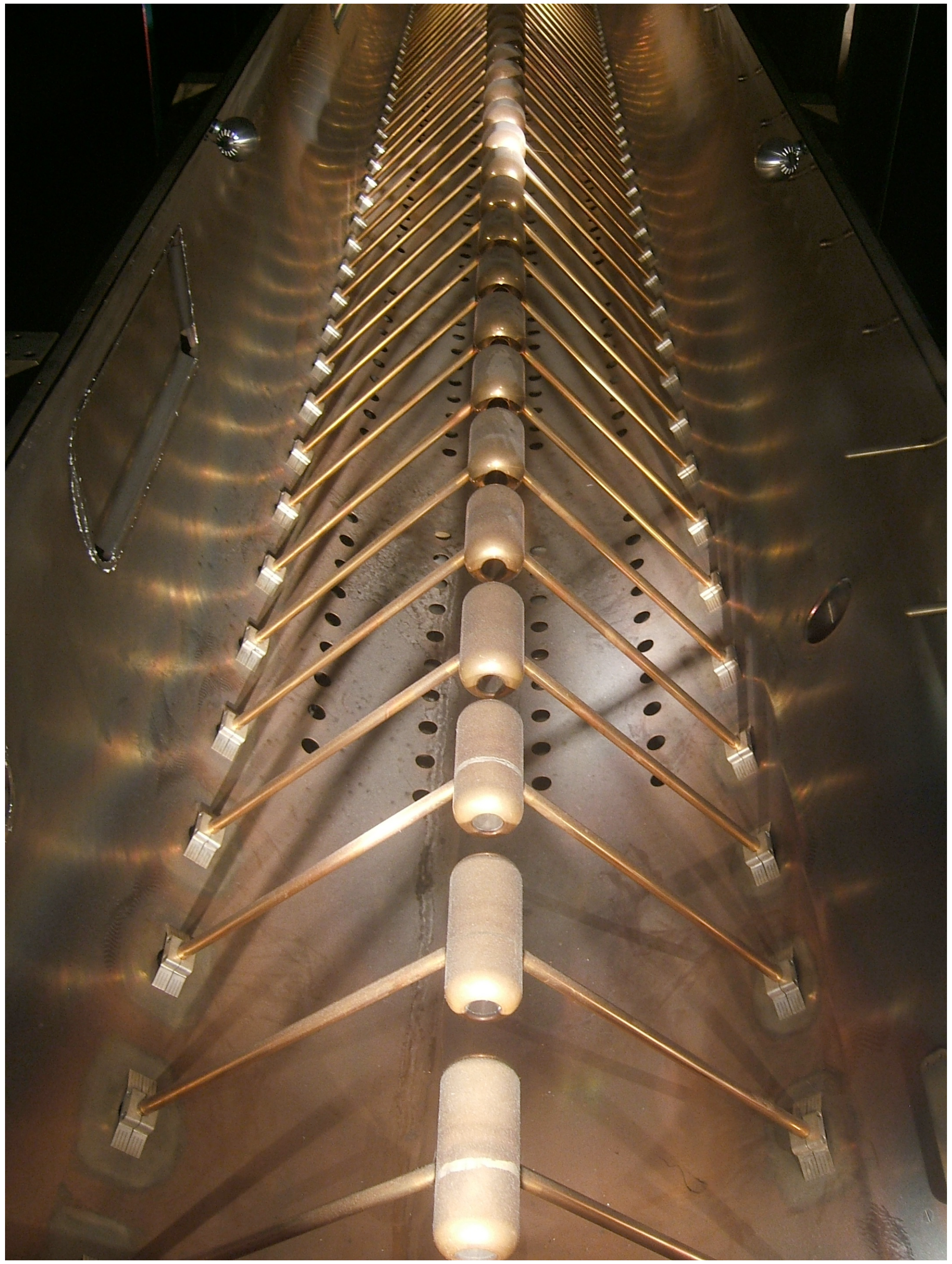


- In 1947, Luis Alvarez (Berkley) designed the first modern DTL (4-32 MeV) using a resonant cavity at 200 MHz (TM_{010}).



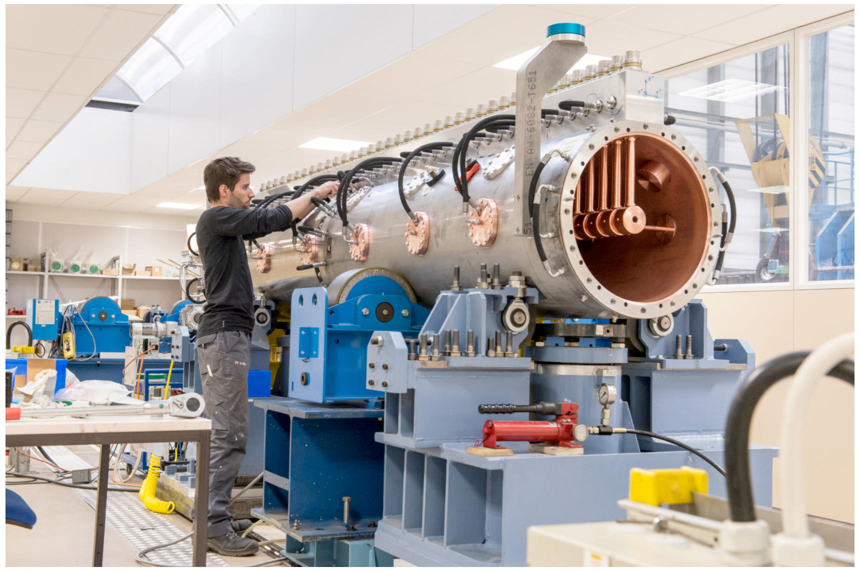
Electric field Magnetic Field

DRIFT TUBE LINAC II



Linac1, 202.56 MHz

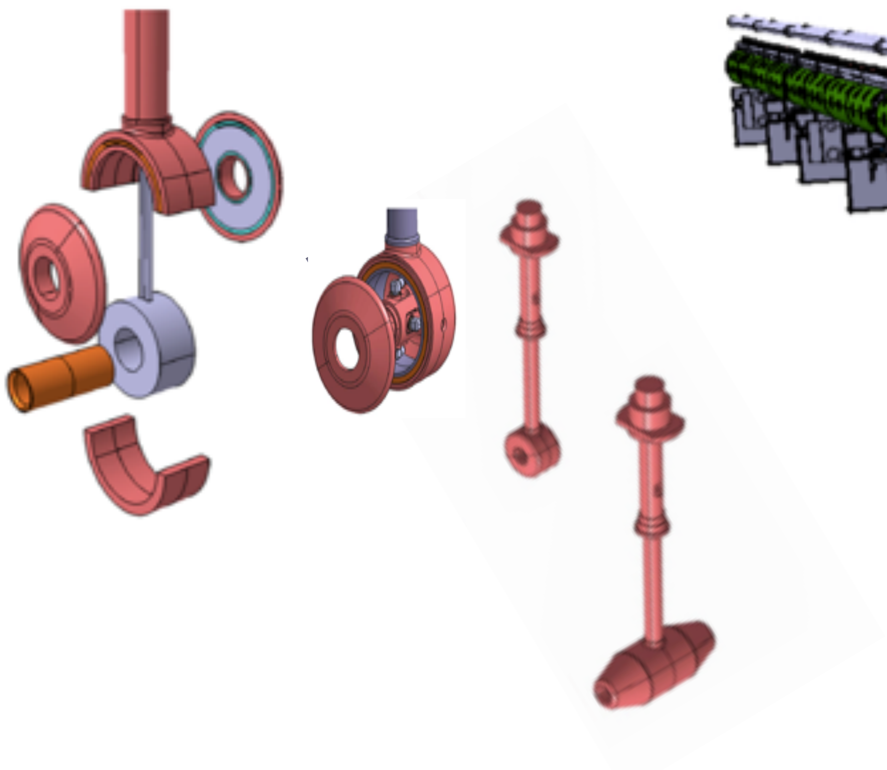
SuperHilac, 70.285 MHz

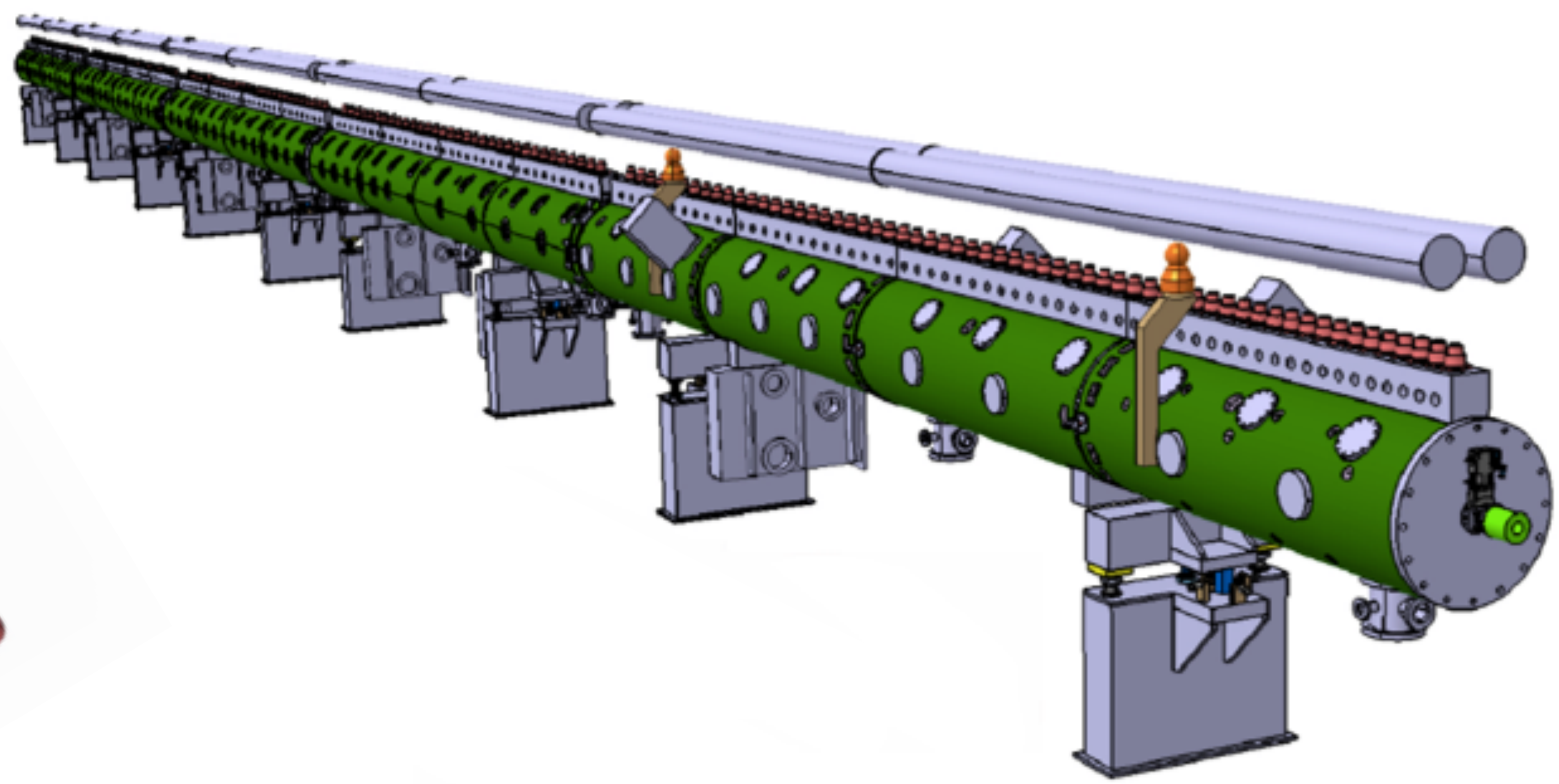


Linac4, 352.21 MHz

ESS DTL

- DTL accelerates the beam from 3.62 to ~90 MeV





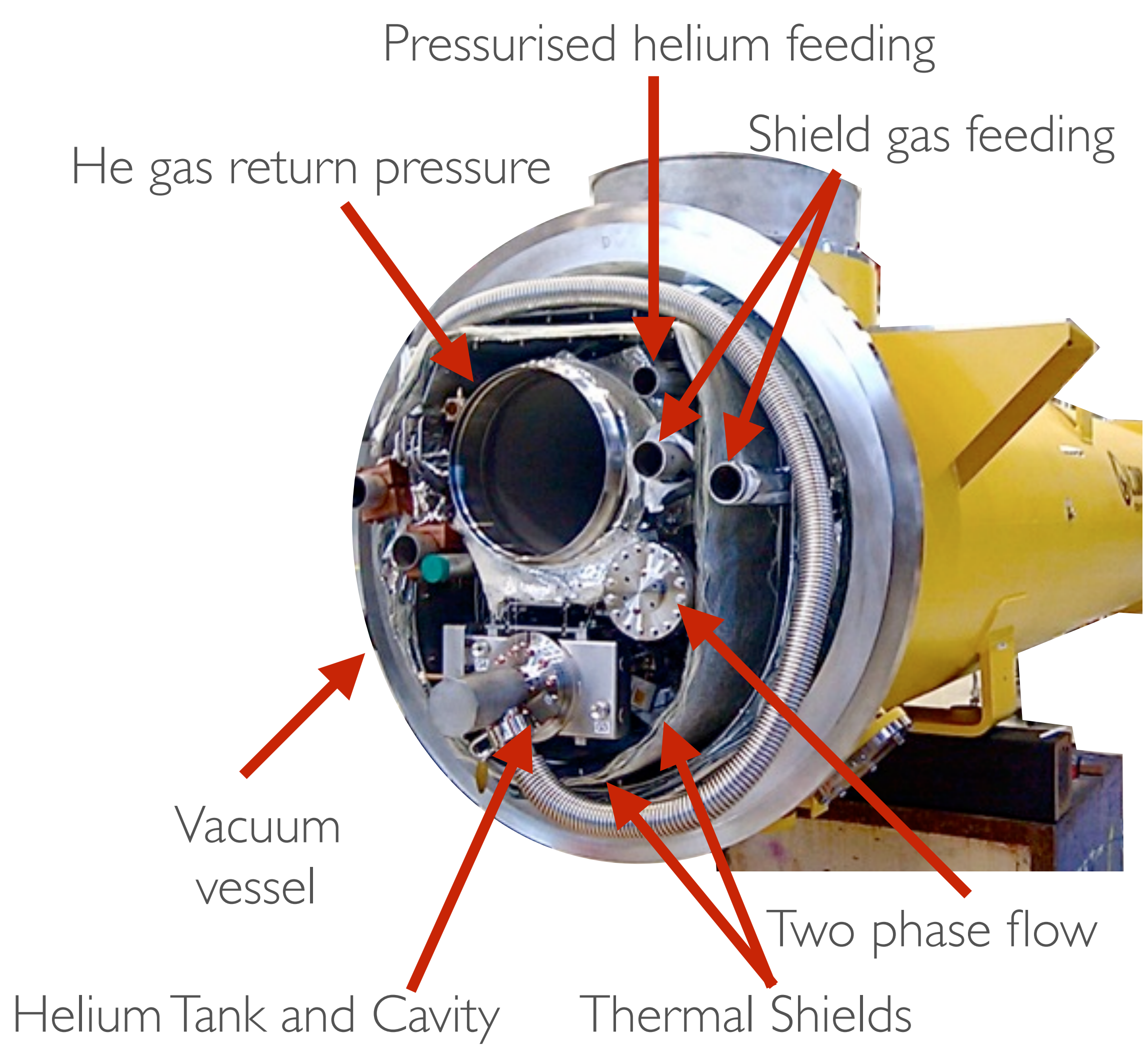
TRANSITION TO SC CAVITIES

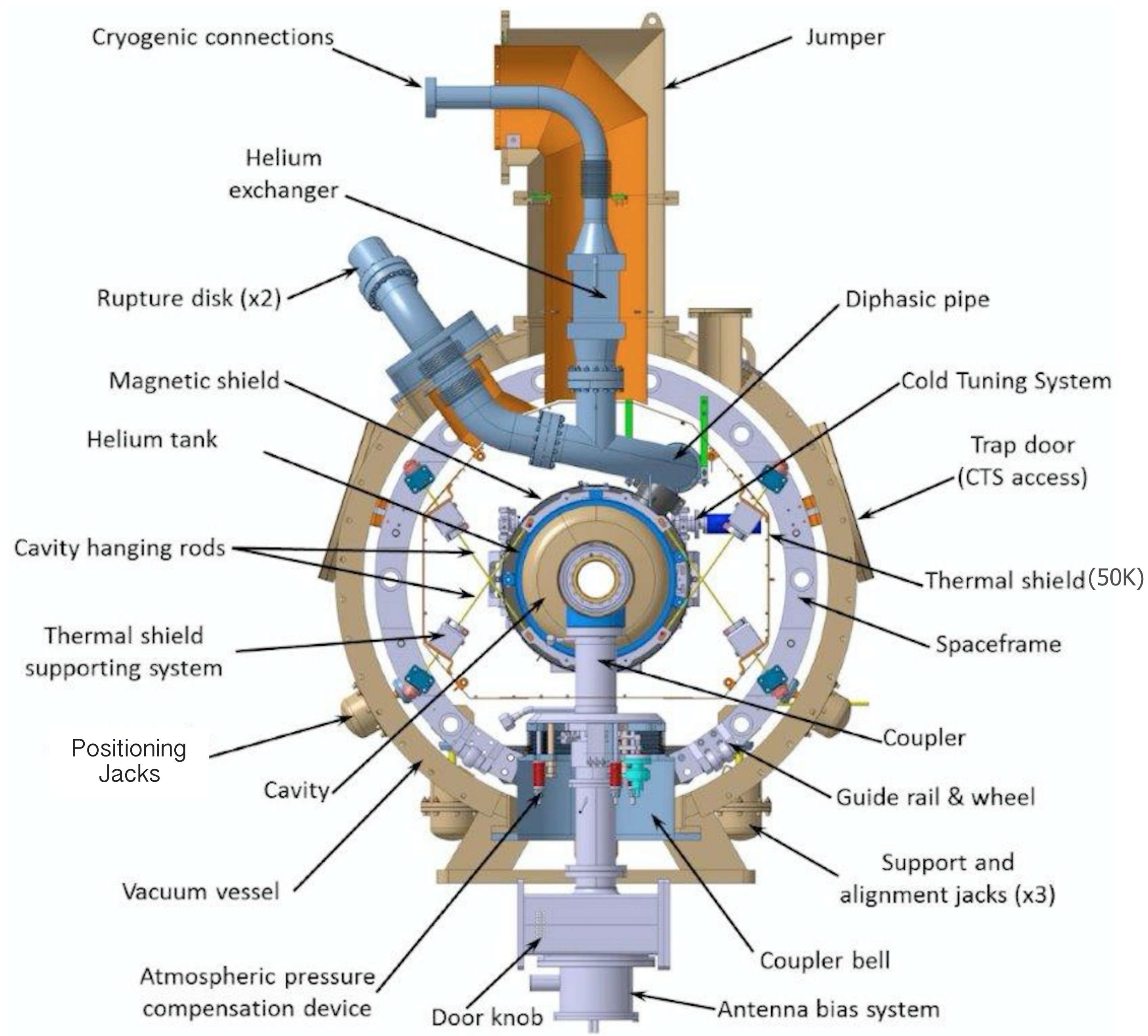
- In normal conducting cavities, a significant part of the RF power is wasted in the copper structure in the form of ohmic loss.
  - For the ESS linac, after several rounds of optimizations, the ratio of wasted power to beam power is 1 to 1, i.e., half the provided power is wasted!
- Superconducting cavities do not have this problem, they have other problems!
- RF efficiency of SC cavities is ~100% (compare to 50% in the ESS DTL).
- But, they operate at cryogenic temperatures (few Kelvin), and cooling them down to that temperature requires significant energy.
- Depending on the duty factor, energy, … one chooses the optimized transition energy.

SUPERCONDUCTIVITY

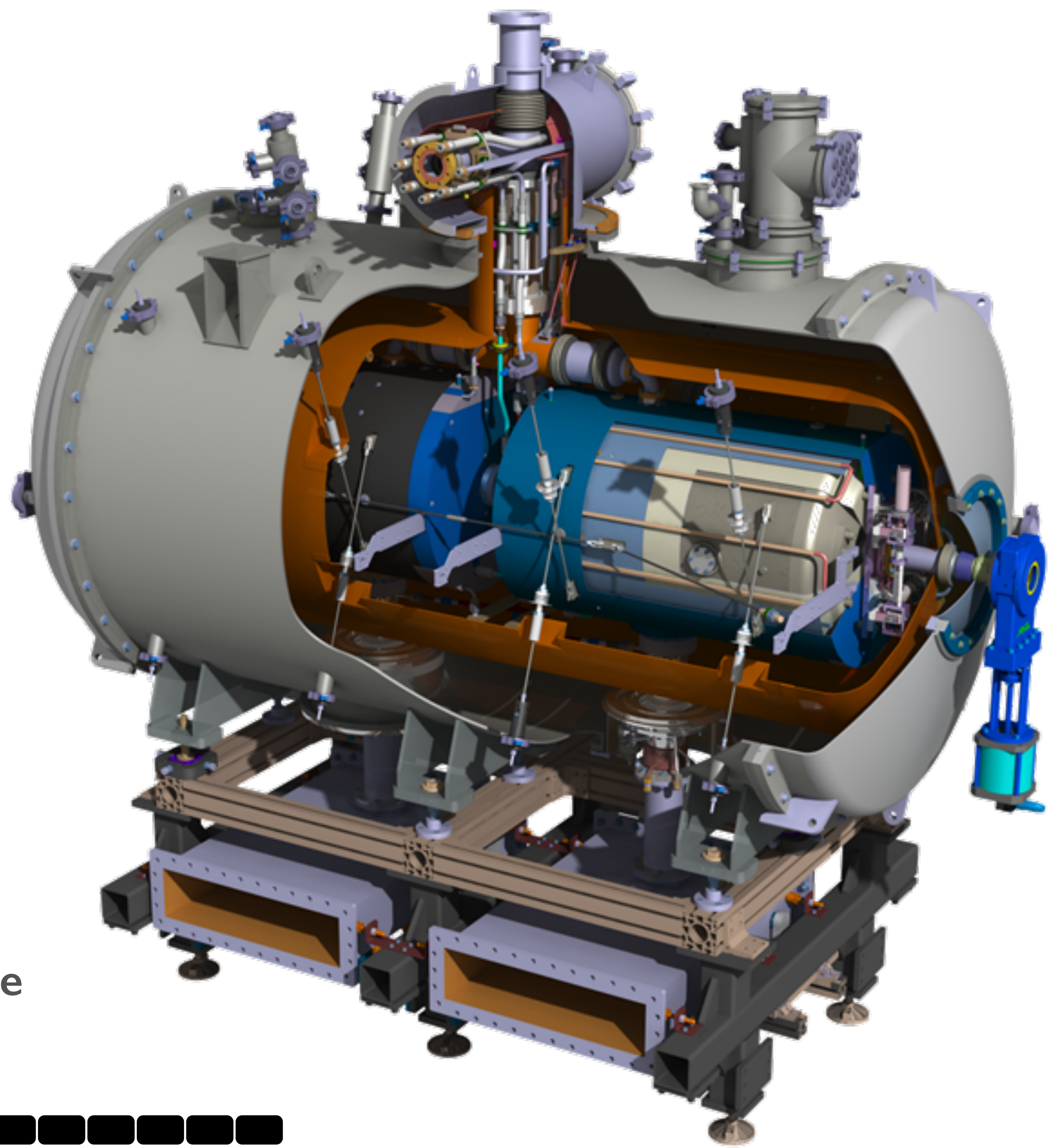
CRYOMODULE

- ILC cryomodule (left) with integrated He distribution line and ESS cryomodule (right) with separated He distribution line.



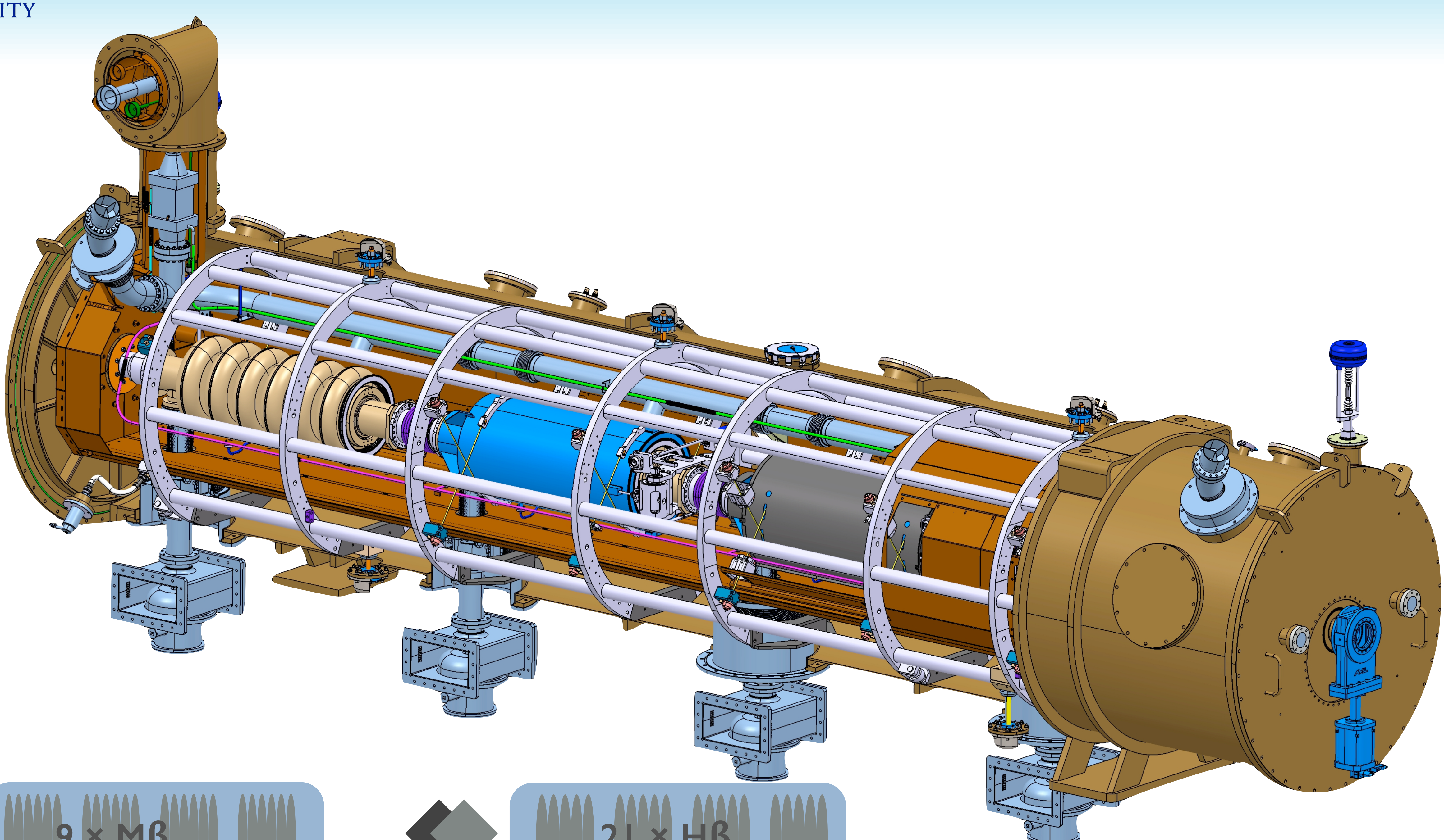


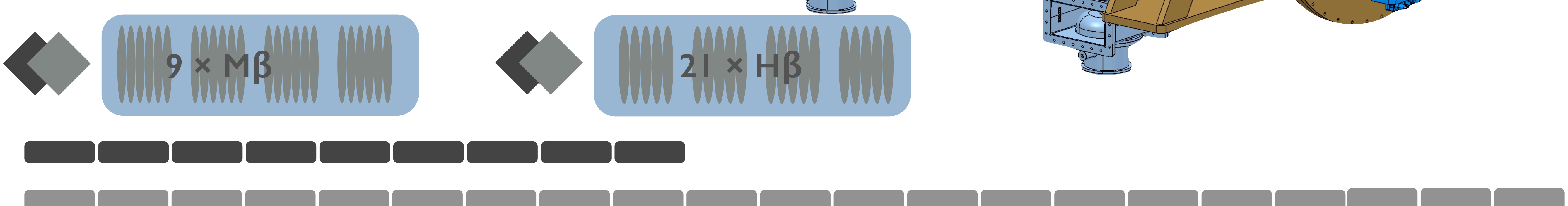
ESS SPOKE CRYOMODULE



13 × Spoke

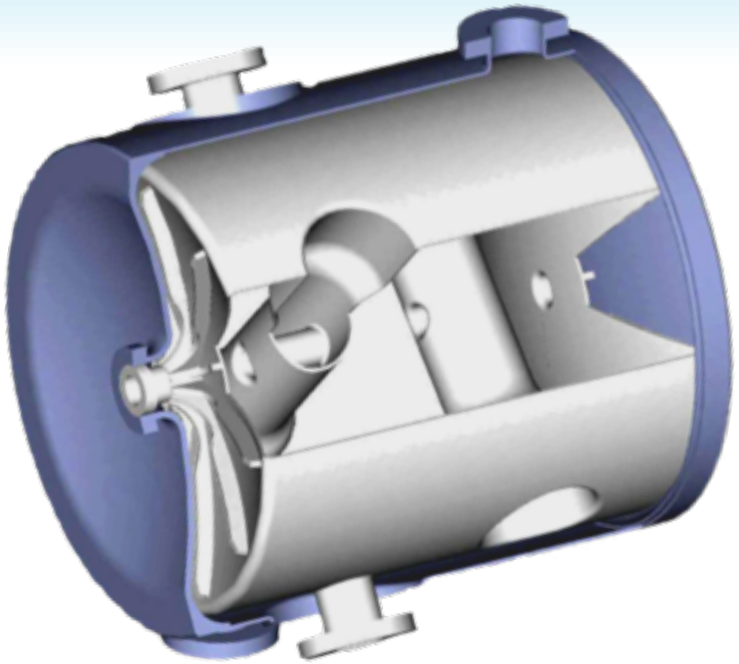
ESS ELLIPTICAL CRYOMODULE

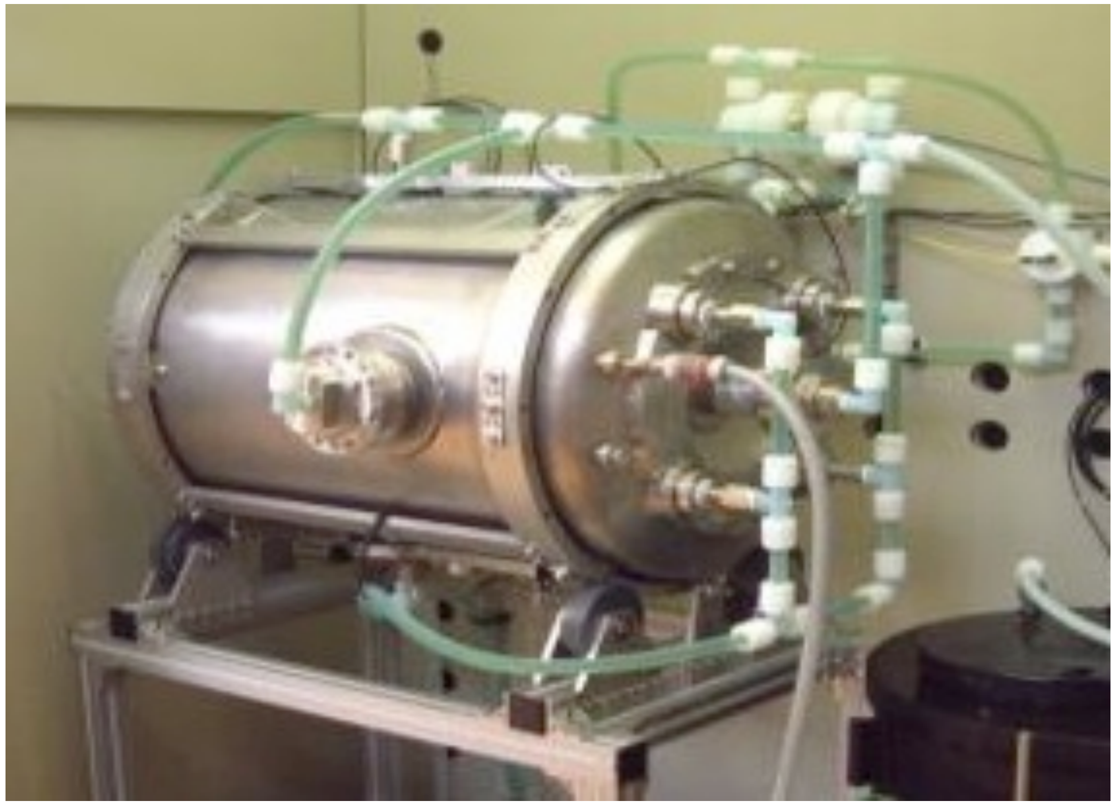


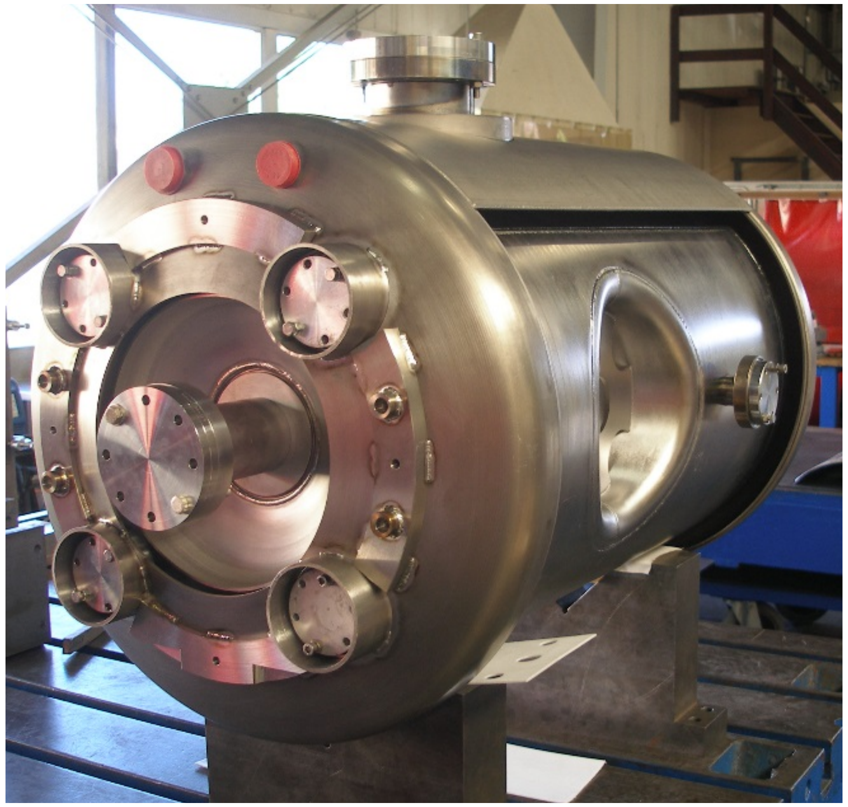


ESS SPOKE CAVITY

- Quadrupole Doublet Focusing
- Starts with a differential pumping section (LEDP)
- Accelerates the beam from 90 to 216 MeV
- Double spoke, $\beta_{opt} = 0.5$, $E_{acc} = 9$ MV/m

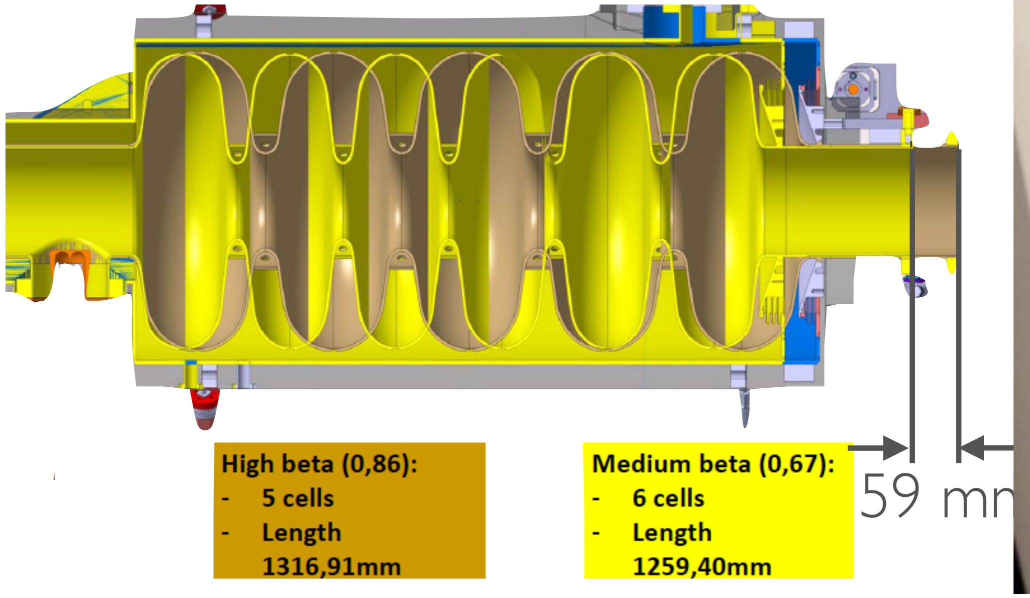






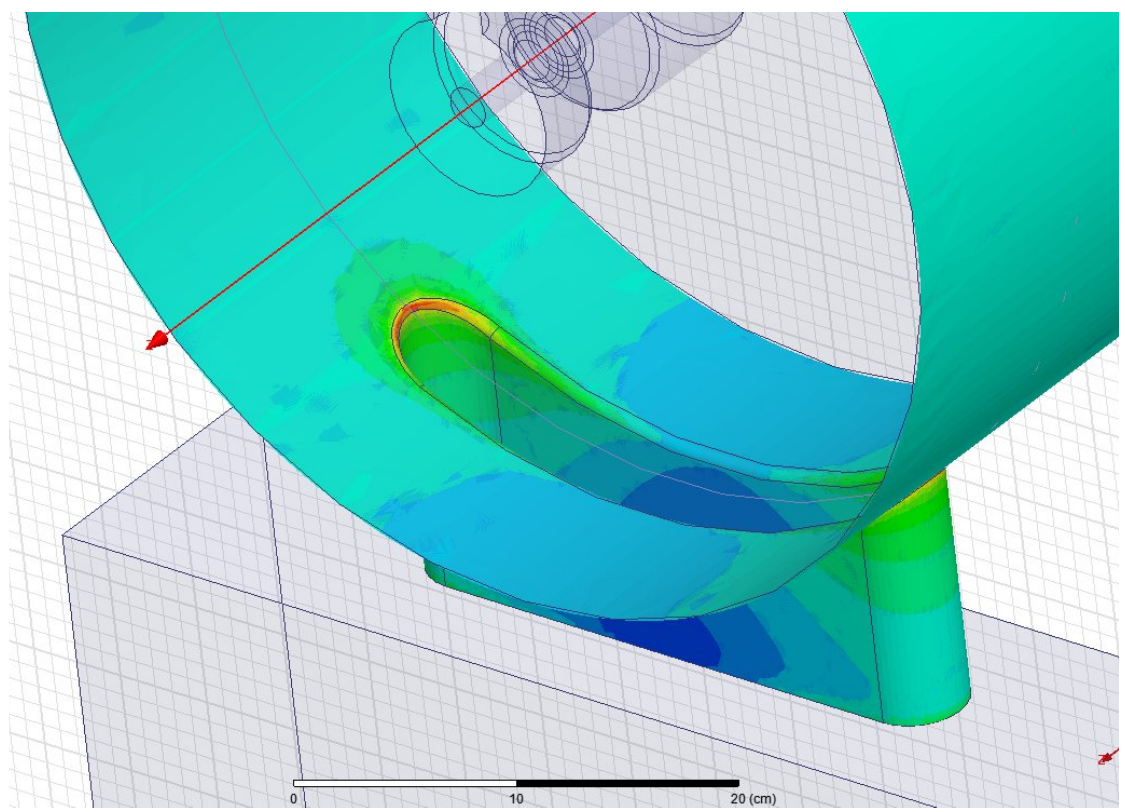
ESS ELLIPTICAL CAVITY

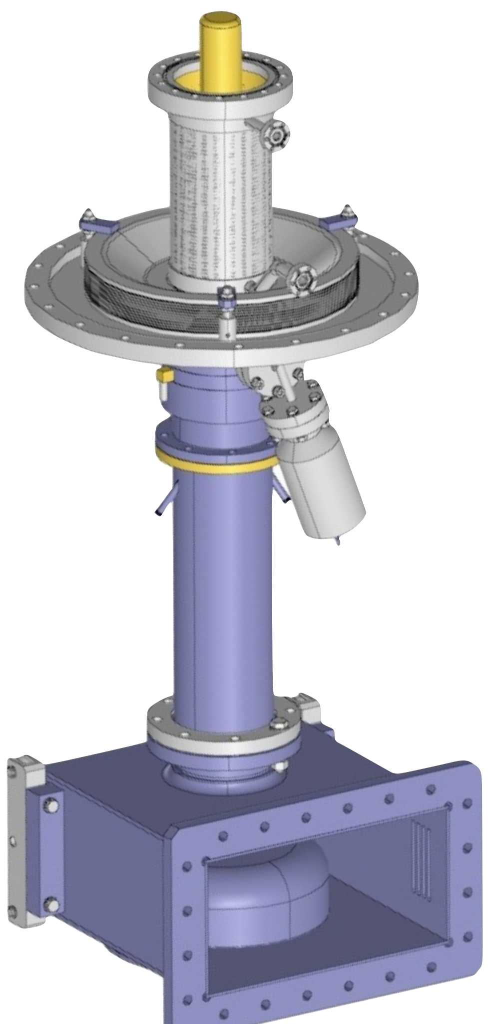
- Quadrupole Doublet Focusing
- Accelerates the beam from 216 MeV to 571 MeV to 2 GeV in Two families:
  - 6-cell, $\beta g = 0.67$, $E_{acc} = 16.7$ MV/m
  - 5-cell, $\beta g = 0.86$, $E_{acc} = 19.9$ MV/m



COUPLER

- The RF power generated by the RF sources is fed to the cavity through couplers.
- Coupler should stand very high voltages, preserve the vacuum and convert the waves from waveguide geometry to cavity geometry.

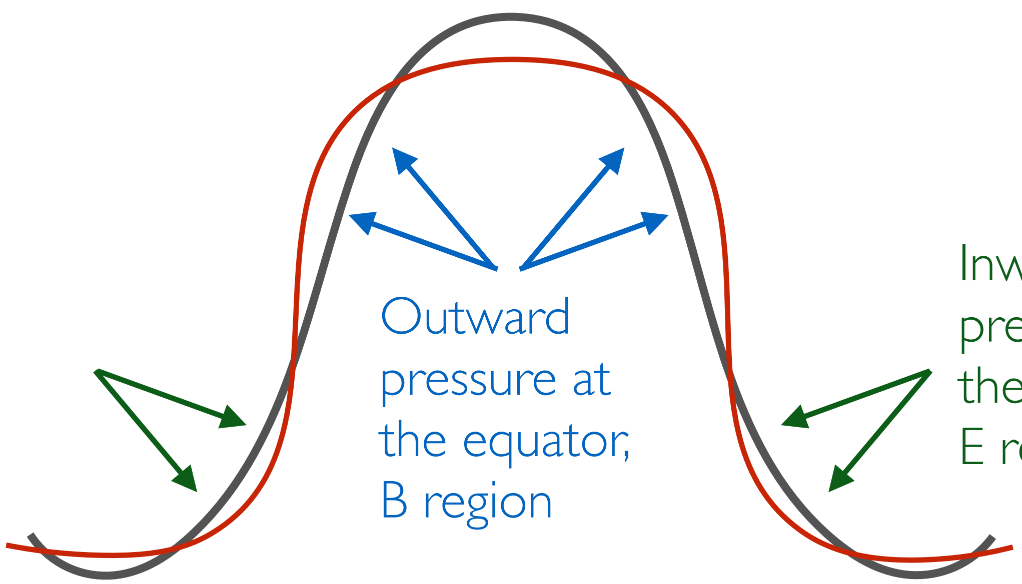




RADIATION PRESSURE

The electromagnetic field in the cavity causes a radiation pressure on the cavity surface:

$P = (\mu_0 H^2 - \varepsilon_0 E^2)/4$



This deforms the cavity shape which causes a frequency shift in the cavity:

$\Delta f = KL \times E^2_{acc}$

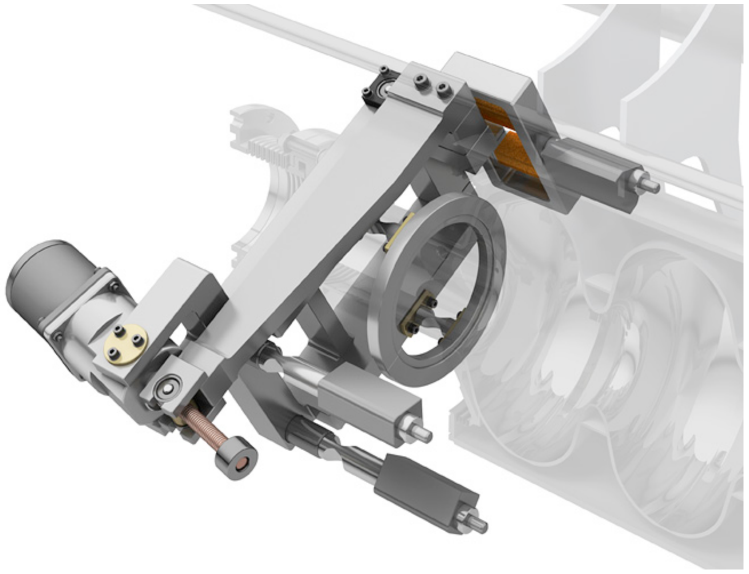
Temperature changes in normal conducting cavities can also alter cavity volume.

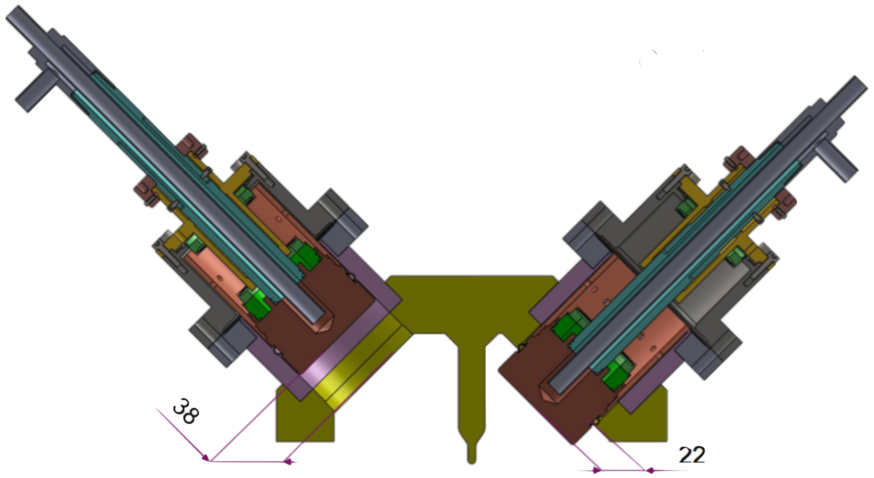
TUNERS

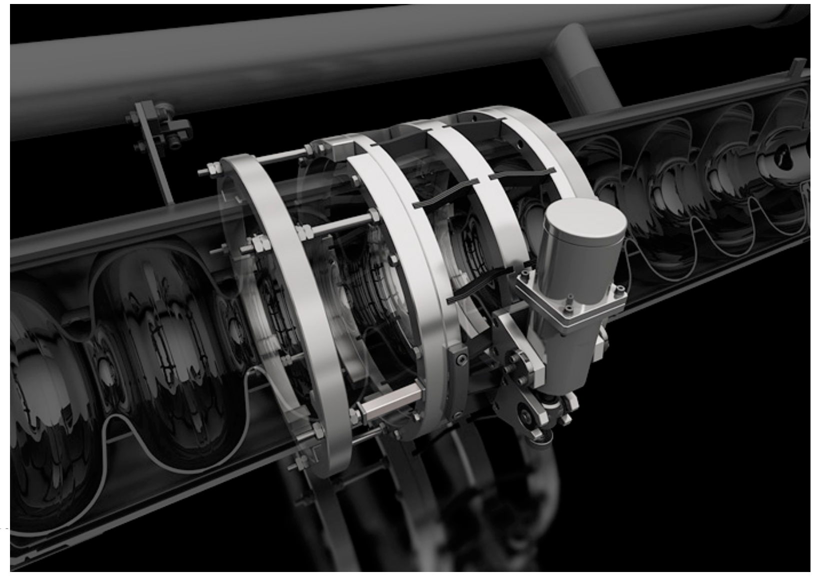
Any deformation of the cavity (either due to radiation pressure, or thermal expansion) will alter the resonant frequency of the cavity.

Tuners are used to adjust the frequency of the cavity to the desired resonant frequency.

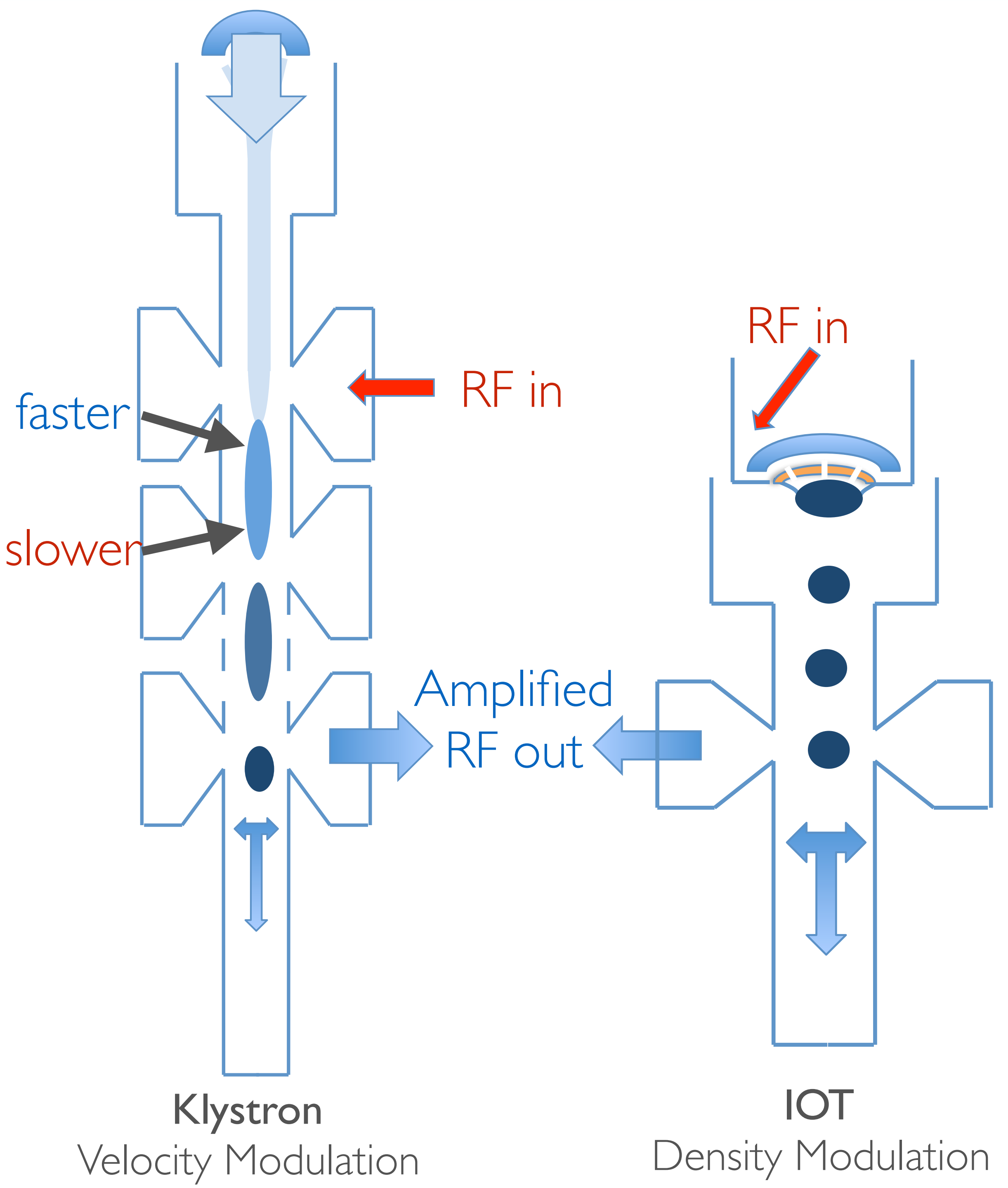
The principle is to change the volume of the cavity where the electric or magnetic fields are non-zero (Slater perturbation theory).

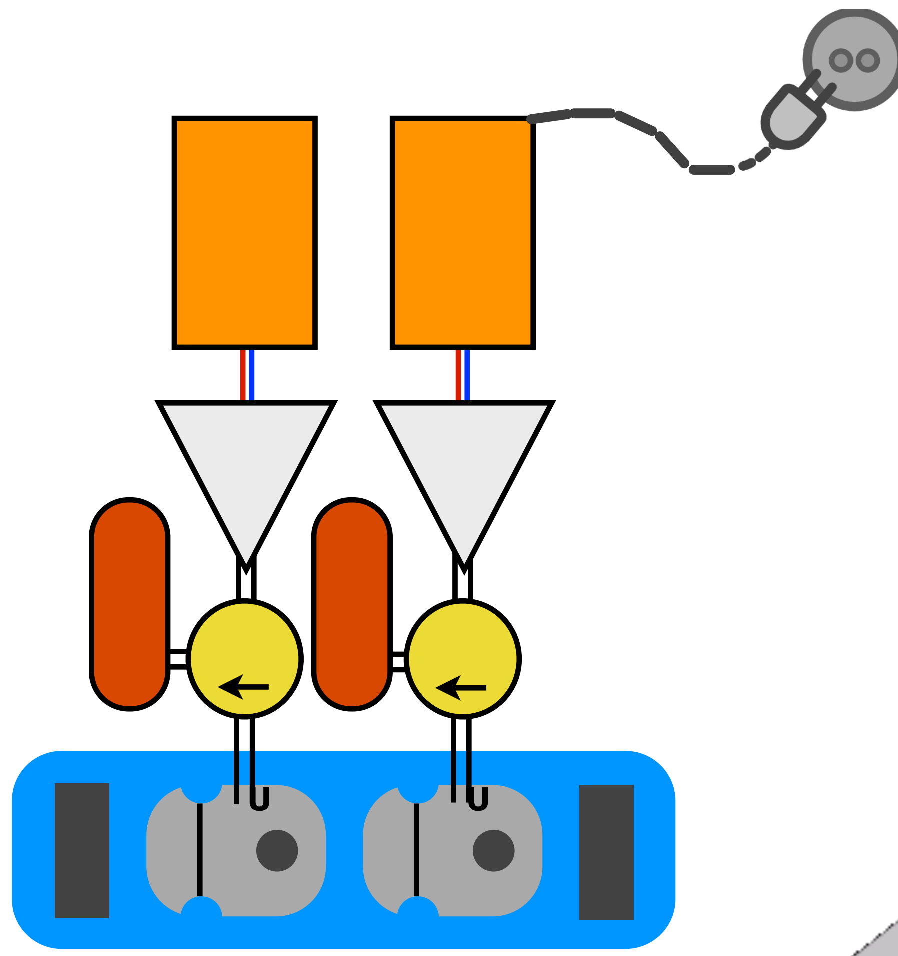


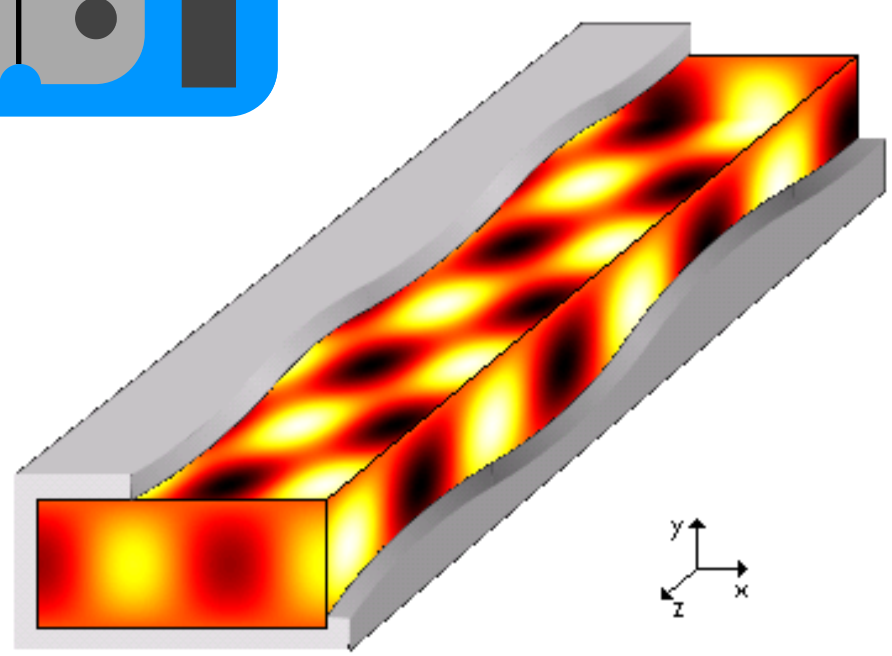




RF GENERATION

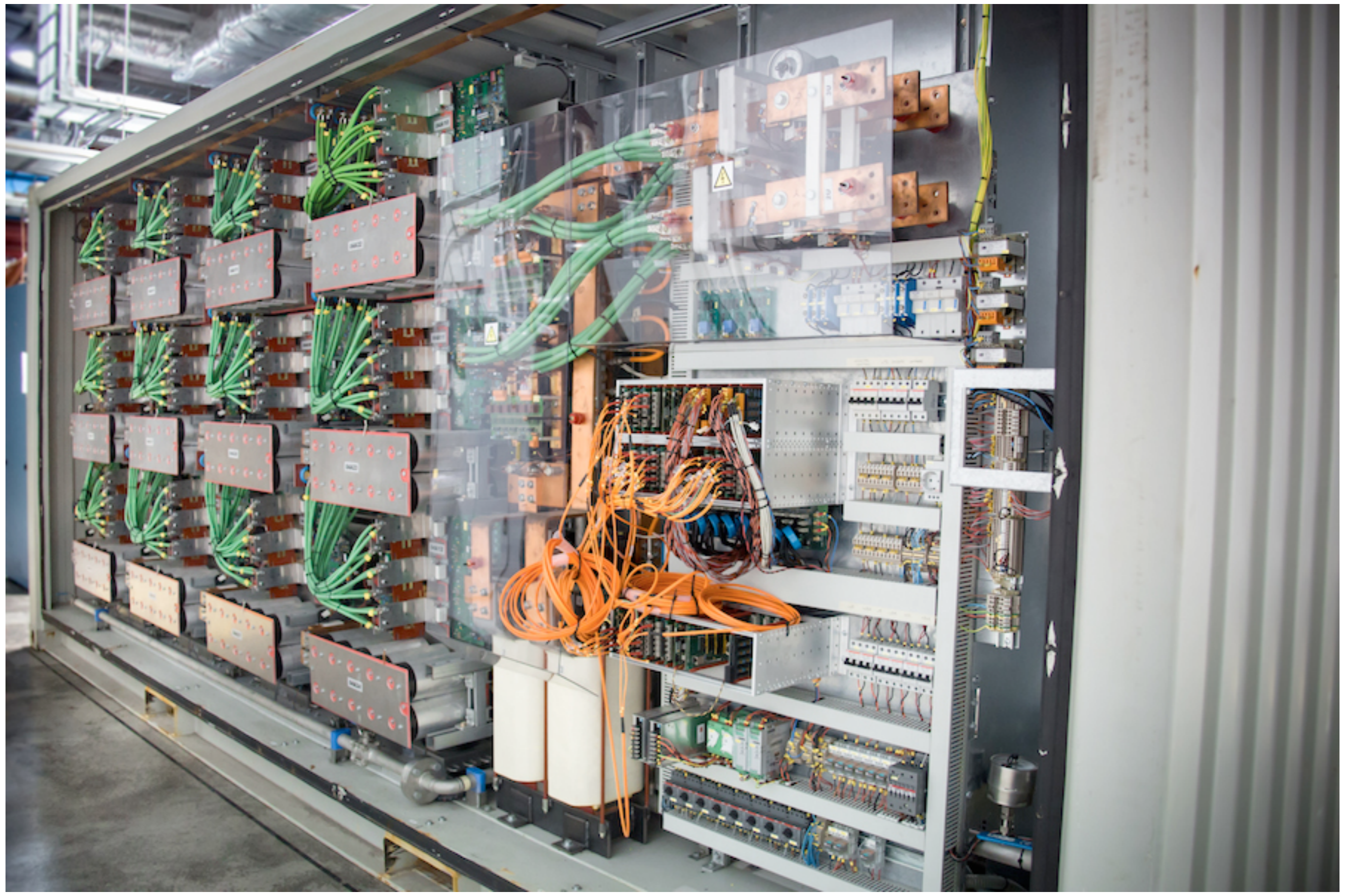




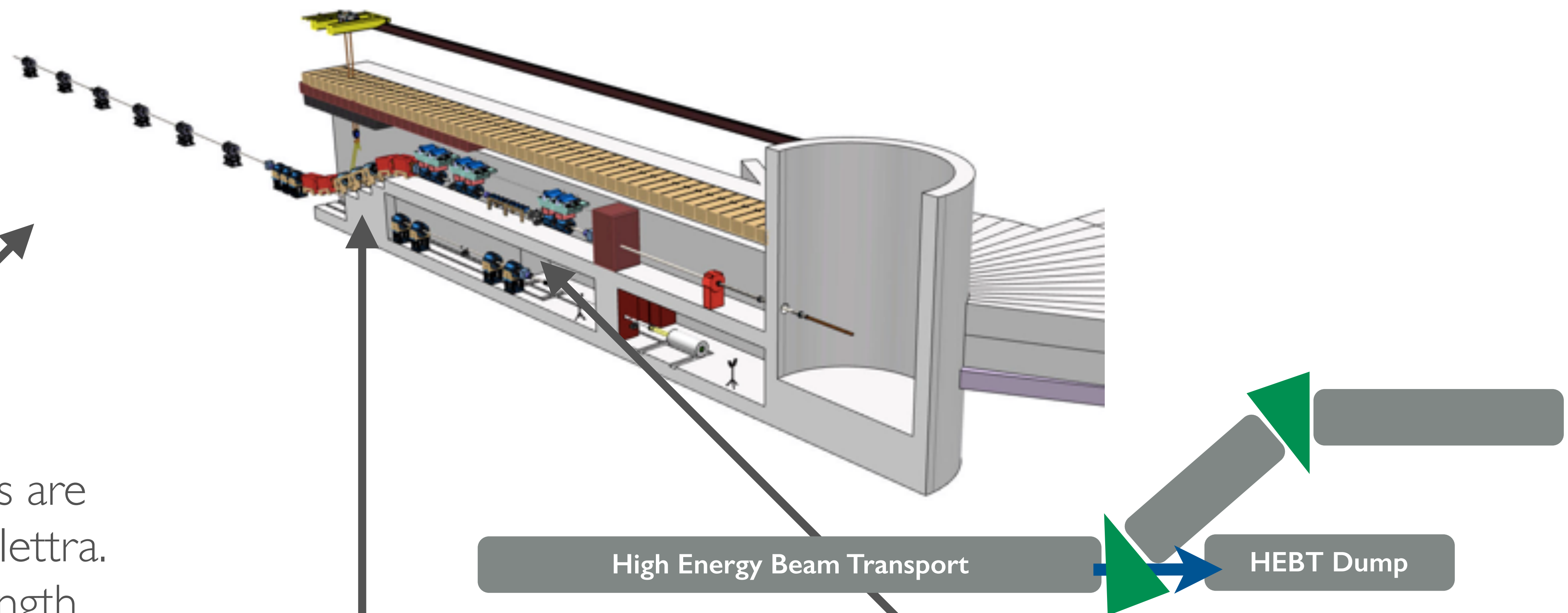


MAJOR RF PIECES

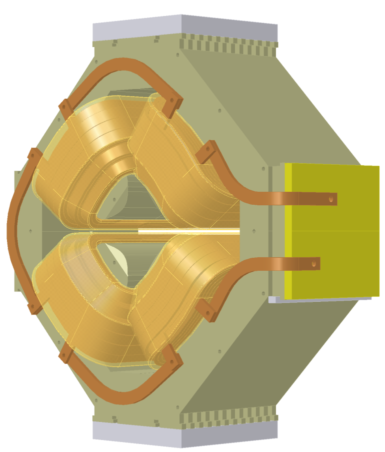
Radio **F**requency



HEBT



HEBT, Magnet doublets are designed and built in Elettra. 12 periods, identical length to HB cryomodules



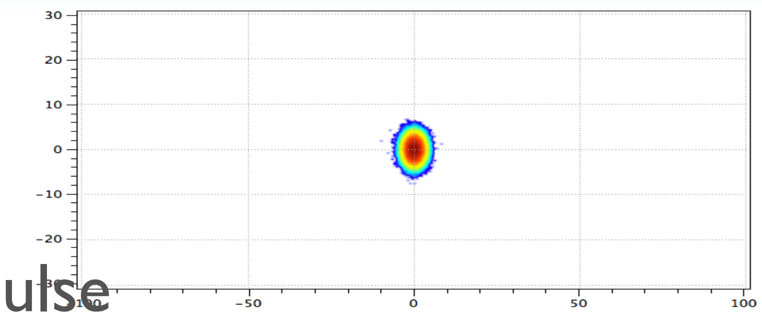
A2T (DogLeg), Magnets are designed and built in Elettra. 6 periods, achromat.

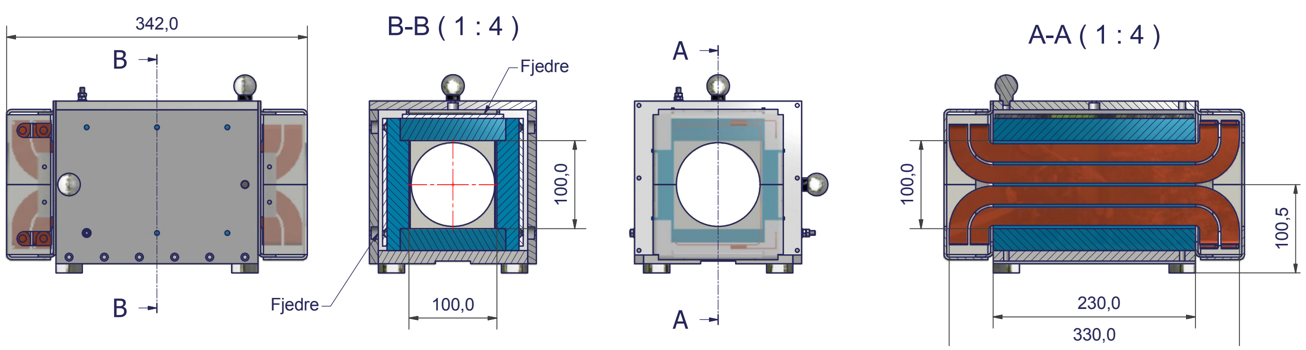
A2T Quadrupoles doublets are designed and built in Elettra, and Raster magnets are designed and built in Aarhus University

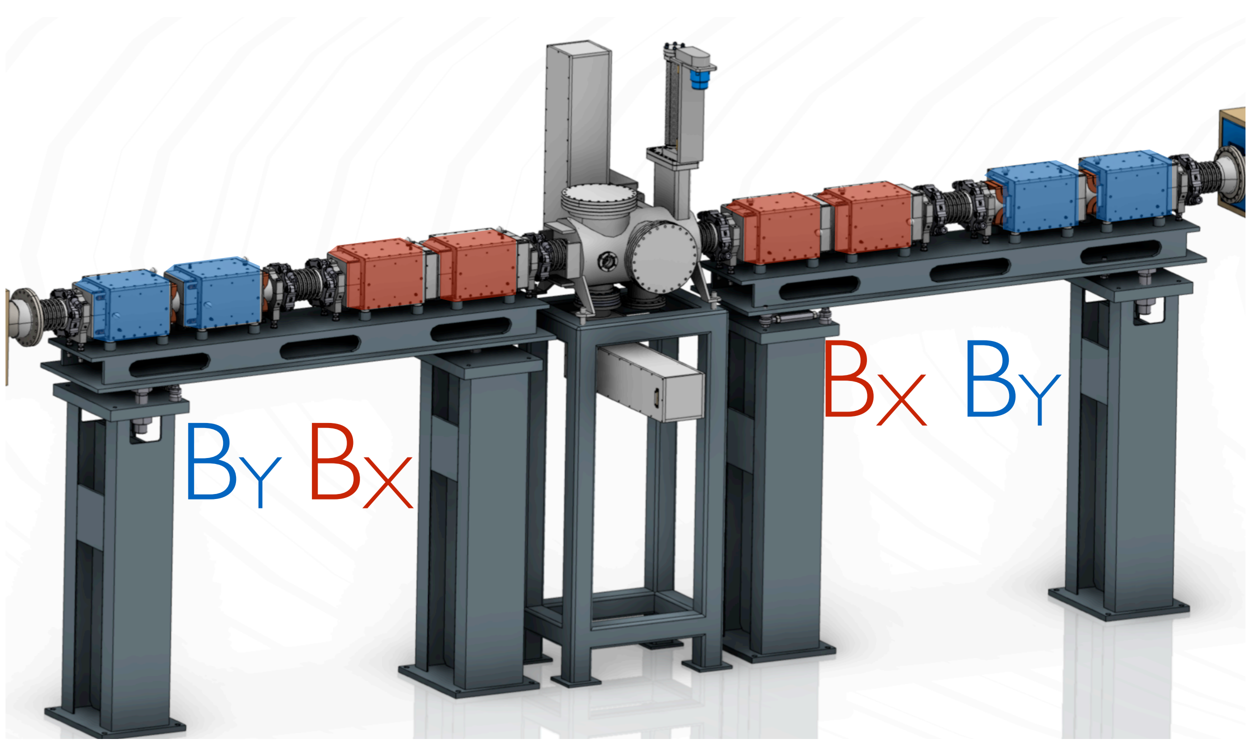
12 × Contingency

THE RASTER SYSTEM

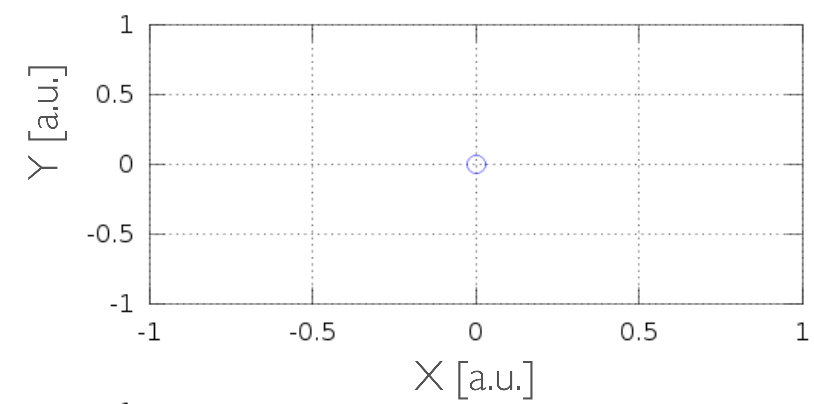
- Raster system sweeping beam in 2D pattern
- 8 colinear magnets, 8 dedicated, identical supplies
- Crosshatch pattern (fx/fy, ϕxy, ax, ay) within 2.86 ms pulse

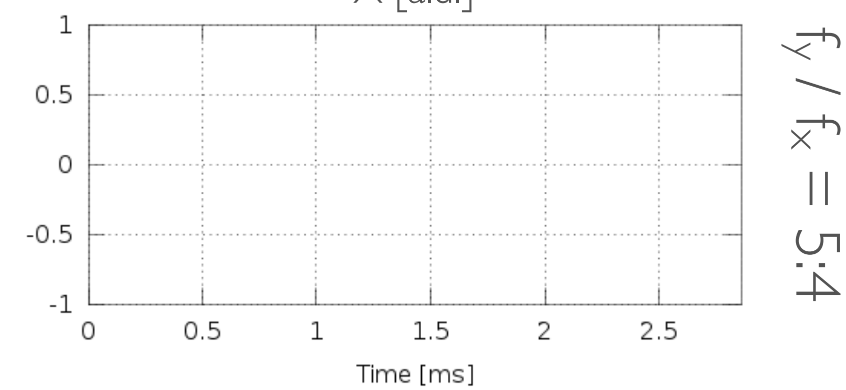


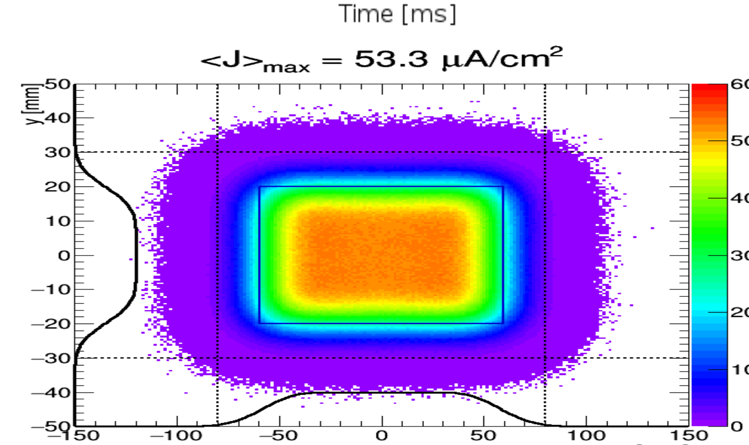




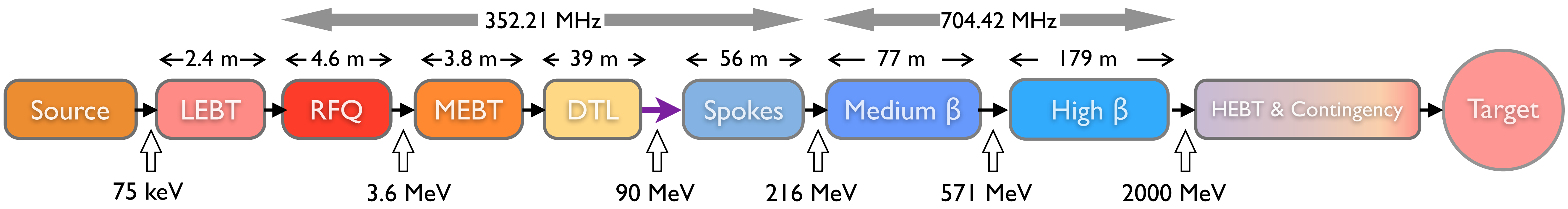
Courtesy: **Heine Dølrath Thomsen**







ESS LINAC, AGAIN



	Length (m)	W_in (MeV)	F (MHz)	β Geometric	No. Sections	T (K)
LEBT	2.38	0.075	--	--	1	~300
RFQ	4.6	0.075	352.21	--	1	~300
MEBT	3.81	3.62	352.21	--	1	~300
DTL	38.9	3.62	352.21	--	5	~300
LEDP + Spoke	55.9	89.8	352.21	0.50 (Optimum)	13	~2
Medium Beta	76.7	216.3	704.42	0.67	9	~2
High Beta	178.9	571.5	704.42	0.86	21	~2
Contingency	119.3	2000	704.42	(0.86)	14	~300 / ~2

LINAC DESIGN

- **RF design :**
  1) Control the field pattern inside the cavity,
  2) Minimize the ohmic losses on the cavity walls.
  3) Optimize (minimize) the total energy consumption.

- **Beam dynamics design :**
  1) Choose the right phase, and keep it right during acceleration,
  2) Choose the right focusing scheme and strength.
  3) Optimize the design for best beam quality and minimised losses.

FURTHER READING

- CAS lectures
- RF Linear Accelerators, T. P. Wangler
- Charged Particle Beams, M. Reiser
- Classical Electrodynamics, J. D. Jackson
- Linear Accelerators, P. M. Lapostolle
- Engines of Discovery, Sessler and Wilson
- The Physics of Particle Accelerators, Klaus Wille

ACKNOWLEDGMENTS

- I have used several images, drawings and illustrations which have been provided by our partner labs or other international labs.