

1) Injection into a storage ring

- a. When transferring particle beams from one accelerator to another, e.g., from an injector linac into a storage ring, one is confronted with a fundamental dilemma: the incoming (or injected) beam is initially outside the storage ring chamber and its trajectory needs therefore to be bent in such a way as to bring it into the chamber. However, it is not enough to steer the injected beam into the physical space delimited by the storage ring chamber at the injection point (IP) - in fact, the angle of the injected beam trajectory with respect to the nominal orbit in the storage ring must also be brought within certain limits - otherwise the incoming beam will simply hit the walls of the storage ring vacuum chamber at some point downstream from the IP and be lost. This requirement on steering the incoming beam can be expressed as a requirement for *bringing the injected beam into the storage ring acceptance*.
- b. Consider an electron storage ring with 528 m circumference, injected from a linear accelerator. Electrons are brought from the linac and conducted through a transport line up to the storage ring where the transfer of electrons (the injection process) happens on the horizontal plane, i.e. we assume that (at the IP) the incoming beam from the linac lies at the same vertical coordinate as the storage ring nominal orbit, with zero vertical angle. The incoming beam horizontal position at the IP is -13.5 mm^1 away from the nominal storage ring orbit and the incoming beam's horizontal trajectory angle with respect to the nominal storage ring orbit is zero. Assume that the storage ring vacuum chamber has inner dimensions of mm , that the maximum horizontal betatron function over the whole ring is m and that horizontal betatron function at the IP is m . Finally, assume that the injection point is a dispersion-free, symmetry point in the storage ring lattice, so that $\alpha = 0$ and $\beta = \beta_{\text{max}}$. **Calculate the horizontal acceptance (in $\text{mm} \cdot \text{mrad}$) defined by the storage ring vacuum chamber aperture. Draw a phase space diagram at the IP indicating the ring acceptance as well as the incoming beam coordinates.**
- c. In order to bring the incoming beam into the storage ring acceptance, one may add a magnet that deflects the incoming beam horizontally² at some point downstream of the IP - let us call this the "kick point" or KP³. Assume that the betatron phase advance between the IP and the KP is π and that the betatron function at the KP is β_{KP} .

¹ The negative sign is meant to indicate the incoming beam is located on the inner side of the ring circumference.

² The deflecting magnet needs therefore to generate a vertical magnet field component.

³ Note that the storage ring vacuum chamber needs to be made a little wider in this particular region between the IP and KP to make sure the incoming beam is not lost!

- m. Calculate the required bending angle at the KP to bring the incoming beam all the way to the center of the storage ring acceptance. Calculate the minimum required bend angle at the KP to just bring the incoming beam into the storage ring acceptance (i.e. barely touching the border of the acceptance).
- d. If we were to implement the bending of the incoming beam by means of a DC magnet, i.e., by a magnetic field that is constant in time, then after one revolution around the ring, the incoming beam would once again suffer the same bend. **Draw a sketch that shows that the incoming beam would be lost (i.e. would again be outside the storage ring acceptance) after one turn in case it is brought to the center of the acceptance in the first pass through the kicker.**
- e. A solution to the problem above is to use a fast pulsed magnet (also called a *kicker*) instead of a DC magnet, so that the incoming beam sees no field at all when it passes by the KP after one turn around the storage ring. **How fast does our pulsed magnet need to be ?**
- f. Once injected and captured in the storage ring, the incoming beam circulates and emits synchrotron radiation - the emission process leads to damping (i.e. acts as "friction") of the betatron oscillations and, after a number of damping times (few tens of ms), the injected beam cools down to follow the storage ring nominal closed orbit. The beam will no longer have any memory of the injection process but its dimensions will instead be determined by the equilibrium between damping mechanisms (emission of light) and excitation mechanisms (due to the quantum nature of the light emission process). This leads to the so called equilibrium emittance - Assume that the storage ring equilibrium horizontal emittance is 300 pmrad and indicate the stored beam at the IP in your phase space diagram. **What is the ratio between the amplitude of the incoming beam oscillations and the stored beam size ?**
- g. An additional problem arises when one wants to fire a succession of pulses from the injector linac into the storage ring so that one can *accumulate* the charge from many linac shots in the storage ring. In that case, one needs to remember that a stored beam already exists in the storage ring when a new incoming beam pulse is fired. In the scheme described above, the kicker magnet deflects both the incoming beam and the stored beam. **How should we choose the kicker deflection angle to make sure that the incoming beam is captured and that, at the same time, the stored beam is not thrown away from the storage ring acceptance?**
- h. The discussion above assumes that the kicker magnet produces a homogeneous field, i.e., that the bending angle the kicker imparts to the beam is independent of the transverse position at which the beam crosses the kicker, meaning that both the incoming and stored beams are deflected by then same angle. The kick given to the stored beam produces a transient perturbation on the position of the light source points seen by the storage ring users - even though this perturbation disappears after only a few damping times, it is of interest to make the injection

process fully transparent by devising a way to kick the injected beam without perturbing the stored beam. Put together a conceptual design for a kicker magnet based on four current-carrying wires arranged along the direction of propagation of the stored beam and with current polarities chosen in such a way that the vertical component of the magnetic field at the horizontal center of the magnet is zero whereas the vertical component of the magnet field at the periphery (say at -6 mm from the center) is large enough to kick a 3 GeV beam by 3 mrad. Assume that the longitudinal space available for the kicker is 30 cm and that the minimum vertical full aperture of the magnet is 8 mm and calculate the required current in the four wires. What happens if the stored beam crosses the kicker slightly off-center ? Can we use this kicker with the same phase advance from the IP to the KP and same initial conditions for the incoming beam as in item c above ?

- i. Can you propose other ways of generating adequate spatial distributions of the kicker fields that lead to transparent injection ?

2) Characteristics of Undulator Radiation

- a. Determine the period length required to achieve an on-axis first-harmonic photon energy of 1.69 keV at an electron beam energy of 3 GeV with a peak undulator field of 1.131 T.
- b. Use the computer code Spectra to determine the spectral angular flux density (in photons/sec/mrad²/0.1%) as a function of photon energy (from 1.5 to 2.0 keV) for an undulator with the parameters above for different values of undulator length (or, equivalently, different values of the number of periods). Scan a range from about 0.5 m to 4.0 m undulator length. (Note: **assume the electron beam has zero emittance and zero energy dispersion**).
 - i. What is the scaling law for the peak spectral angular density as a function of number of periods? Comment.
 - ii. What is the scaling law for the spectral peak width as a function of number of periods? Comment.
- c. Take now a fixed undulator length (4.016 m) and determine the spectral angular flux density as a function of the electron beam energy spread - starting from 0 and up to 5 % spread (in reality the energy spread is typically on the order of 0.1%) .
 - i. How does peak angular flux density change with the energy spread ? Comment.
 - ii. How does the spectral peak width scaled change the energy spread ? Comment.
- d. Determine (still using Spectra) the spectral peak width for the first, third and fifth harmonics. How does the peak width scale with the number of the harmonic ? What about the relative spread ?
- e. Since the width of the spectral lines of an undulator are affected by the electron energy spread, an observation of the undulator spectrum can be used to estimate the energy spread. From the results you obtained in items c and d above, what is the best choice of harmonic for performing such as experiment?
- f. The calculations above only take into account radiation produced along the axis of the undulator. In reality, detectors have a finite angular acceptance and photons emitted off-axis will also be detected. Use Spectra to determine the photon energy dependence (around the third harmonic) of the spectral flux integrated on a rectangular aperture. Start with a small aperture (50x50 microrad²) and gradually increase to 100x100 microrad. In what way is the observed broadening of the

spectral peak different from the broadening observed when the energy spread is introduced ? Can you explain why ?