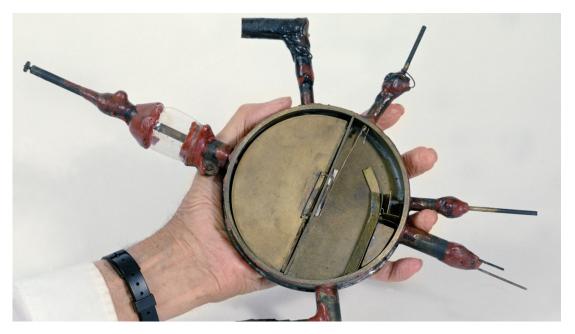
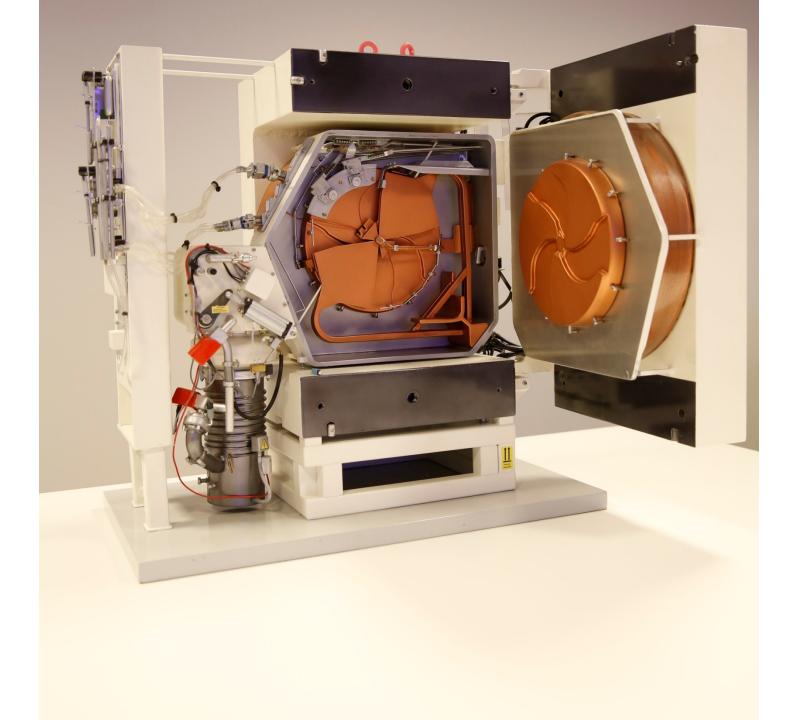
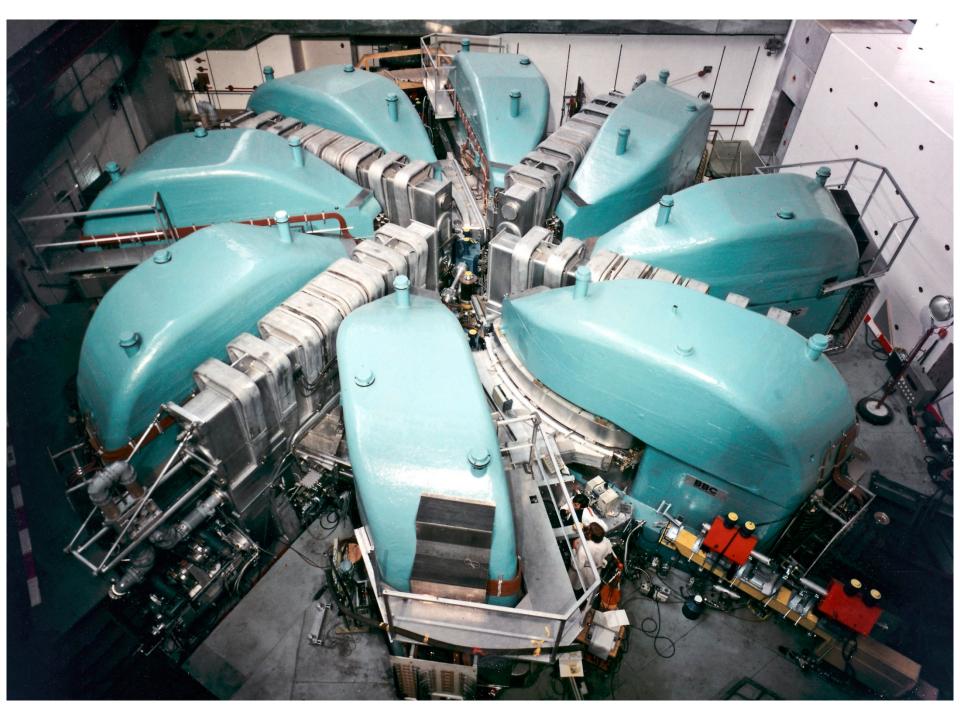
Cyclotrons

- Classical cyclotron
- Synchrocyclotron
- Isochronous cyclotron

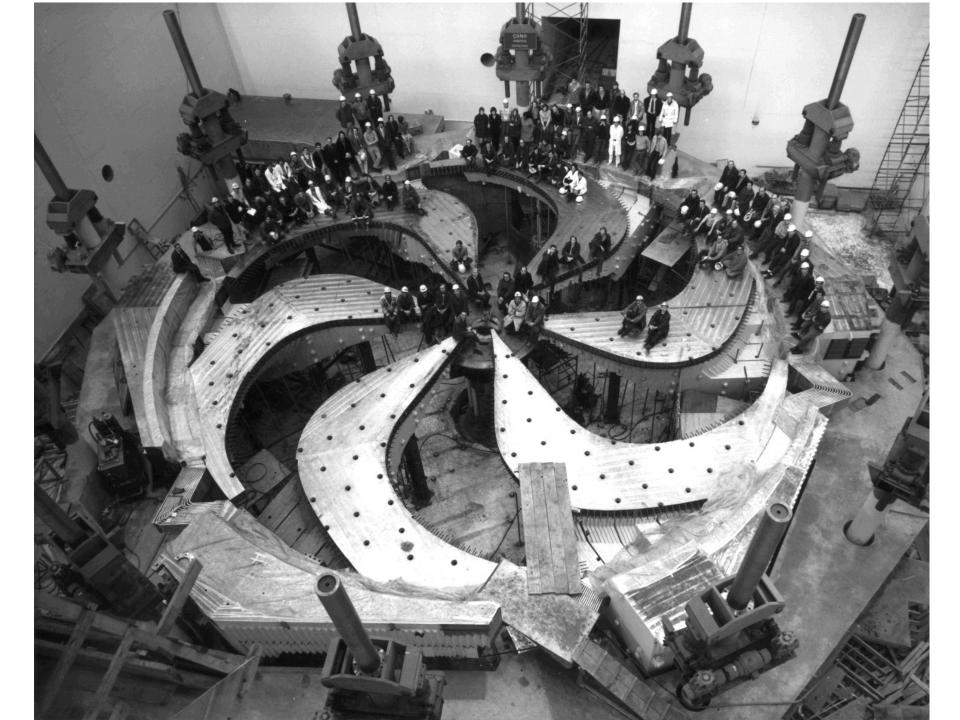


Ernest O. Lawrence, 1932 Nobel Prize, 1939









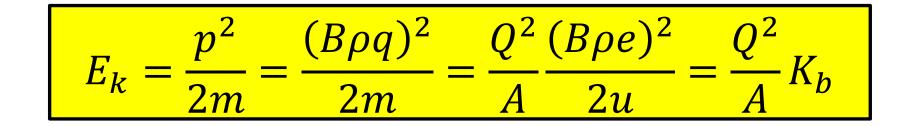
Classical Cyclotron

(13 x constant)

-a charged particle (q.m) in a magnetic field (\mathcal{F}) VLB centripetal force = magnetic force $\frac{mv^2}{r} = qVB \iff Br = Bg = \frac{P}{q}$ $(=) \frac{\sqrt{2}}{r} = \frac{\sqrt{2}}{m}$

w=we = cyclotion frequency WEF to = we = accelerating frequency =

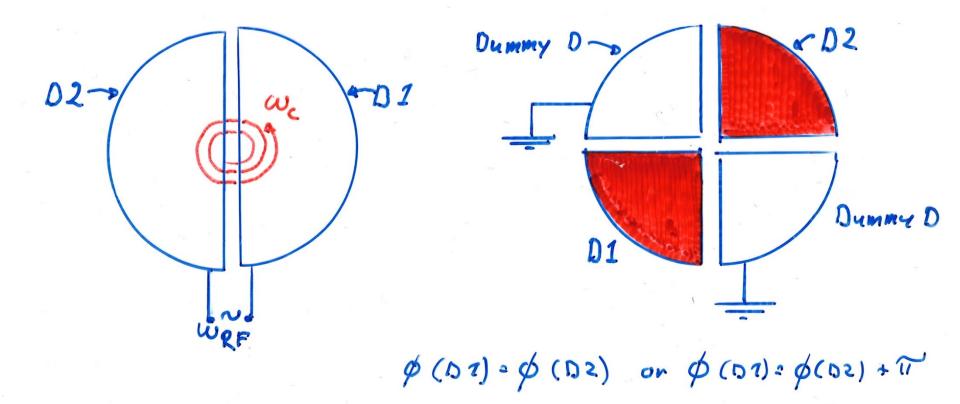
Non-relativistic bending limit:



Example: $\rho = 1 \text{ m}$ B = 1.7 T $K_b = 139 \text{ MeV}$

Also $\omega_{RF} = h \omega_{C}$ $h = 1, 2, 3, \cdots$

harmonic number



· polarity of electrode changes when the particle is inside dec - p new acceleration in next gap compare with Videroe BSZT -> proton: fRF 530.5 MAZ example 1. B=1.5T, r=43 cm example 2. $= \frac{p^2}{2m} = \frac{(B_S)^2 e^2}{2m} = \frac{20}{4} \text{ MeV}$ ~ not possible in a classical cycl. Energy limit in classical cyclotrons 1 = Ex x 1% - Ep ~ 10 Hel (Eo= 938 MeV)

betatron oscillations Focusing 2 FEVERg median plane - Focusing in 2-direction if B decreases as r increases - B= constant : no focusing - Bincreases ! defocusing Let's look at the tocusing conditions in more detail,

Kerst-Serber equations
Consider a moving particle in a classical cyclotton

$$B \neq B(\theta)$$
, $B = (r, z)$
near equilibrium orbit. (E0)
 $On E0$ (bending radius = g)
 $\frac{mv^2}{s} = q B_0 V$ $\Longrightarrow \frac{q}{m} B_0 = \frac{V}{s} = w_c$
Write: $B = B_0 (1 + k \frac{x}{s}) = -B_2$ (gradient normalized with g)

1º Radial focusing

Radial focusing

$$m\ddot{r} = -qBv + m\frac{V^{2}}{r} \qquad r = S + x \rightarrow \ddot{r} = \ddot{x}$$

$$m\ddot{x} = -qVB_{0}\left(1 + k\frac{x}{s}\right) + mv^{2}\left(S + x\right)^{-1}$$

$$= -qVB_{0}\left(1 + k\frac{x}{s}\right) + mv^{2}\frac{1 - \frac{x}{s}}{s}$$

$$= -qVB_{0}\left(1 + k\frac{x}{s}\right) + mv^{2}\frac{1 - \frac{x}{s}}{s}$$

$$= -qVB_{0}k\frac{x}{s} + m\frac{V^{2}}{s} - m\frac{V^{2}}{s}\frac{x}{s}$$

$$= -qVB_{0}k\frac{x}{s} - qVB_{0}\frac{x}{s}$$

$$\ddot{x} = -\left(\frac{4B}{m}k\frac{v}{s} + \frac{qB_{0}}{m}\frac{v}{s}\right)x$$

$$\widetilde{w}_{c} \quad \widetilde{w}_{c} \quad \widetilde{w}_{c} \quad \widetilde{w}_{c}$$

 $\Rightarrow \ddot{x} + (w_c^2 k + w_c^2) x = 0$ $\ddot{x} + \omega_c^2 (1+k) x = 0$ - P X= A cos (Vikk wet + 4.) limited if k >-1

2° Axial focusing

m==-quBx VXB=0 = DB2 DBX Bx=2 DB2 =-2. K J. më quek B./g (a)z · m Bo g kz 2 - wc2k2 = 0 2

FOCUSING IN BOTH PLANES IF -1 < K < 0

Note! Often in the literature $-k = n = -\frac{3}{5} = -\frac{r}{5} \frac{40}{4r}$

· · · o < n < 1

field index

n, k

 (Q_r, Q_2) Betatron frequencies Vr. V2 $Y_{r} = \frac{\omega_{r}}{\omega_{c}} = \sqrt{1+k}$ $V_{z} = \frac{\omega_{z}}{\omega_{c}} = \sqrt{-k}$ Particles oscillate around EO with frequencies w, and Wz Vr periods /turn radially

V2 - 11 -

axially

Synchro cyclotron

Ex increases - Pm increases - Pwc decreases

(We decreases also due to focusing condition)

- Decrease accelerating frequency with increasing energy = synchrocyclotron

ADVANIAGES

+ higher energy + possibility for better axial focusing

DISADUANTAGES

- only one (tow) pulse at the time can be accelerated - r intensity goes down

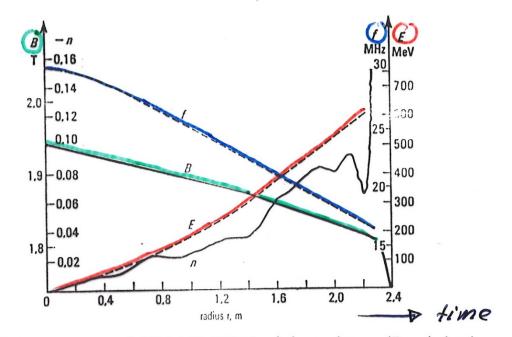
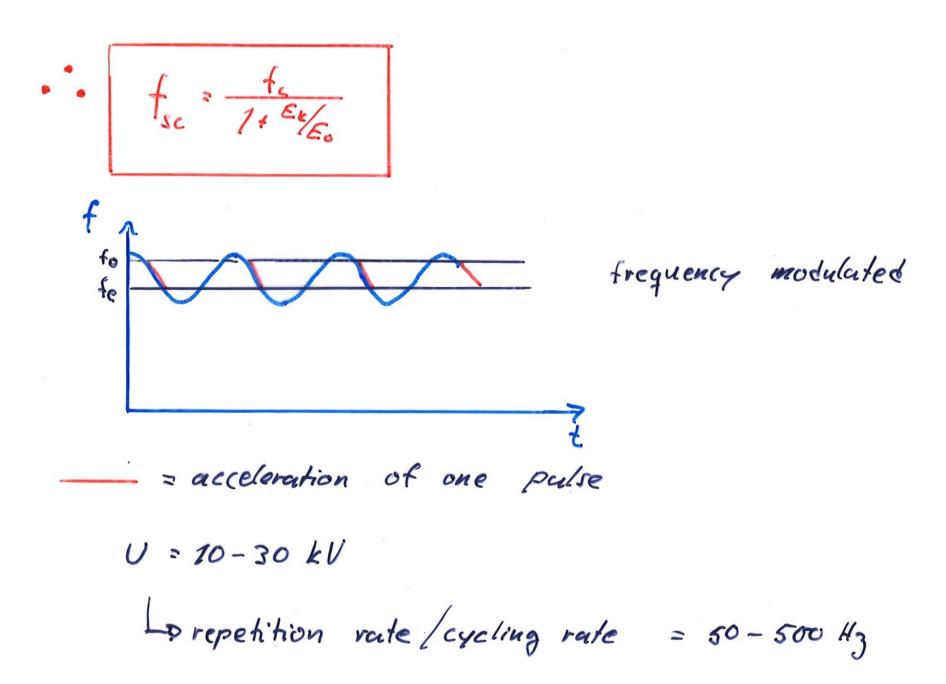


Figure 3.7 Parameters of 600-MeV CERN synchrocyclotron (B-- induction, E - proton energy, f - accelerating-voltage frequency, n field index)

Synchro cyclotron frequency

 $\omega_{e} = \frac{q}{m_{o}} B$

Wsc We 1 Eo Eo = 1+ Ex/E. 2 m



Isochronous cyclotron = sector focusing cyclotion = AVE cyclotion (Azimullaty Varying Field) Another way to compensate for the mass increase or frequency decrease is to increase magnetic field with radius (energy) BUT ! Kerst-Serber : axial defocusing => axial focusing must be increased by modifying the magnetic field so that the synchronous condition is fulfilled · cannot be done radially (symch. condition)

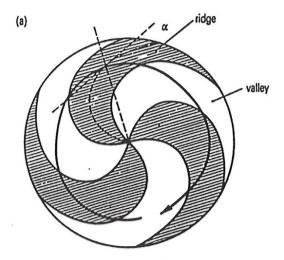
- P Question: Can axial tocusing be increased modifying the field azimuthally so that <BY, corresponds to synchronous field?

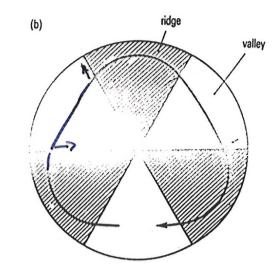
Answer: YES

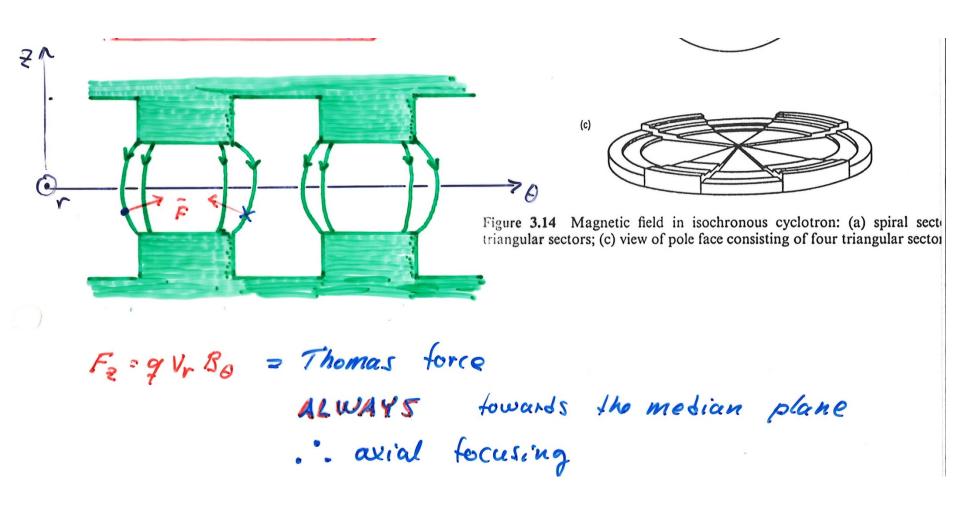
Try sectors and examine the components of Lorenz force

,

ISOCHRONOUS CYCLOTRONS







Spiral effect

At the sector edge (2+0) Br+0 Juto hill (sector) Br70 Out from hill Br<0

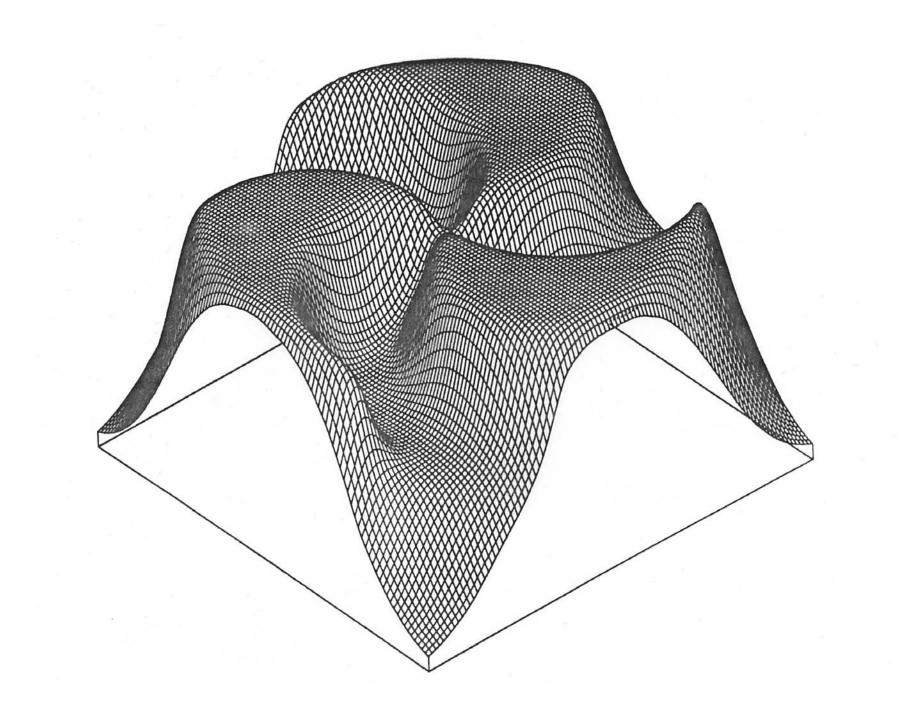
50:

"Areal focusing that was lost due to the rochronous condition was gained back with (spiral) sectors"

Spiral angle a

Define FLUTTER F: F= <B27 - <B72 "normalized variance" \Rightarrow $V_2^2 \simeq -k + \frac{N^2}{N^2 - 1} F (1 + 2 \tan^2 x) + \cdots$ N= number of sectors 3 sector cyclotron: Vr = 1+k + 0.675 F(1+ famex) + ... Note! Adding sectors decreases flutter

N-2 ac

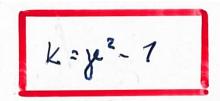


Synchronous condition:

 $B = \frac{m}{q} w_c$ = y m. wc = y B. $=\frac{B_{o}}{\sqrt{1-\left(\frac{v}{c}\right)^{2}}}=\frac{B_{o}}{\sqrt{1-\left(\frac{r\omega_{c}}{c}\right)^{2}}}$

Field shape (radially)?

K= r dB $= \frac{rB_{e}}{B} \frac{d}{dr} \left(1 - \frac{\omega^{2}}{c^{2}}r^{2}\right)^{-2}$ $= \frac{rB_{0}}{B} \frac{\omega^{2}r}{c^{2}} \left(1 - \frac{\omega^{2}r^{2}}{c^{2}}\right)^{-\frac{1}{2}}$ = B. B y3 1/ye 1- 1/22



field index corresponding to isochronous field

V2=-K+F(1+2+am2x)

= 1-y2 + F (1+2+am2x) > 0

F(7+2, tom'a) > ye2-7 Focusing condition



 $\gamma = \frac{E_k + m_0 c^2}{m_0 c^2}$

For room temperature cyclotrons (B < 2 T)

- Flutter F does not depend on B
 - So, maximum γ (v or E/A) limited by magnet geometry

For superconducting cyclotrons (B >> 2 T) iron is saturated

- <B²> ² = constant (given by sector geometry)
 - Hence, Flutter decreases as 1/B²
 - Focusing limit:

$$\frac{E}{A} = K_f \frac{Q^2}{A^2}$$

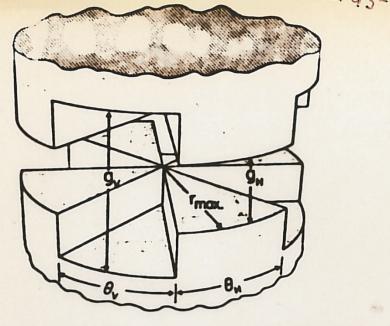


Fig. 2. Schematic drawing of the pole tip geometry assumed in our calculations. The hillgap "gh" and the valley gap "g_v" are c-crywhere uniform. Likewise the hill angular width " θ_h " and the valley ang far width θ_v " are indent of radius although in spice case the ingular service of the hill edge varies with radius (sect. 4). The sector number is given by $N = 300 / (\theta_h + \theta_v)$.

The pole outer radius is designated "rman".

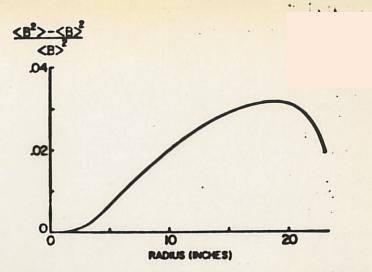


Fig. 3. Flutter [eq. (1)] vs radius for the "standard case" pole tip, which has $\theta_h = \theta_v = 45$, $g_h = 3$, $g_v = 36$, $r_{max} = 24$ " and $\sim B^{\sim}$ as in eq. (3) (≈ 3.5 T). The focusing is adequate for about $\approx MeV/nucleon$.

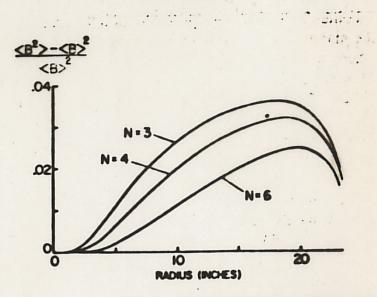
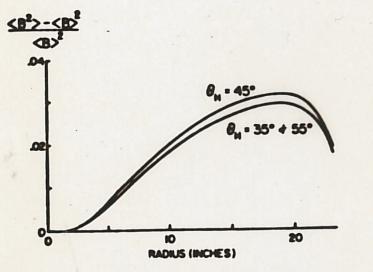


Fig. 4. Flutter vs radius for the standard four-sector case, compared with values for three sectors and six sectors. In all cases $\theta_{\rm B} = \theta_{\rm v}$ and other parameters are the standard case values.



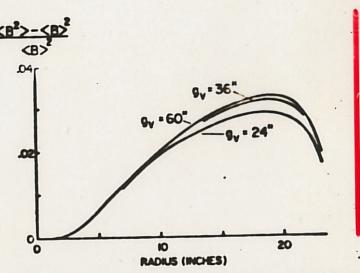
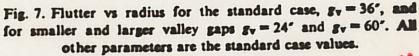
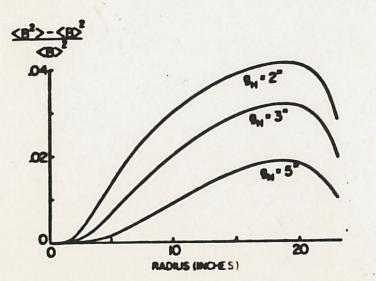


Fig. 5. Flutter vs radius for the standard case $\theta_h = 45^\circ$, and for narrower and wider hills, $\theta_h = 35^\circ$ and 55° (flutter identical). In all cases $\theta_v + \theta_h = 90^\circ$ and other parameters are the standard case values.





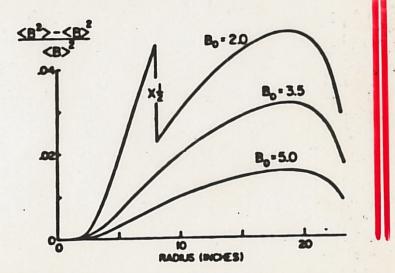
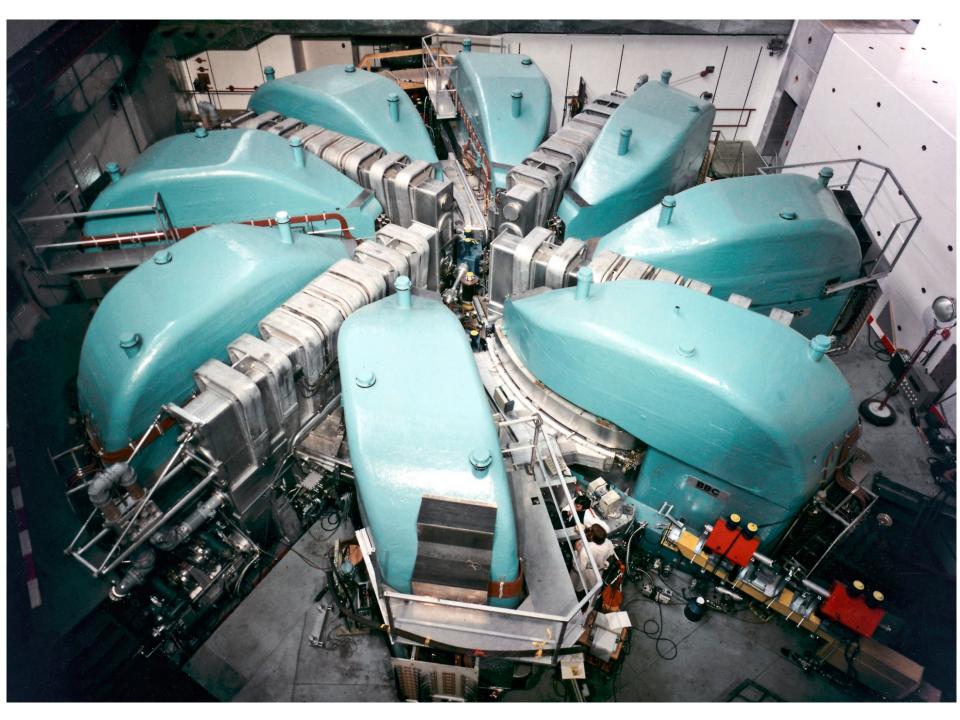


Fig. 6. Flutter vs radius for $1 \text{ stan}^{3/2-3}$ case, $p = 3^{\circ}$, and for smaller and larger hill gaps, $g_h = 2^{\circ}$ and $g_h = 5^{\circ}$. All other parameters are the standard case values.

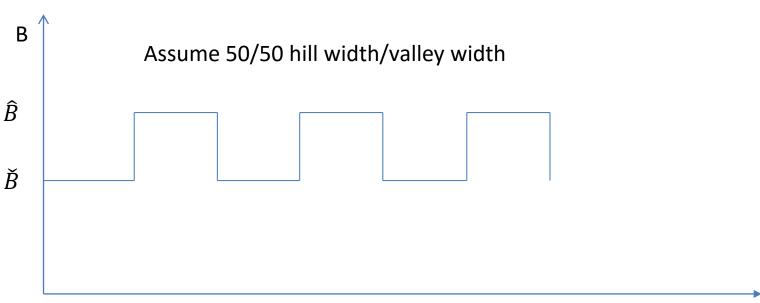
Fig. 8. Flutter vs radius with the central field at the 3.5 T standard case value, and lowered and raised to 2.0 T and 5.0 T. All other parameters are the standard case these the peak value for the 2.0 curve is 0.05.

Separated sector cyclotrons

- For higher energy light ions, axial focusing sets the limit
 - Increase spiral angle
 - Increase flutter F
 - Zero field in the valleys
 - Separated sectors
 - » Space for equipment between the sectors (dipoles)
 - Effective resonators for high accelerating field



Flutter

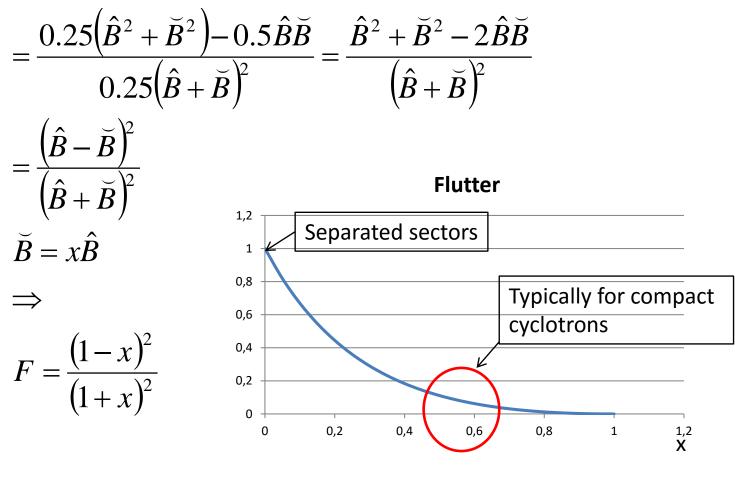


$$\left\langle B^2 \right\rangle = 0.5\hat{B}^2 + 0.5\breve{B}^2 = 0.5\left(\hat{B}^2 + \breve{B}^2\right)$$

$$\left\langle B \right\rangle^2 = \left(\frac{\hat{B} + \breve{B}}{2}\right)^2 = 0.25\left(\hat{B}^2 + \breve{B}^2 + 2\hat{B}\breve{B}\right) = 0.25\left(\hat{B}^2 + \breve{B}^2\right) + 0.5\hat{B}\breve{B}$$

$$\left\langle B^2 \right\rangle = \left\langle B^2 \right\rangle^2 = 0.25\left(\hat{B}^2 - \breve{B}^2\right) = 0.25\left(\hat{B}^2 - \breve{B}^2\right) + 0.5\hat{B}\breve{B}$$

$$F = \frac{\langle B^2 \rangle - \langle B \rangle^2}{\langle B \rangle^2} = \frac{0.5(\hat{B}^2 + \breve{B}^2) - 0.25(\hat{B}^2 + \breve{B}^2) - 0.5\hat{B}\breve{B}}{0.25(\hat{B} + \breve{B})^2}$$



Remember: $F(1+2\tan^2\alpha) > \gamma^2 - 1$

For high E/A, choose separated sectors



Passive focusing channels

Central region + inflector

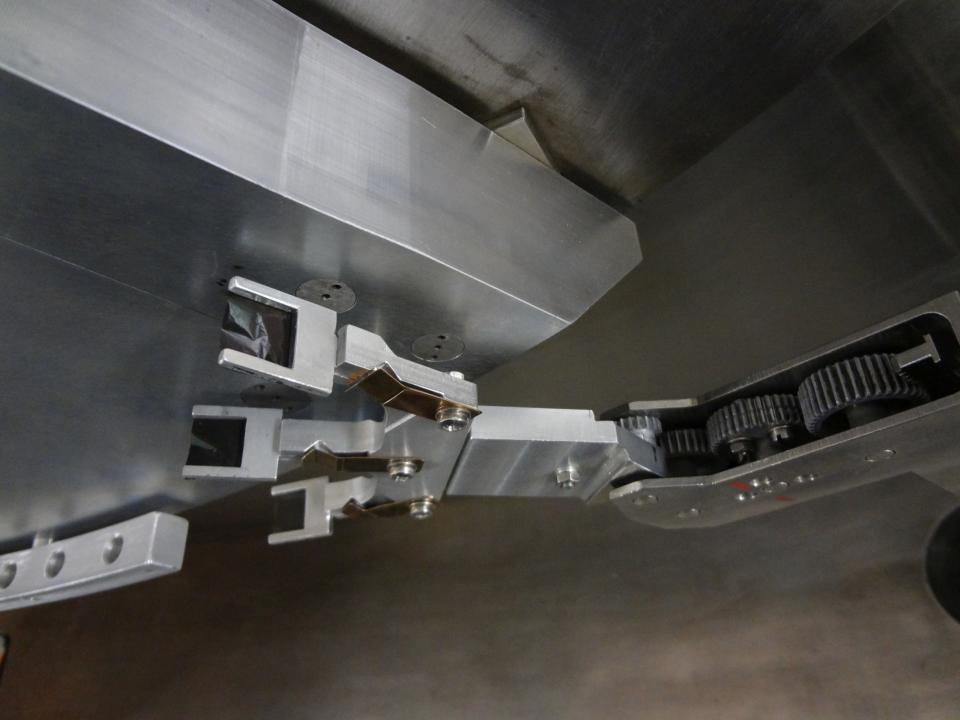
Electromagnetic channel

Deflector

Spiral inflector







Injection/central region and extraction

Injection

•External ion source

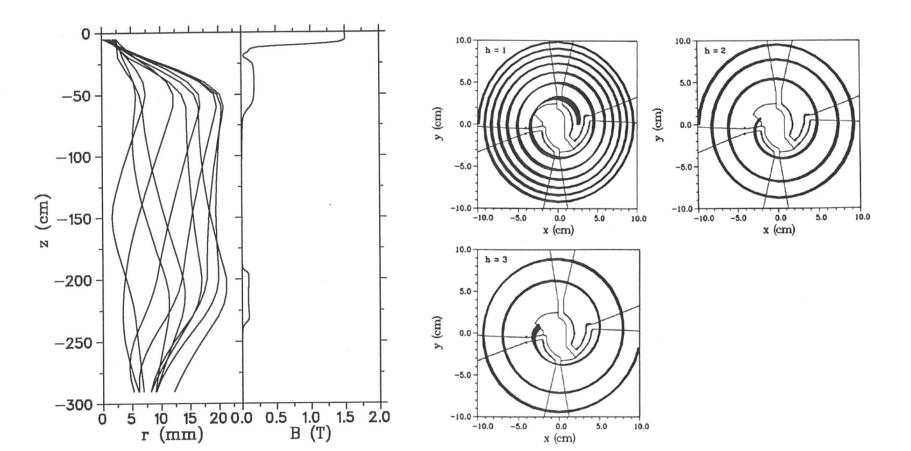
•Matching the beam into the cyclotron's

Central region acceptance

•Accelerated equilibrium orbit "eigen ellipses"

Low-energy beam

Possible space charge limitation



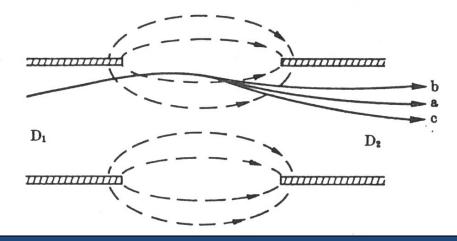
Forces in the cyclotron

$$\vec{F} = q \left(\vec{E} + \vec{v} \times \vec{B} \right)$$

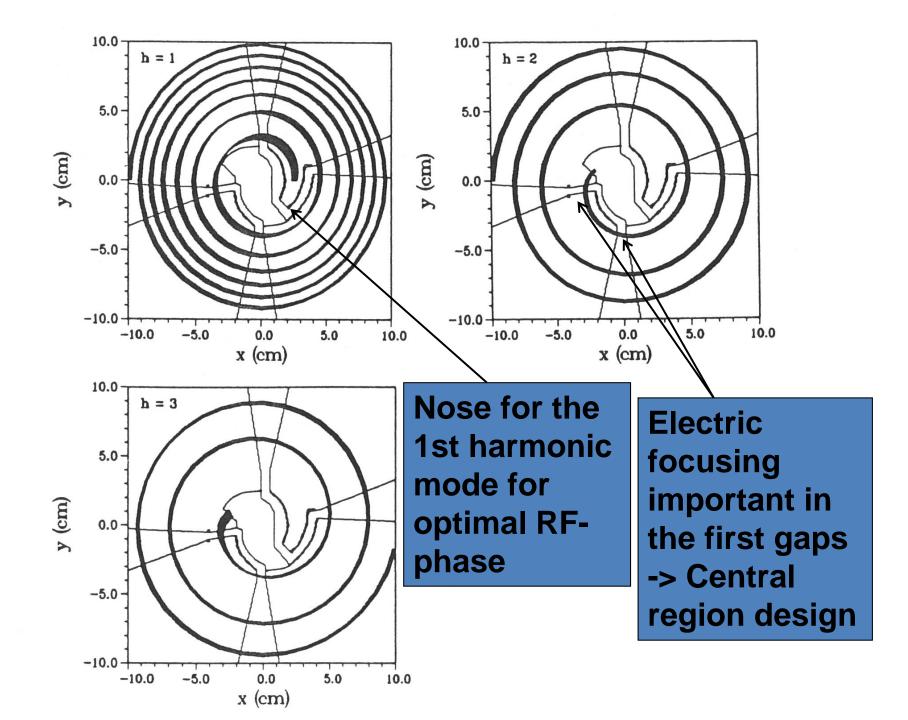
Typically $\widehat{E} \cong 10 \text{ MV/m}$ $B \cong 1.5 \text{ T}$ $F_E = F_B$ $\Rightarrow v \cong 0.02 c$ $\Rightarrow \frac{E}{A} \cong 200 \text{ keV/n}$

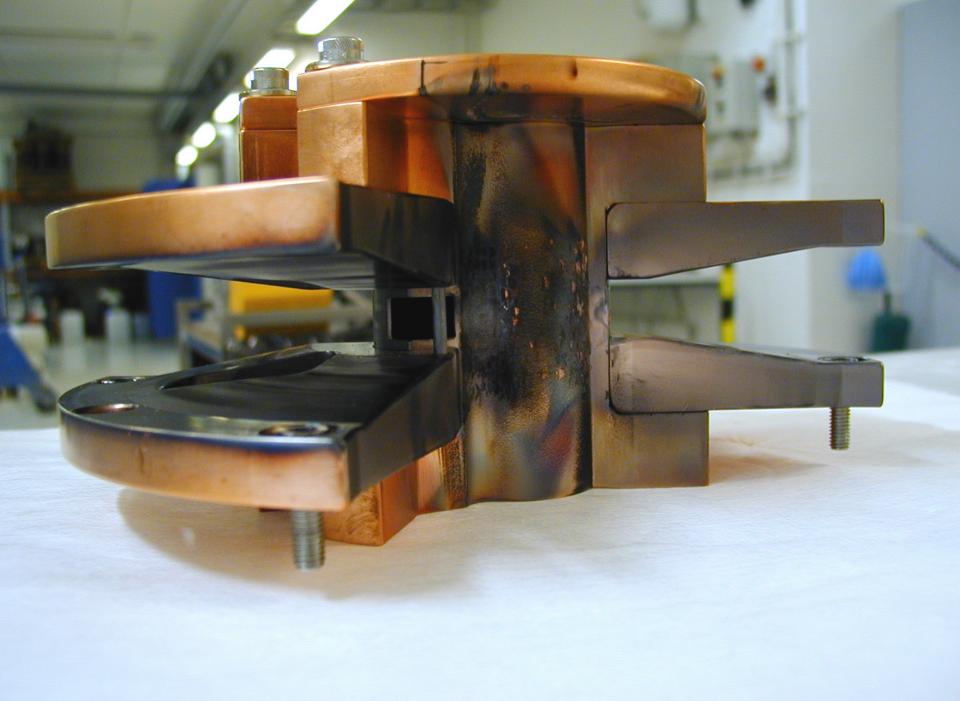
This energy is reached during 1 – 2 turns

Outside the central region only magnetic forces (bending, focusing) are relevant. However, electric focusing is important along the first 1 – 2 turns.

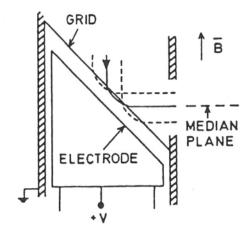


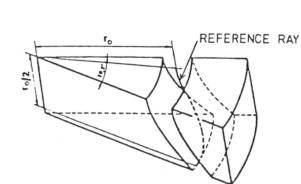
Transit time effect in an accelerating gap in a) static field, b) increasing field and c) decreasing field. The effect is exaggerated

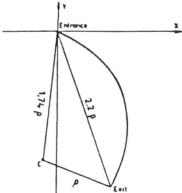




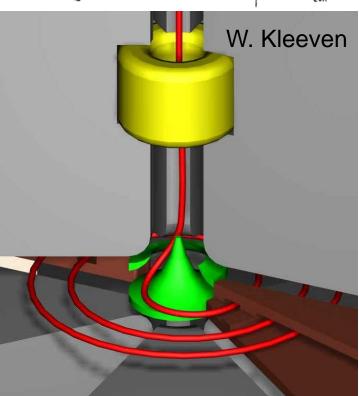
Inflectors



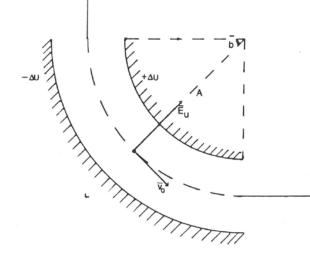




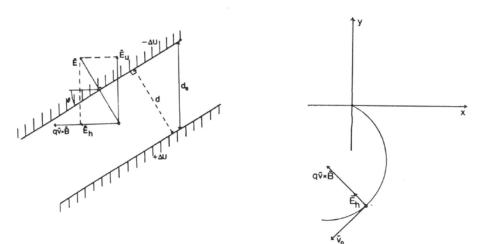




Spiral inflector



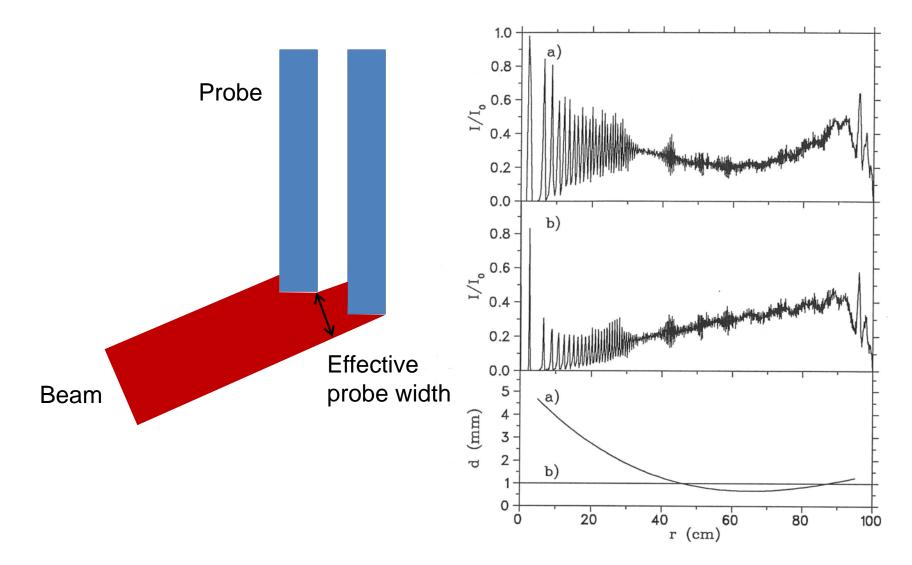
Beam bending without magnetic field



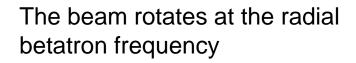
a) Cross-section of the spiral
 electrodes and b) beam projection on
 xy plane

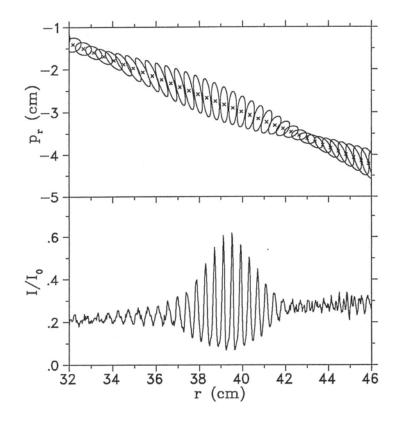


Injection has an effect on beam behaviour in the cyclotron

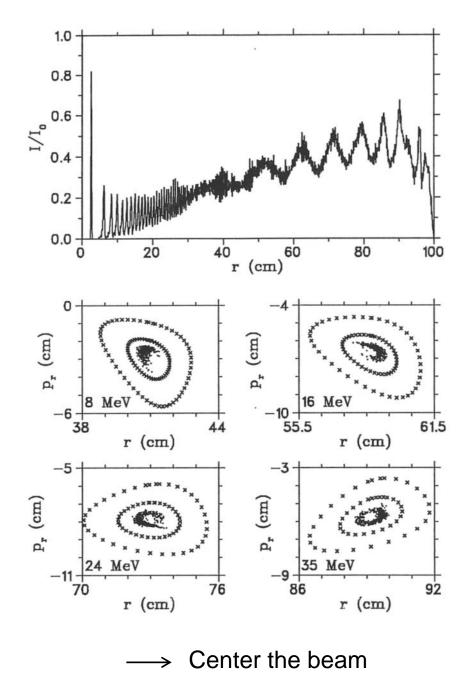


Differential probe scan with a) a changing effective probe width and b) with a constant effective probe width.

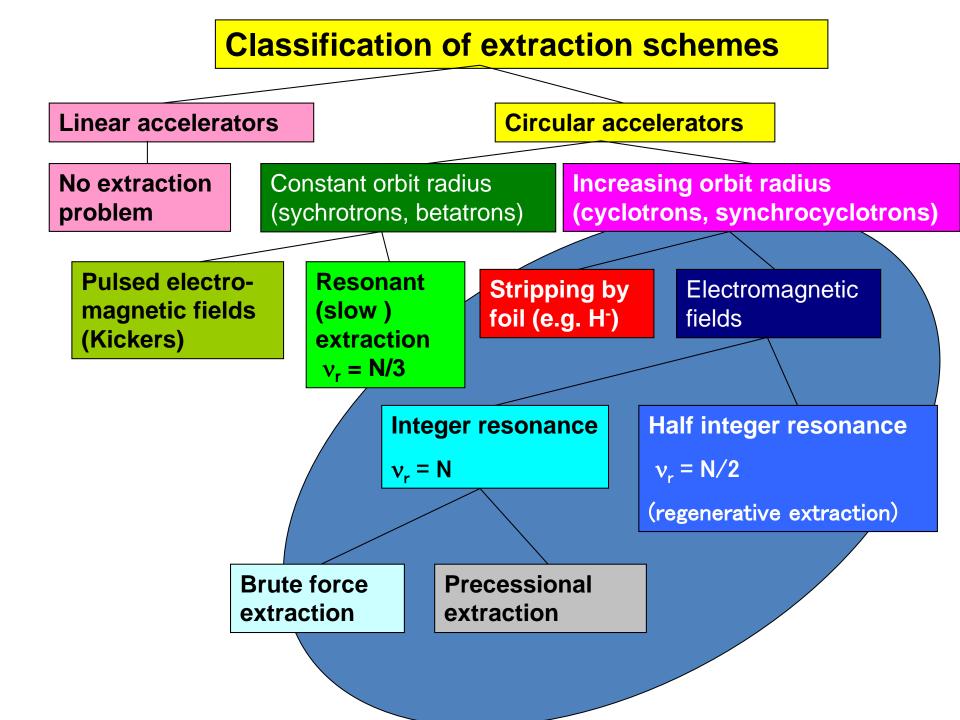




 Match the beam into the acclerated equilibrium orbit eigen ellipses with quadrupoles (4)



Extraction from cyclotrons



Extraction by acceleration

Radial increase of the orbit

- •By acceleration
- •By magnetic pumps

$$\frac{dR}{dn} = \frac{dR}{dn} (\operatorname{accel}) + \frac{dR}{dn} (\operatorname{magn})$$

$$\frac{dR}{dn}(\text{accel}) = R\frac{E_{\text{g}}}{E}\frac{\gamma}{\gamma+1}\frac{1}{\nu_r^2}$$

Three ways to get a high extraction rate

- 1. Build cyclotrons with a large average radius (without increasing the maximum energy)
- 2. Make the energy gain per turn as high as possible
- 3. Accelerate the beam into the fringe field, where v_r drops

This also calls for high energy gain, since phase slip in the fringe field must be kept small

Item 1. Remember that for the same maximum field and the same energy gain per turn

 $\frac{dR}{dR}$ (accel) $\propto \frac{1}{D}$

Item 3. especially important for high energy cyclotrons



Remember: for an isochronous field

$$k = \frac{r}{B} \frac{dB}{dr}$$
 Field index
= $\gamma^2 - 1$

And e.g. for a 3-sector magnet

$$v_r^2 \approx 1 + k + 0.675 F (1 + \tan^2 \alpha) + \dots$$

So, e.g. for the PSI 580 MeV cyclotron in the isochronous extraction region

 $v_r = 1.6$ and at the extraction in the fringe field $v_r = 1.1$ Factor of 2 in turn separation

Resonant extraction

Normally the radial gain per turn by acceleration in not enough

•Magnetic perturbations to enhance the turn separation

The integer resonance $v_r = N$

Brute force

Bump in the axial field ΔB

$$\Delta B(r,\theta) = b_N \cos N(\theta - \theta_N)$$

 v_r close to $N \longrightarrow$

The beam is driven off centre, maximum additional radial gain per turn being

$$\frac{dR}{dn}$$
(brute force) = $\pi R \frac{b_N}{NB_0}$

For a typical conventional cyclotron (B_0 =1.7 T) a bump of 0.1 mT introduces a radial gain of about 0.2 mm!!

To get a desired turn separation bigger bumps are needed

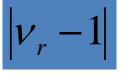
•"Brute force"

•This method has been used for example in the AEG compact cyclotron

Precessional extraction

The beam goes through $v_r=1$ resonance with a first order perturbation

•Beam starts to oscillate around its equilibrium orbit with a frequency



 $\cdot v_r$ decreases with radius

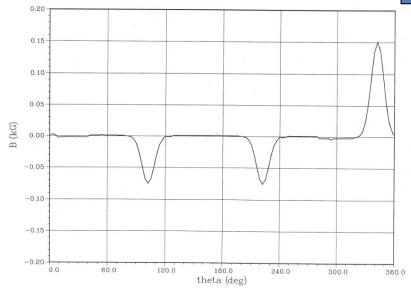
•Two consecutive turns oscillate with a slightly different frequency

•Phase difference between the turns increases

•Turn separation increases

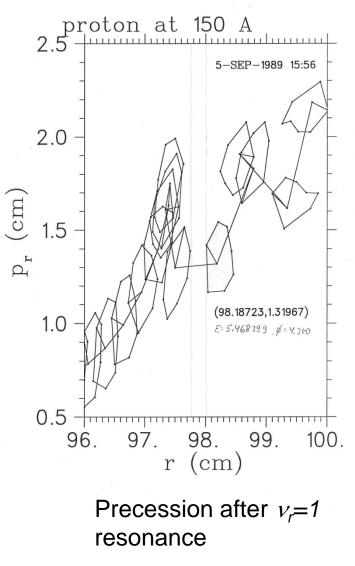
HCD FIELD AT 90 CM





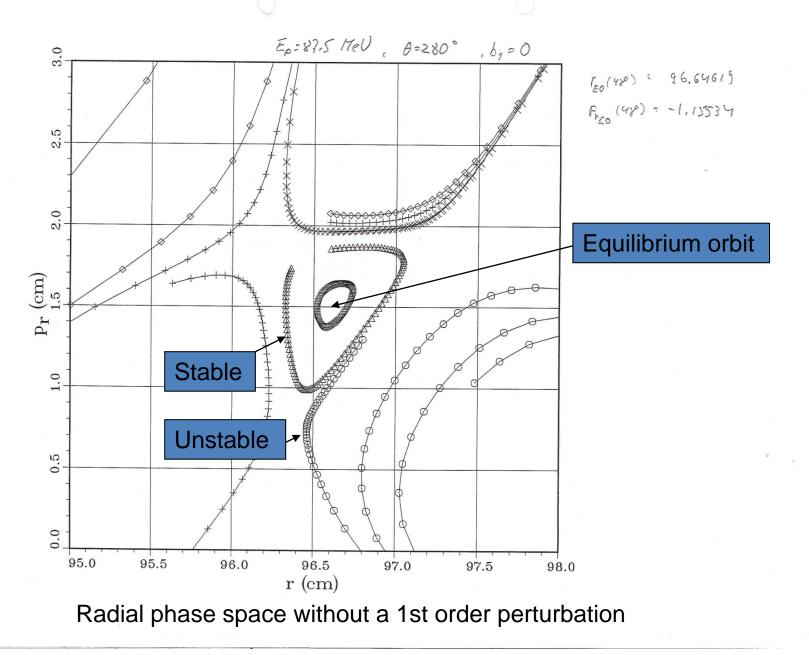
Contribution of harmonic coils in three valleys





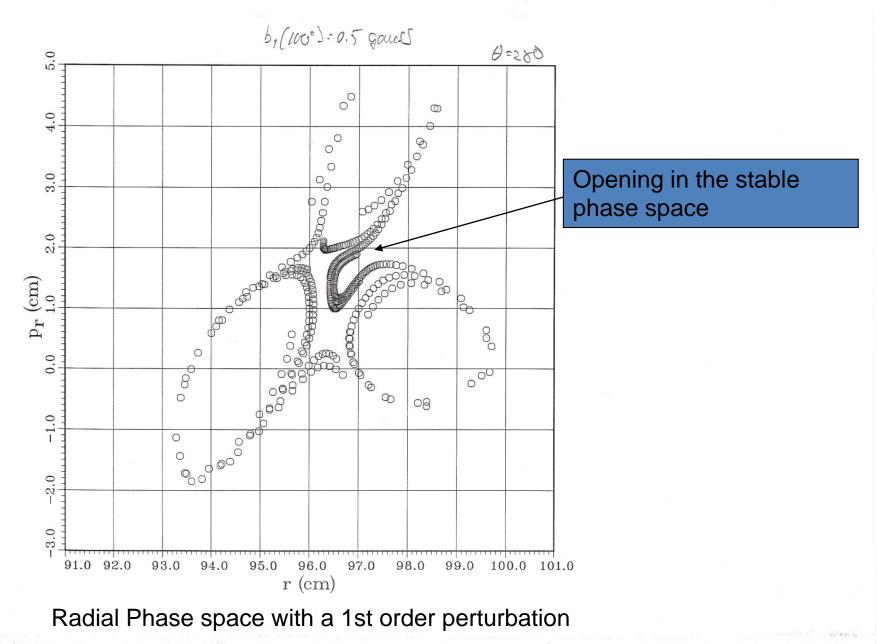
Betatron osc. around static EO

4.10.88 PH



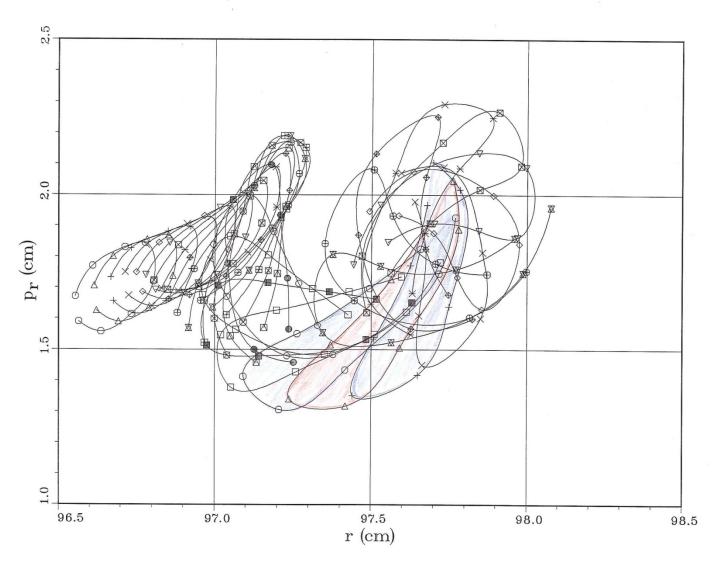
5.10.72 PH

Betatron osc. around static EO



 $\mathrm{B}_1 = 1.0~\mathrm{gauss} \; / \; \theta_\mathrm{b1} = 160 \; / \; \theta = 280$

10.10.82 1011

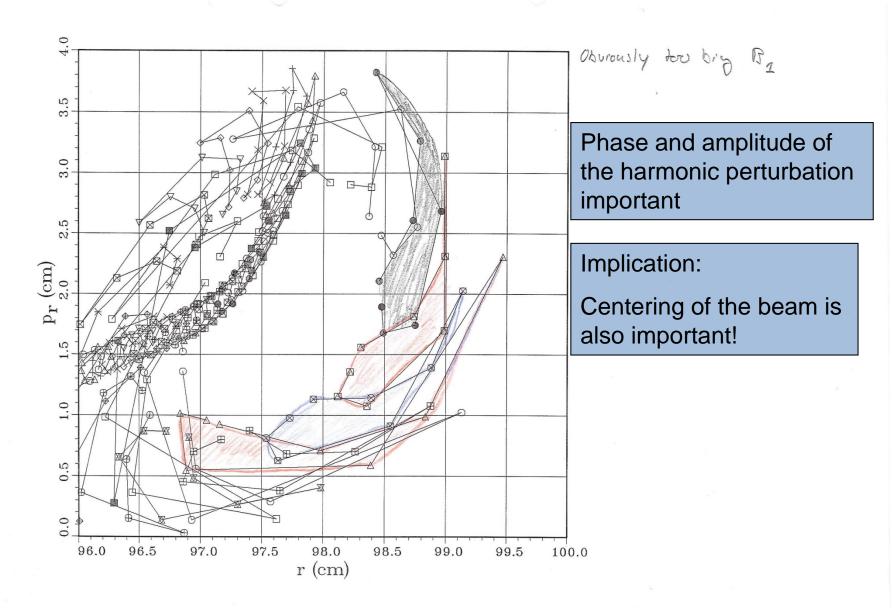


Phase and amplitude of the perturbation is important!

The state of the second

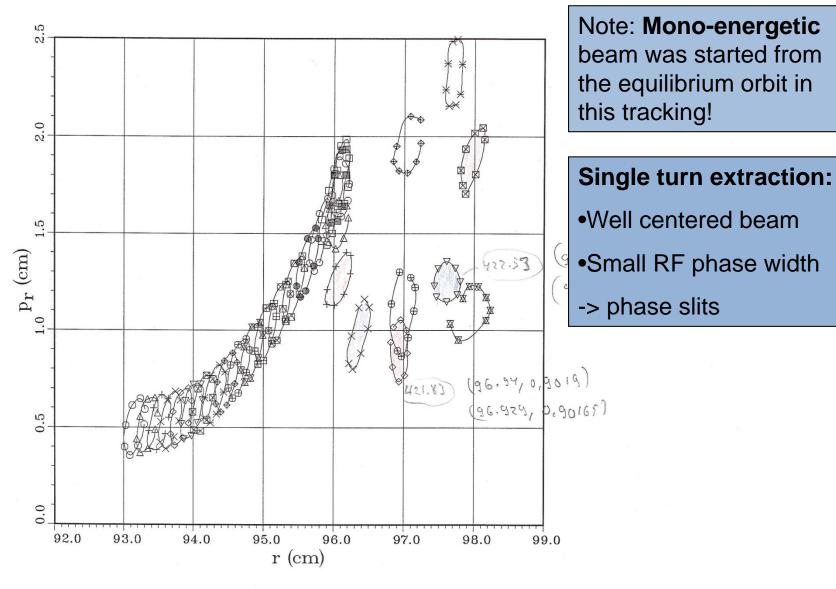
 $B_1=2~gauss$ / $\theta_{b1}=100$ / $\theta=280$

7.10.18 FN



$$B_1 = 3.0 \text{ gauss} / \theta_{b1} = 100 / \theta = 280$$

12.10.88 PH

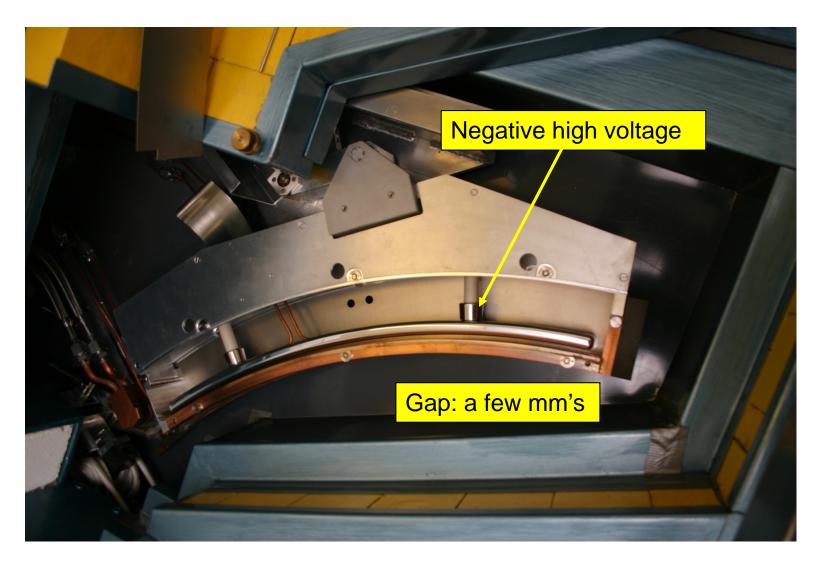


Nice behavior with a proper 1st harmonic perturbation

Extraction elements

- (Harmonic coils)
- Electrostatic deflector
- Electromagnetic channel
- (Passive) focusing channels
- Stripper

Electrostatic deflector



Electromagnetic channel

Electrostatic deflector

V-shape entrance for the septum

• effective thickness 0 mm

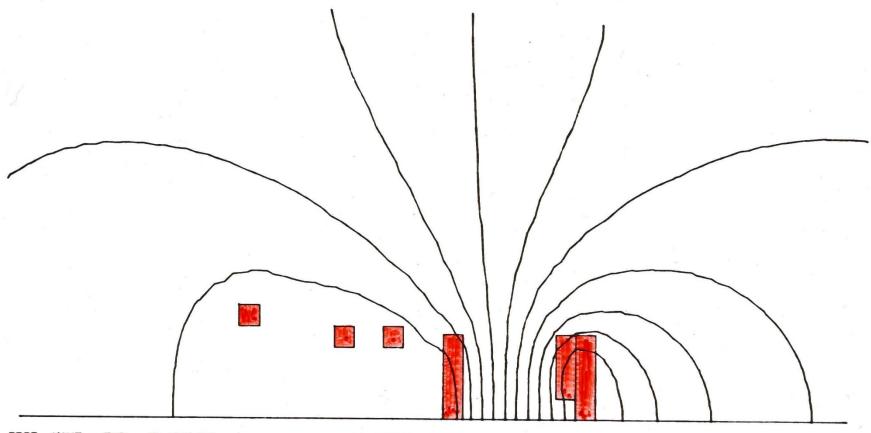
Distribute heat

Electromagnetic channel



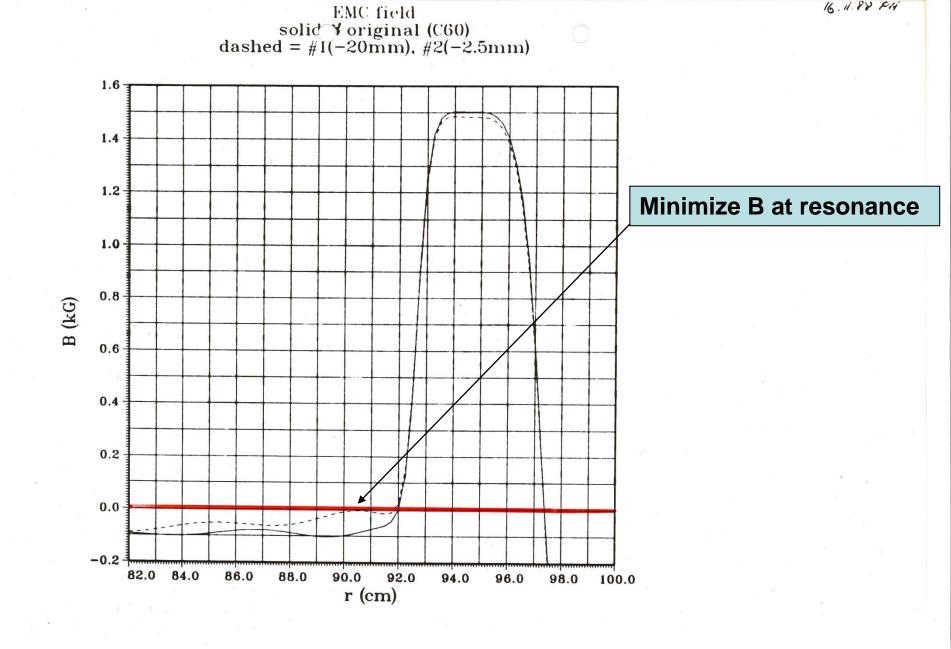
High current in the EMC coil

•Main coil current + booster current

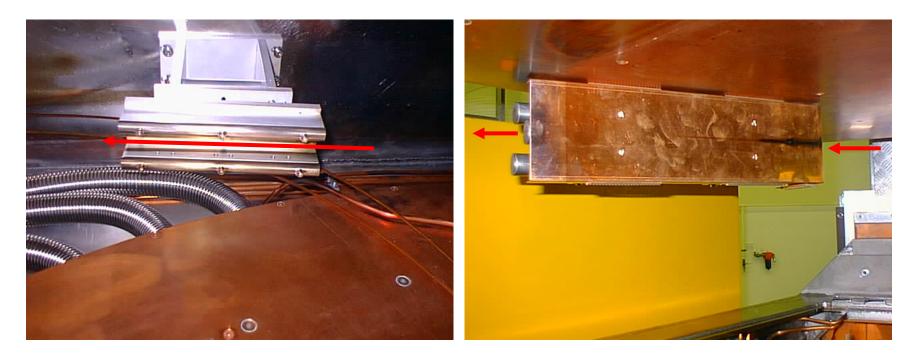


PROB. NAME = EMC - 19-NOV-88

CYCLE = 5160



Passive focusing channel



Vertically focusing

Horizontally focusing

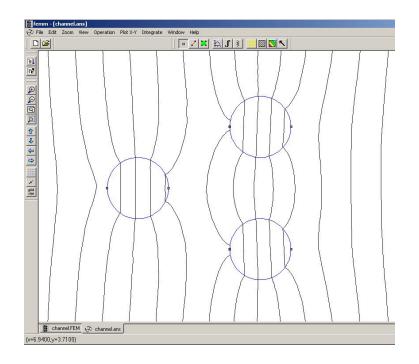
Iron bars are magnetized by the cyclotron magnetic field

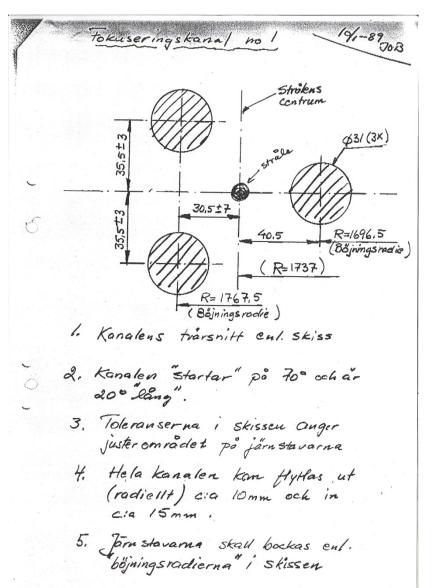
Focusing channel

•Extracted beam travels in the fast decreasing fringe field

•Horizontally defocusing

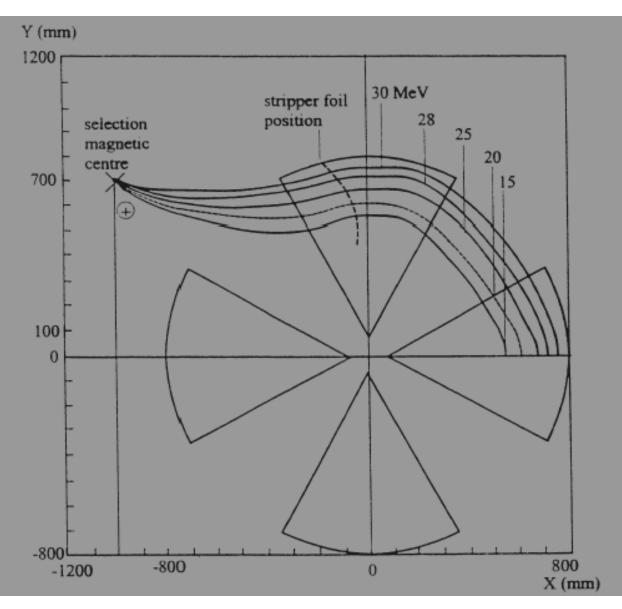
•More focusing by shaping the field (gradient) by passive channels





Stripping extraction

- The extraction efficiency for deflector + EMC is typically 50 90 %.
 - For high intensities activation, vacuum and melting problems
- For negative ions (H⁻, d⁻) stripping
 - $-1-2 \ \mu m$ carbon foil strips both electrons away
 - Charge state -1 -> +1
 - Efficiency close to 100 %
 - Short distance in the fringing field
 - Less focusing problems
- Caution! Electromagnetic stripping at high B and high velocity
 - Electron affinity (binding energy) for H^- is 0.75 eV



IBA Cyclone 30 Extraction by stripping

All energies go to one crossover point by proper foil azimuthal position

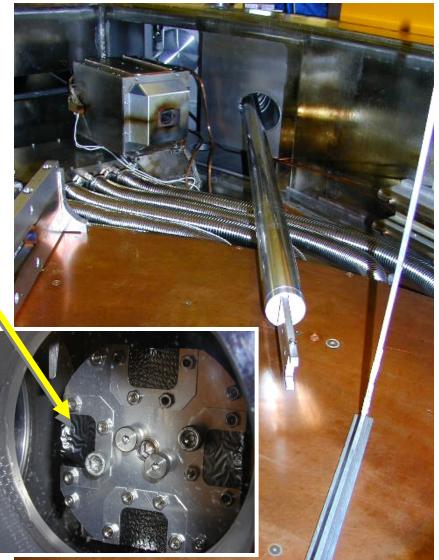
Place combination magnet at crossover Accelerator Laboratory, Department of Physics, University of Jyväskylä

Stripper

•4 carbon foils (16x22 mm)

-thickness 1 and 2 μm







Stripping extraction for heavy ions

- Typically $q_2 = 2q_1(1.4 4)$
 - Initially moderate charge state
 - Limits the maximum energy
 - Motivation
 - High extraction efficiency
 - Naked ion after stripping (no charge distribution)
 - e.g. 300 AMeV Cyclotron proposal (INFN)
 - Easy
 - If not fully stripped then a distribution
 - » Only one charge state has the right trajectory
 - e.g. Dubna

