



Cavities and waveguides

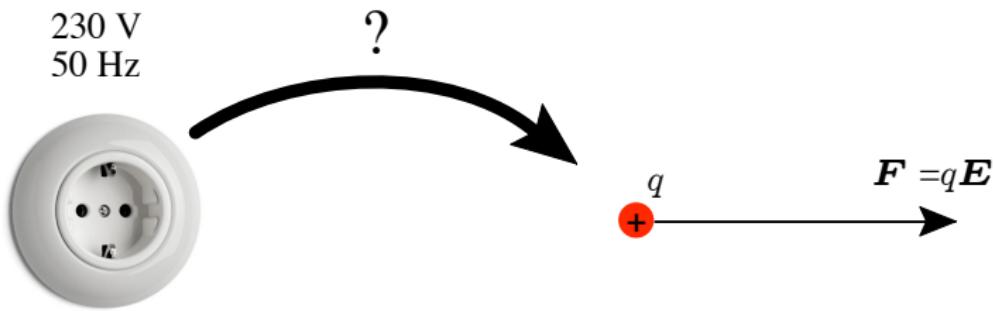
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Department of electrical and information technology

Acceleration of particle

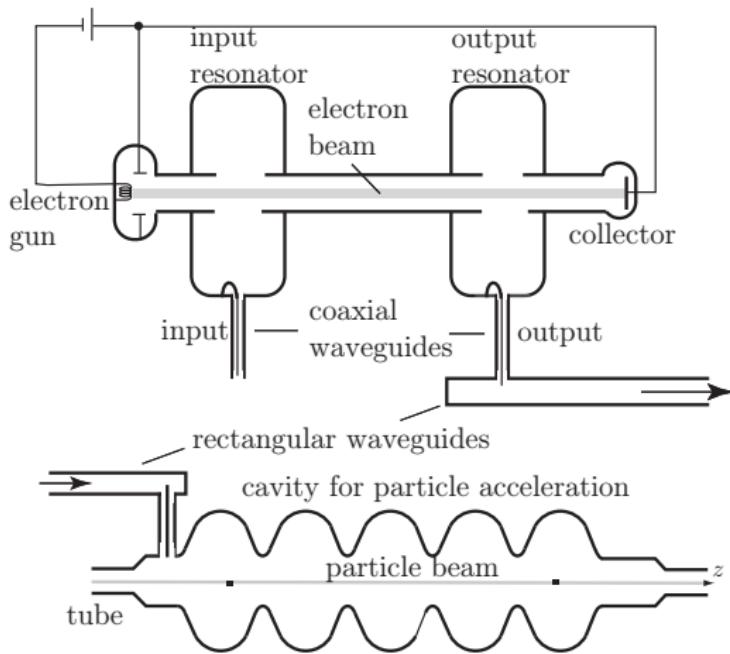


Acceleration of particle

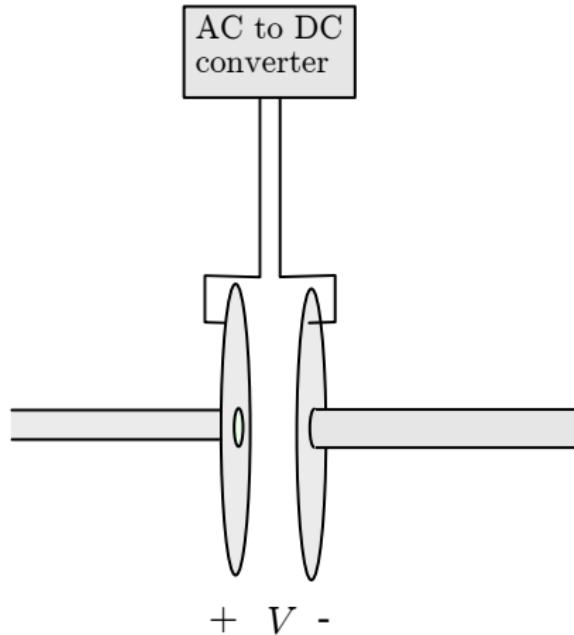


- ▶ Particle energy. Beam intensity.
- ▶ Efficiency
- ▶ Size

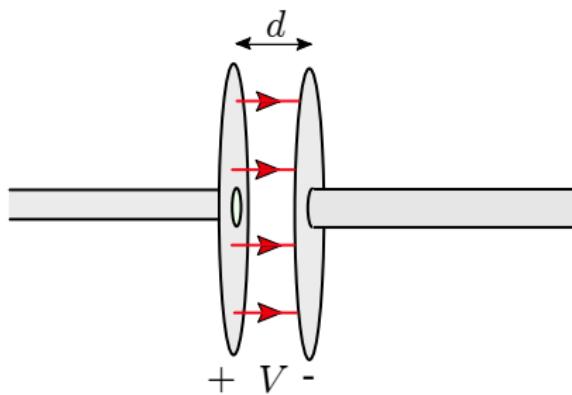
A system



A simple system

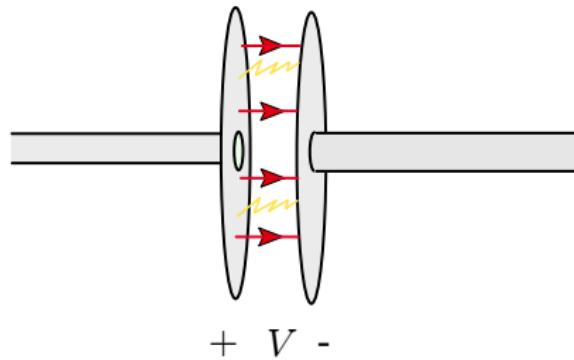


A simple system



Electric field $E = \frac{V}{d}$. Particle energy qV .

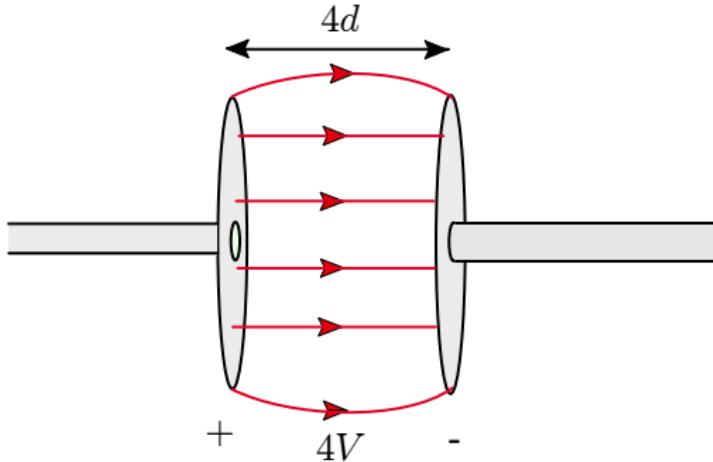
A simple system



Problemo: Discharges!

Solution: Increase distance and voltage.

A simple system



Electric field $E = \frac{4V}{4d}$. Particle energy $4qV$.

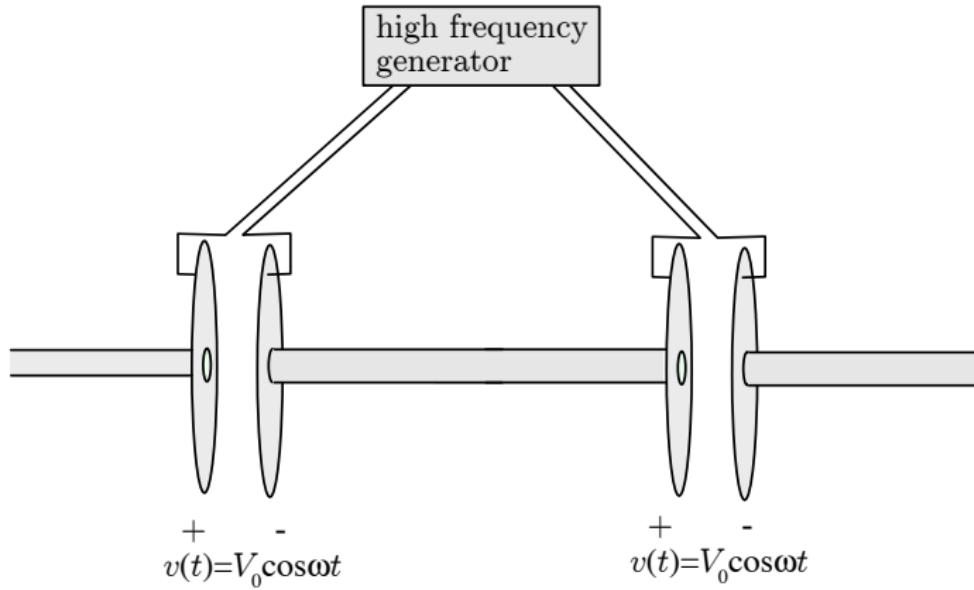
Problemo: Very large distances between plates. Very high voltages.

Electrostatic solutions up to 30 MeV

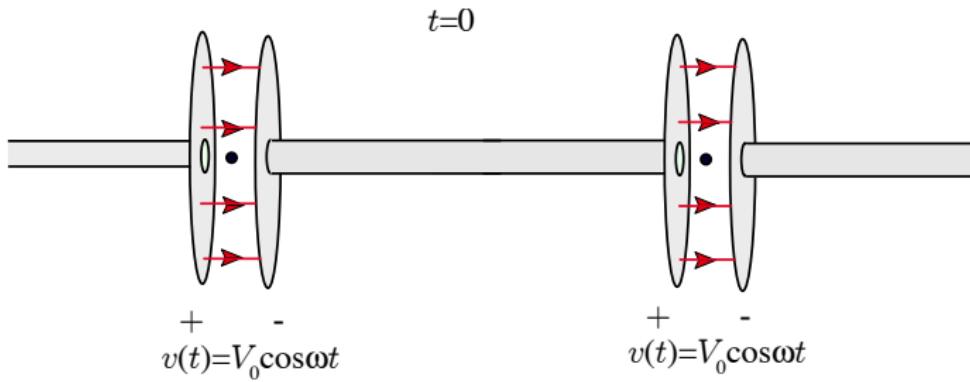
How do we reach 3 GeV (MAX IV, ESS), or 7 TeV (LHC)?

Answer: Use AC!

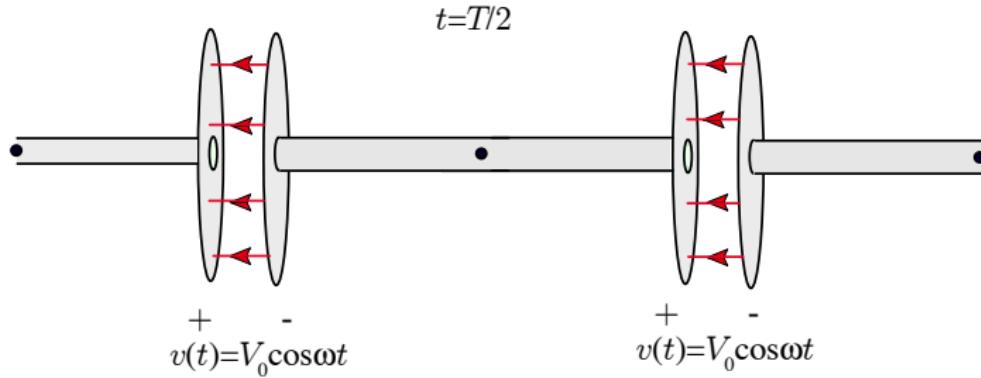
A simple AC system



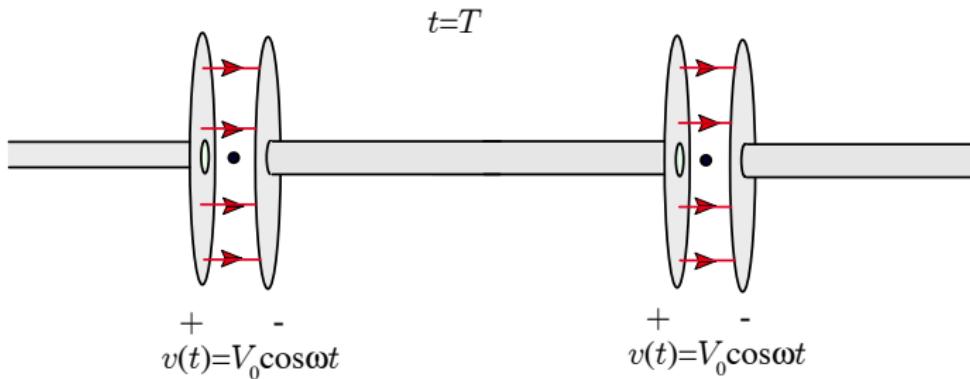
A simple AC system



A simple AC system



A simple AC system

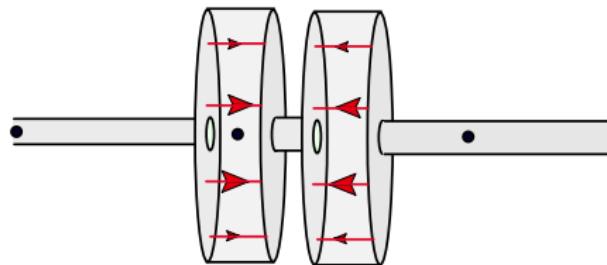


Problemo: At high frequencies the capacitors become antennas and radiate power.

Solution: Put walls on the capacitors so that they become pillboxes.

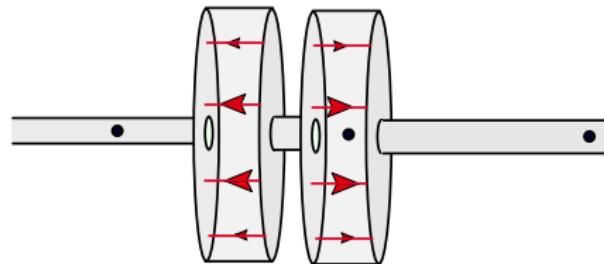
The cylindric cavity

$t=0$



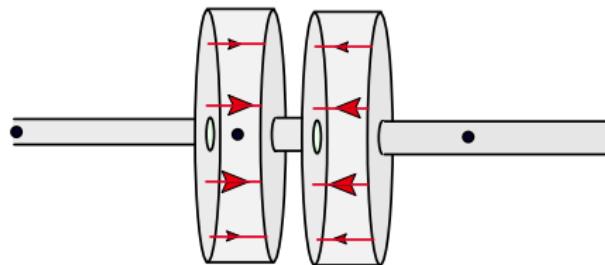
The cylindric cavity

$t=T/2$

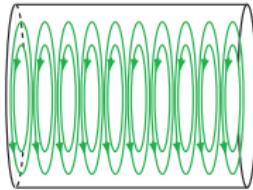
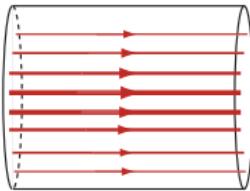


The cylindric cavity

$t=T$



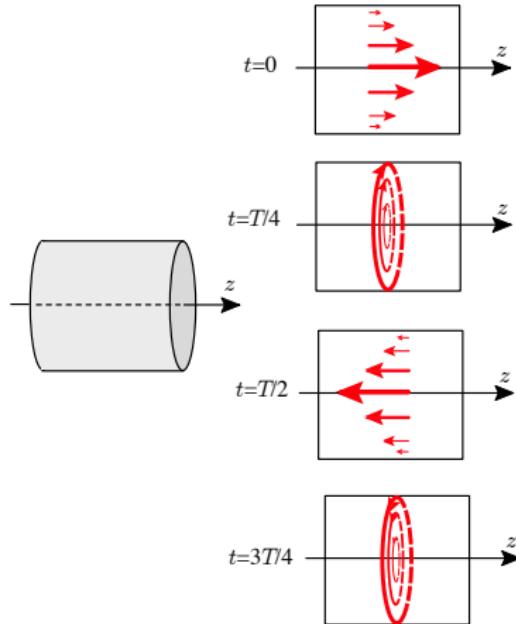
The cylindric cavity: TM₀₁₀ mode



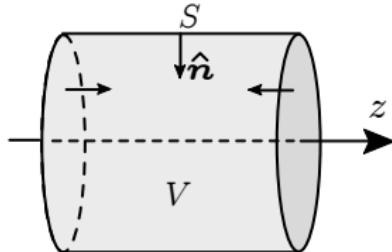
$$\mathbf{E}(\rho, t) = E_0 J_0(k\rho) \cos(\omega t) \hat{\mathbf{z}}$$

$$\mathbf{H}(\rho, t) = H_0 J_1(k\rho) \sin(\omega t) \hat{\phi}$$

The cylindric cavity: TM₀₁₀ mode



The cylindric cavity



$$\nabla^2 \mathbf{E}(\mathbf{r}) + k^2 \mathbf{E}(\mathbf{r}) = \mathbf{0}, \mathbf{r} \in V$$

$$\nabla \cdot \mathbf{E}(\mathbf{r}) = 0, \mathbf{r} \in V$$

$$\hat{\mathbf{n}} \times \mathbf{E}(\mathbf{r}) = \mathbf{0}, \mathbf{r} \in S$$

$$k = \frac{\omega}{c}$$

Eigenvalue problem!

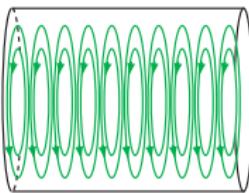
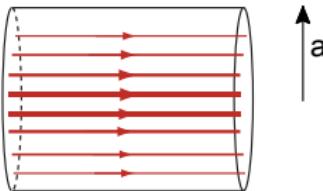
Eigenvalue problem

- ▶ Infinitely many eigenvalues k_n^2
- ▶ All k_n^2 are real and positive
- ▶ The eigenfields $\mathbf{E}_n(\mathbf{r})$ are orthogonal and can be normalized

$$\iiint_V \mathbf{E}_n(\mathbf{r}) \cdot \mathbf{E}_m(\mathbf{r}) dV = \delta_{nm}$$

- ▶ Magnetic field from the induction law: $\mathbf{H}_n = -\frac{i}{\omega_n \mu_0} \nabla \times \mathbf{E}_n$
- ▶ A cavity mode is the fields that belong to a certain eigenwavenumber k_n

The TM₀₁₀ mode



$$\mathbf{E}(\rho, t) = E_0 J_0(k\rho) \cos(\omega t) \hat{z}$$

$$\mathbf{H}(\rho, t) = H_0 J_1(k\rho) \sin(\omega t) \hat{\phi}$$

where $k = 2.405/a$. Resonance frequency $f = \frac{2.405 \cdot c}{2\pi a}$.

The TM₀₁₀ mode

Examples:

MAX IV Linac: $f = 3 \text{ GHz}$ then $a = 3.8 \text{ cm}$

Elliptic cavities at ESS and LHC: $f = 704 \text{ MHz}$ then $a = 16.2 \text{ cm}$

Drift tube linac (DTL) at ESS: $f = 302 \text{ MHz}$ then $a = 34.4 \text{ cm}$

Cavities in storage ring for MAX IV: $f = 100 \text{ MHz}$. Then $a = 114 \text{ cm}!$

The TM_{*mnl*} mode

What is TM₀₁₀?

Transverse magnetic mode with $m = 0$, $n = 1$, $\ell = 0$.

$$\nabla^2 E_z(\rho, \phi, z) + k^2 E_z(\rho, \phi, z) = 0$$

$$E_z(a, \phi, z) = 0$$

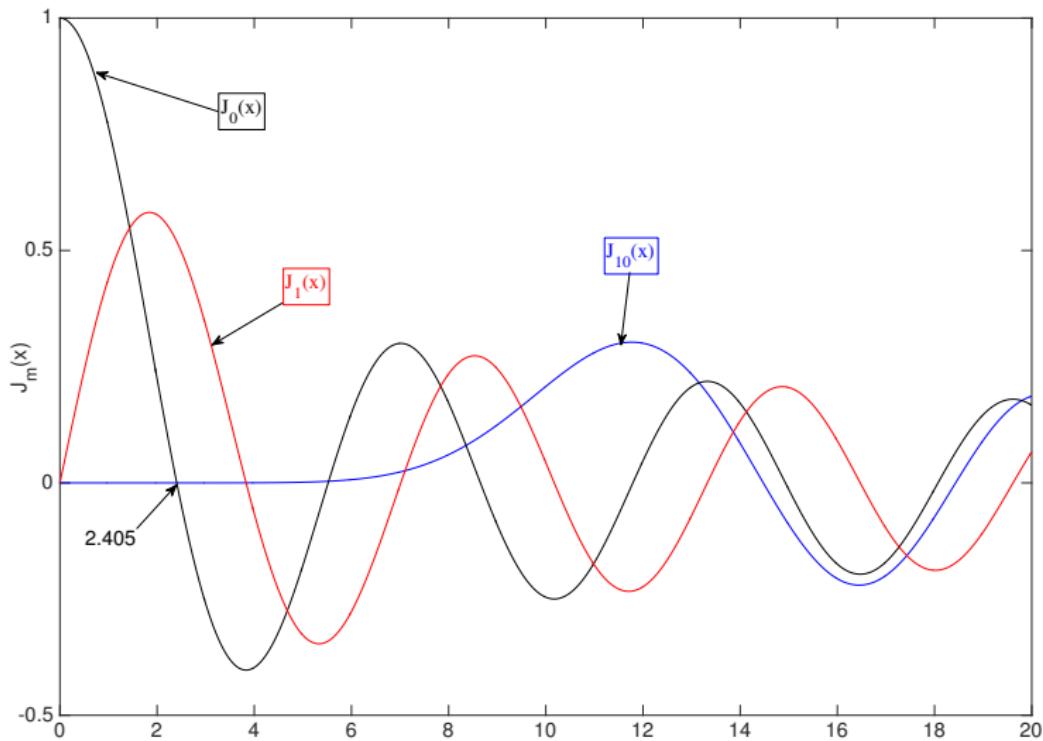
$$\frac{\partial E_z}{\partial z}(\rho, \phi, 0) = \frac{\partial E_z}{\partial z}(\rho, \phi, h) = 0$$

$$E_z(\rho, \phi, z) = E_0 J_m(k_{tmn}\rho) \cos(m\phi) \cos\left(\frac{\ell\pi z}{h}\right)$$

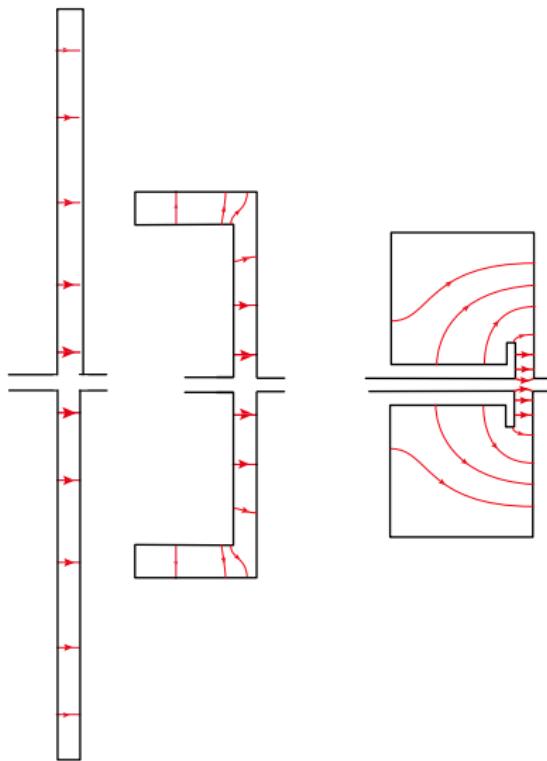
$$m = 0, 1, 2 \dots, n = 1, 2, \dots, \ell = 0, 1, 2 \dots$$

$$J_m(k_{tmn}a) = 0 \text{ and } k^2 = k_{tmn}^2 + \left(\frac{\ell\pi z}{h}\right)^2$$

The $\text{TM}_{mn\ell}$ mode



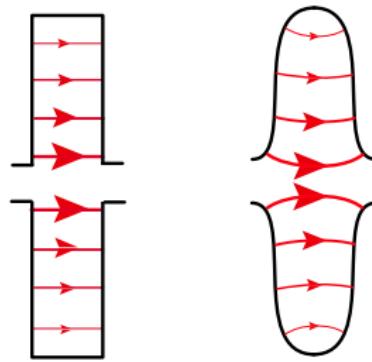
Deformation of the TM₀₁₀ mode



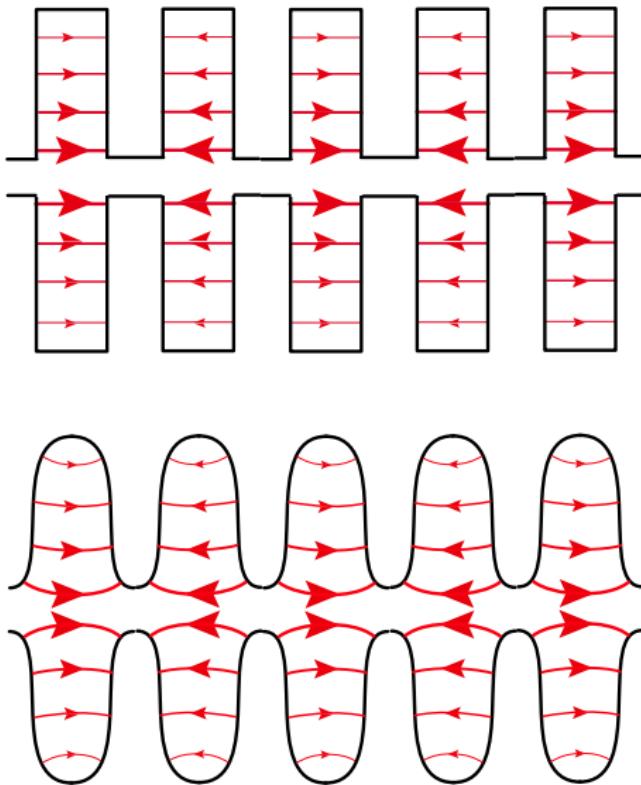
Deformation of the TM₀₁₀ mode



Deformation of the TM₀₁₀ mode



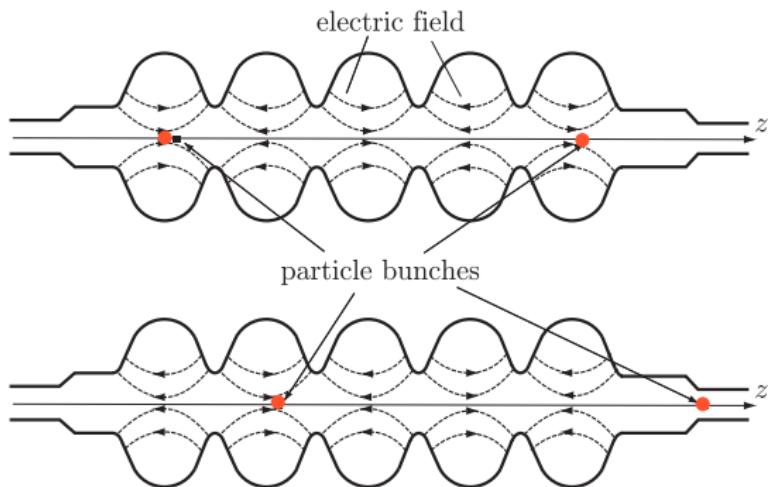
Deformation of the TM_{010} mode



Deformation of the TM₀₁₀ mode

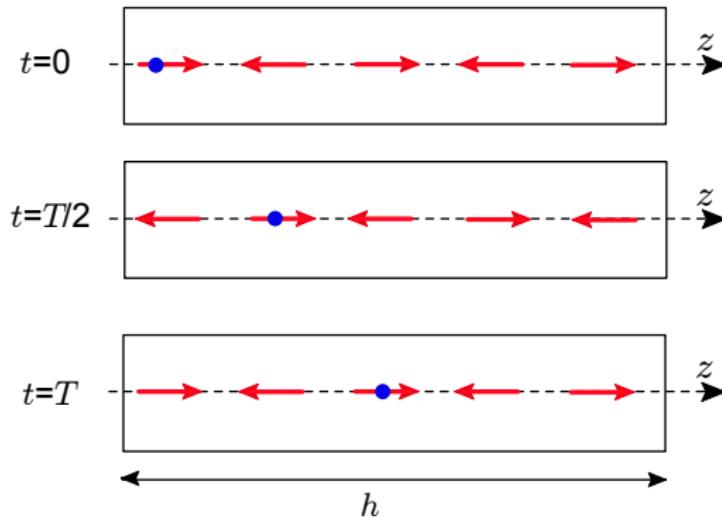


Deformation of the TM₀₁₀ mode



Idea

Use $\text{TM}_{01\ell}$?



Use TM_{01ℓ}?

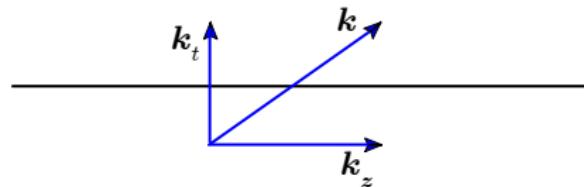
$$E_z(\mathbf{r}) = E_0 J_0(k_{t1}\rho) \cos\left(\frac{5\pi z}{h}\right)$$

$$E_z(\mathbf{r}) = \frac{1}{2} E_0 J_0(k_t \rho) \left(e^{ik_z z} + e^{-ik_z z} \right)$$

$$k_z = \frac{5\pi}{h}$$

Two traveling waves ($\overrightarrow{}$, $\overleftarrow{}$).

Idea



$$k_t = \frac{2.405}{a}, k_z = \frac{5\pi}{h} \text{ and } k = \sqrt{k_t^2 + k_z^2}$$

Phase speed $\equiv v_p = \frac{\omega}{k_z}$.

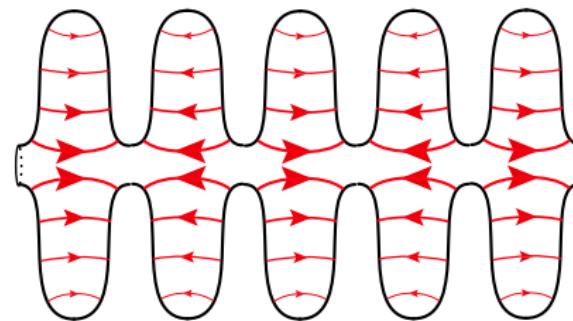
Speed of light $\equiv c = \frac{\omega}{k}$

$$v_p = \frac{k}{k_z} c > c$$

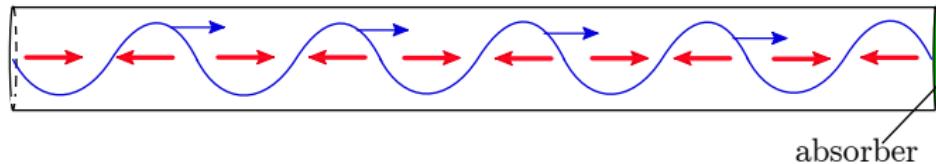
Problemo! We need to slow down the waves so that $v_p =$ speed of the particle.

Using the $\text{TM}_{01\ell}$ mode

Method: Deform the cavity.



Traveling wave cavities



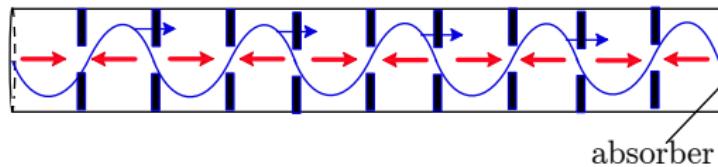
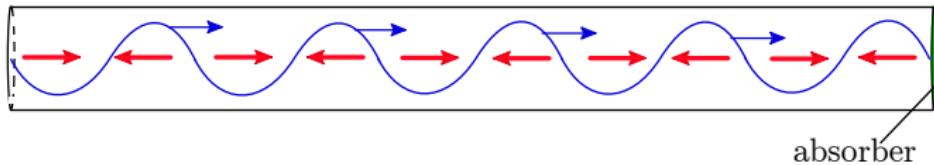
Only right moving wave!

$$E_z(\mathbf{r}) = E_0 J_0(k_t \rho) e^{ik_z z}$$

$$E_z(\mathbf{r}, t) = E_0 J_0(k_t \rho) \cos(\omega t - k_z z)$$

Problemo: $v_p >$ speed of particle

Traveling wave cavities



v_p = speed of particle

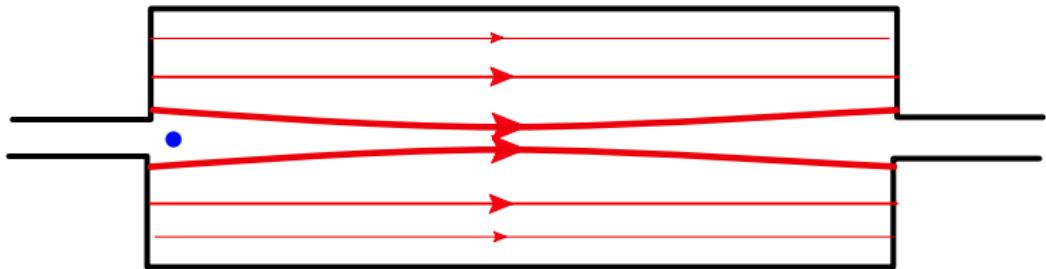
Traveling wave cavities



v_p = speed of particle

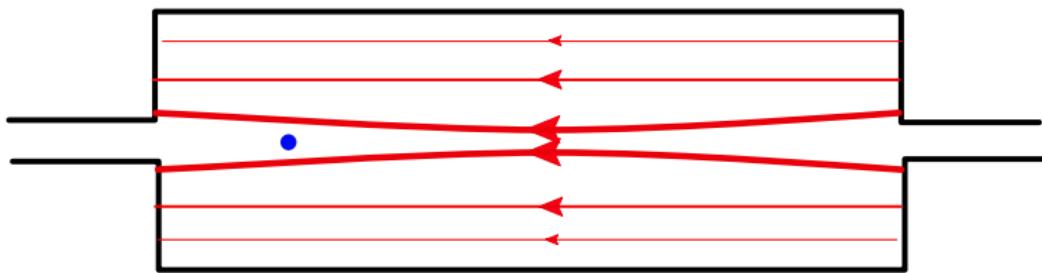
Drift Tube Linac (DTL)

Go back to TM₀₁₀



Drift Tube Linac (DTL)

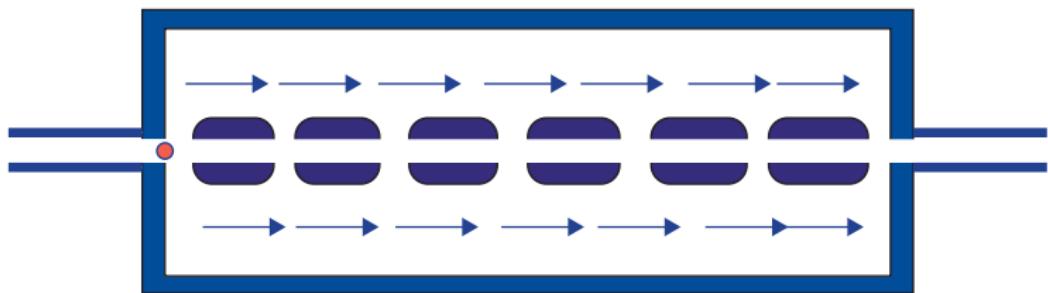
TM_{010}



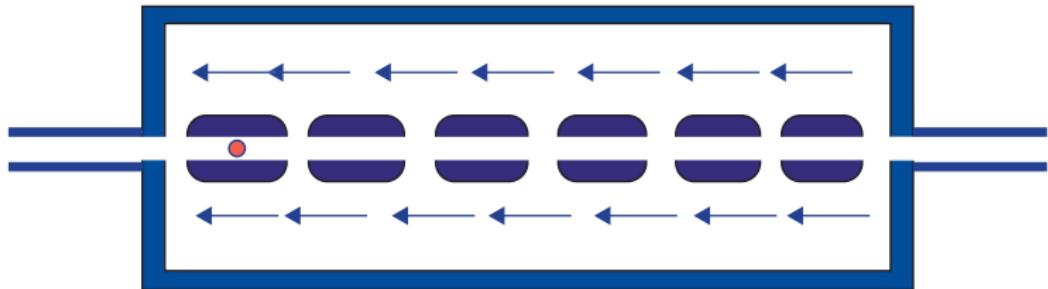
Problemo!

Drift Tube Linac (DTL)

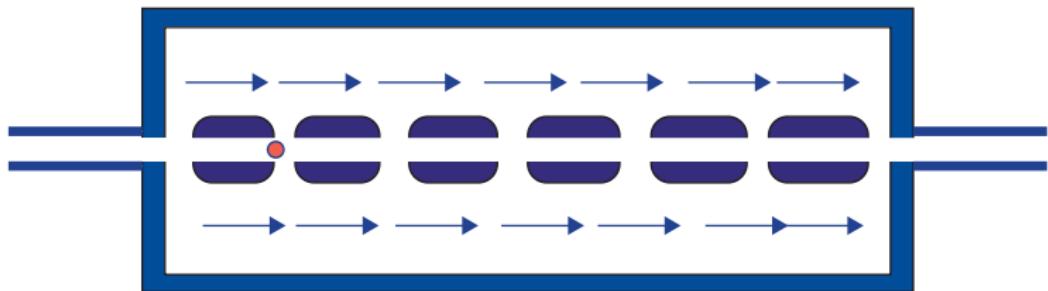
A solution!



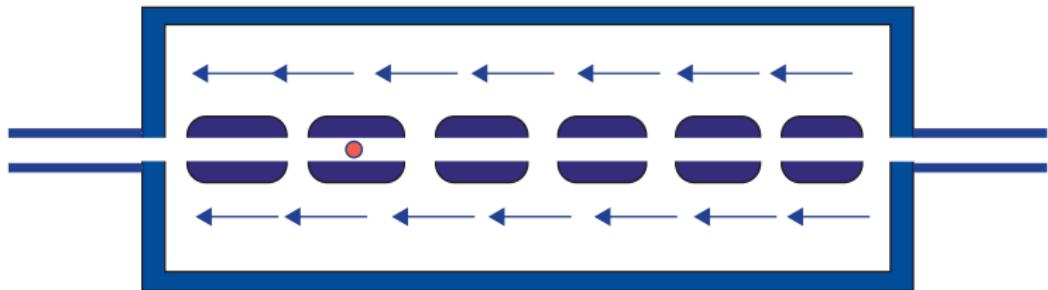
Drift Tube Linac (DTL)



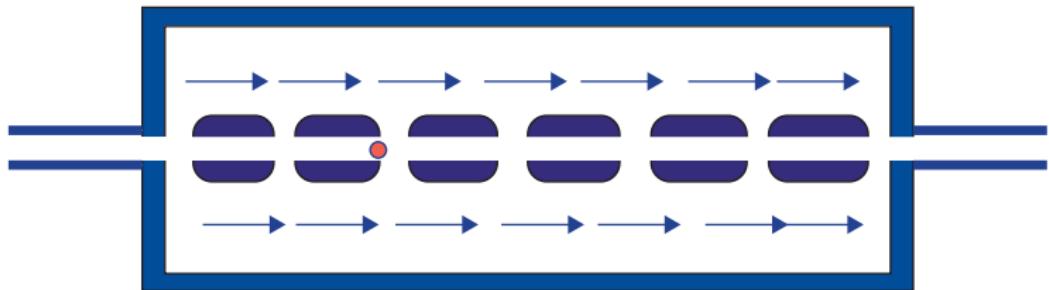
Drift Tube Linac (DTL)



Drift Tube Linac (DTL)



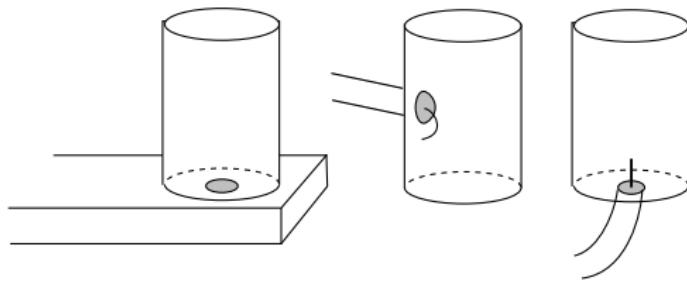
Drift Tube Linac (DTL)



Drift Tube Linac (DTL)

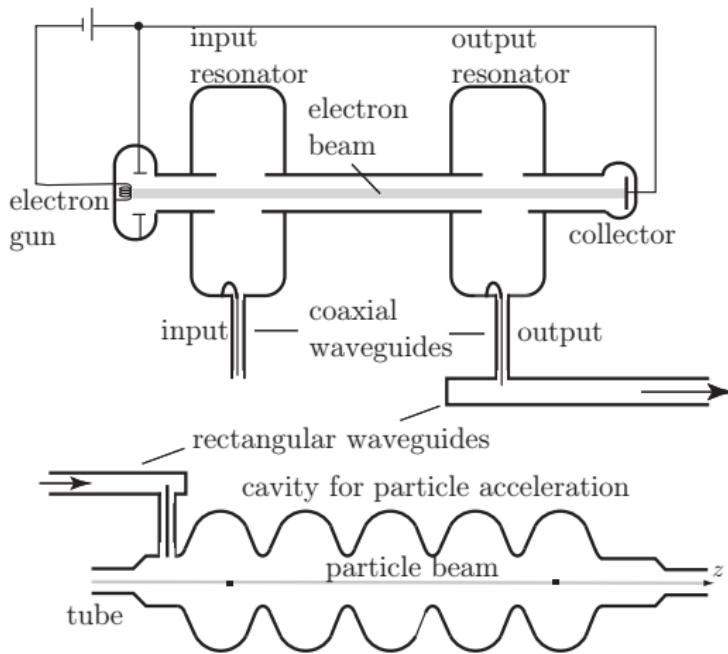


Power coupler

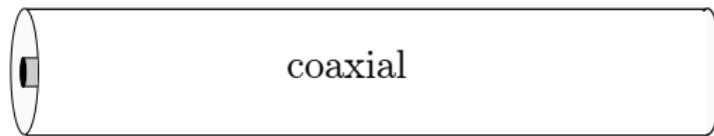


Three examples of feeding power to the TM_{010} mode.

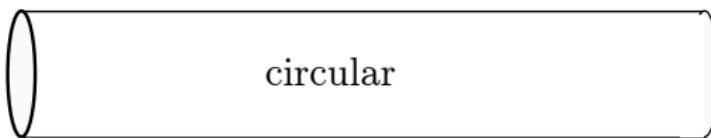
Feed of elliptic cavity



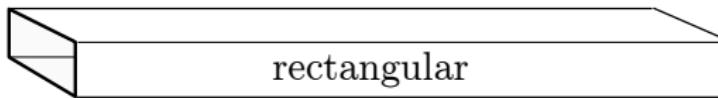
Waveguides



coaxial



circular



rectangular

Coaxial

Advantage: Small radius. Large bandwidth.

Disadvantage: Large losses.

Circular

Advantage: Low losses.

Disadvantages: Large radius. Small bandwidth. Hard to control polarization of waves.

Rectangular

Advantages: Low losses. Easy to control polarization. Easy to fabricate and to build systems of.

Disadvantage: Quite large cross section.

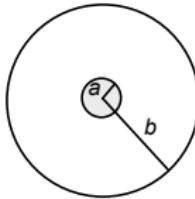
Waveguides

The winners

Coaxial for frequencies up to 200 MHz.

Rectangular for frequencies above 200 MHz.

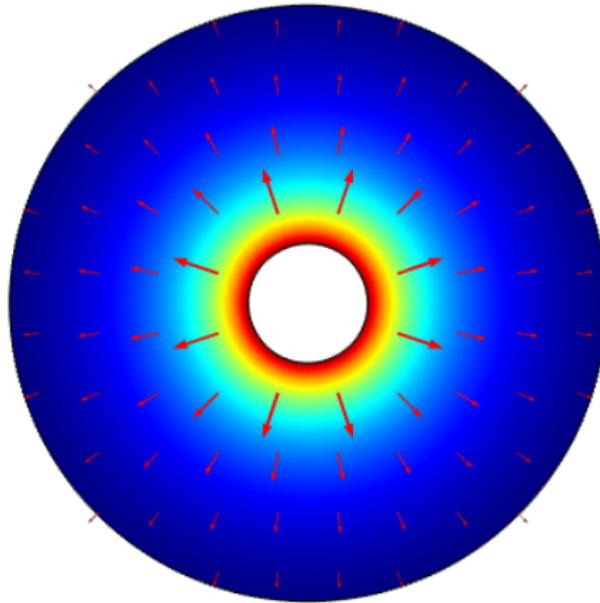
Waveguides: coaxial



Example: $a = 3 \text{ cm}$, $b = 15 \text{ cm}$.

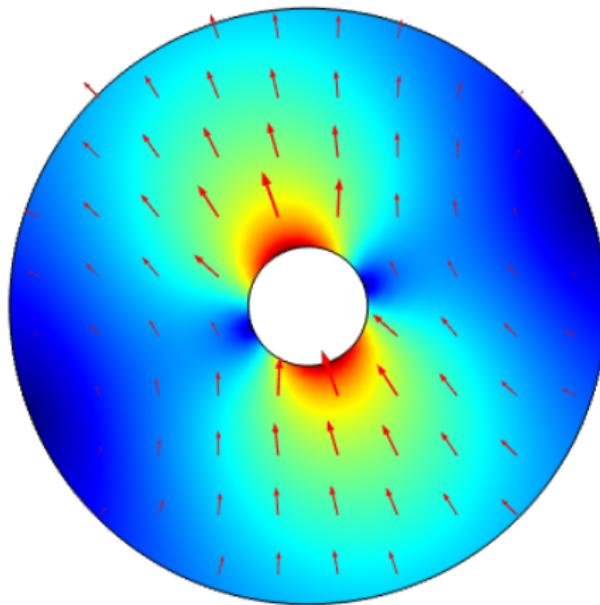
Why is it a problem to use this coax at 600 MHz?

Waveguides: coaxial



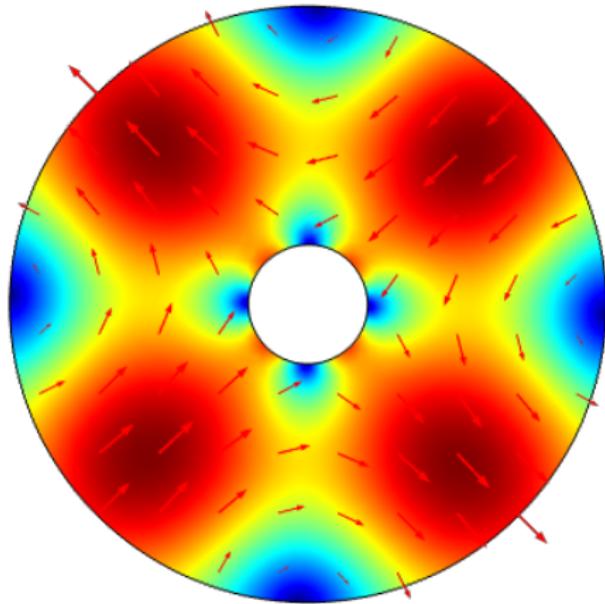
Example: $a = 3 \text{ cm}$, $b = 15 \text{ cm}$.
First mode starts at 0 Hz.

Waveguides: coaxial



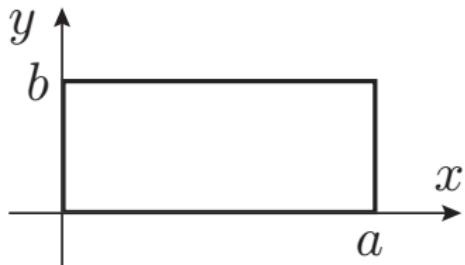
Example: $a = 3 \text{ cm}$, $b = 15 \text{ cm}$.
Second mode at 542 MHz.

Waveguides: coaxial



Example: $a = 3 \text{ cm}$, $b = 15 \text{ cm}$.
Third mode at 965 MHz.

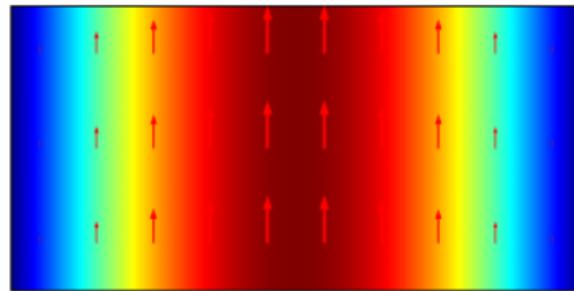
Waveguides: rectangular



Example: $a = 60$ cm and $b = 30$ cm

Question: Why cannot we use this waveguide for $f < 250$ MHz and $f > 500$ MHz?

Waveguides: rectangular



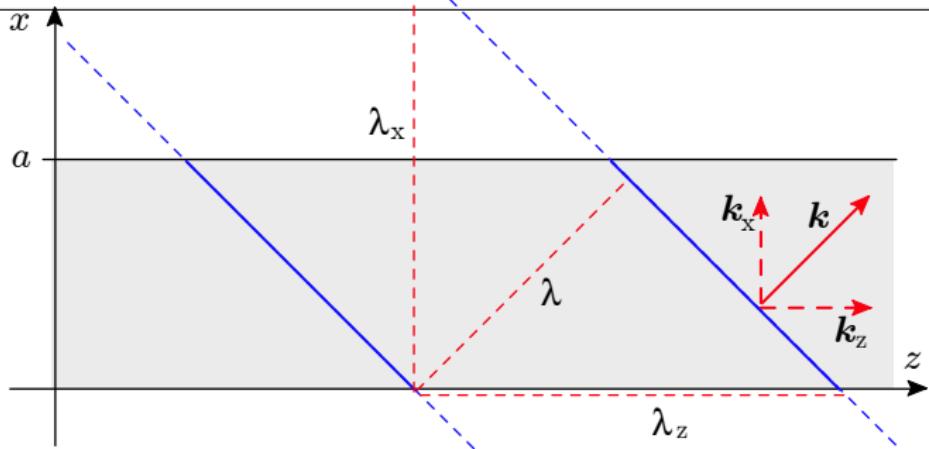
Example: $a = 60$ cm, $b = 30$ cm.

First mode (TE_{10}) starts at 250 MHz.

$$\mathbf{E}(\mathbf{r}) = E_0 \sin\left(\frac{\pi X}{a}\right) e^{ik_z z} \hat{\mathbf{y}}$$

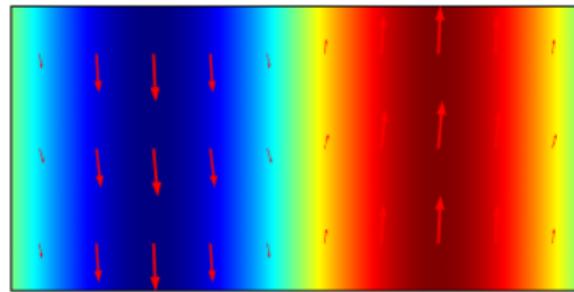
$$k_z = \sqrt{k^2 - (\pi/a)^2}$$

Waveguides: rectangular



- $k = \frac{2\pi}{\lambda}$
- $k_x = \frac{\pi}{a}$
- $k_x = \frac{2\pi}{\lambda_x} \Rightarrow \lambda_x = 2a$
- $k_z = \frac{2\pi}{\lambda_z}$

Waveguides: rectangular



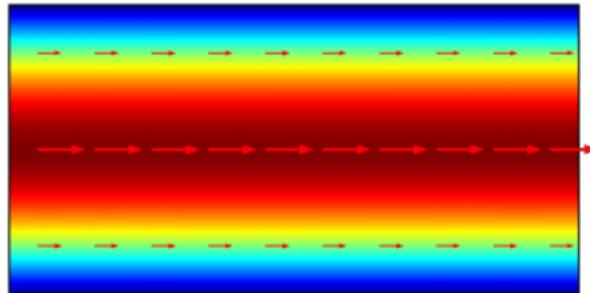
Example: $a = 60$ cm, $b = 30$ cm.

Second mode (TE_{20}) starts at 500 MHz.

$$\mathbf{E}(\mathbf{r}) = E_0 \sin\left(\frac{2\pi x}{a}\right) e^{ik_z z} \hat{\mathbf{y}}$$

$$k_z = \sqrt{k^2 - (2\pi/a)^2}$$

Waveguides: rectangular



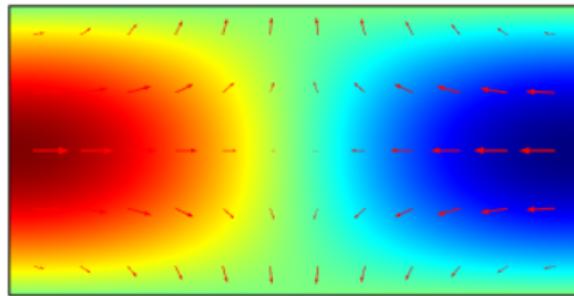
Example: $a = 60$ cm, $b = 30$ cm.

Third mode (TE_{01}) starts at 500 MHz.

$$\mathbf{E}(\mathbf{r}) = E_0 \sin\left(\frac{\pi y}{b}\right) e^{ik_z z} \hat{x}$$

$$k_z = \sqrt{k^2 - (\pi/b)^2}$$

Waveguides: rectangular



Example: $a = 60 \text{ cm}$, $b = 30 \text{ cm}$.
Fourth mode (TE_{11}) starts at 559 MHz.

Rectangular waveguide: Eigenvalue problem TE-modes

Eigenvalue problem for TE-modes

$$\frac{\partial^2 H_z(\mathbf{r})}{\partial x^2} + \frac{\partial^2 H_z(\mathbf{r})}{\partial y^2} + k_t^2 H_z(\mathbf{r}) = 0, \quad \mathbf{r} \in V$$
$$\hat{\mathbf{n}} \cdot \nabla H_z(\mathbf{r}) = 0, \quad \mathbf{r} \in S$$

Solutions:

$$H_{zmn}(\mathbf{r}) = H_0 \cos\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) e^{ik_z z}$$

$$k_{tmn}^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

$$k_{zmn} = \sqrt{k^2 - k_{tmn}^2}$$

$$m = 0, 1, 2, \dots, n = 0, 1, 2, \dots, \text{but } (m, n) \neq (0, 0)$$

Waveguides: Cut-off frequencies TE-modes

Cut-off frequencies for TE-modes

$$k_{zmn} = \sqrt{k^2 - k_{tmn}^2}$$

If $k < k_{tmn}$ then k_z is imaginary and $e^{ik_z z} = e^{-|k_z|z}$. No power can be transported.

If $k = k_{tmn}$ then $k_z = 0$ and $e^{ik_z z} = 1$. Cut-off!

If $k > k_{tmn}$ then k_z is real and $e^{ik_z z}$ is a wave. Power is transported.

The cut-off frequencies are $f_{cmn} = \frac{c}{2\pi} \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$
 $m = 0, 1, 2, \dots, n = 0, 1, 2, \dots$, but $(m, n) \neq (0, 0)$

Waveguides: Cut-off frequencies TE-modes

Example: Cut-off frequencies for TE-modes when $a = 60\text{cm}$ and $b = 30\text{cm}$

$$f_{cmn} = \frac{c}{2\pi} \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

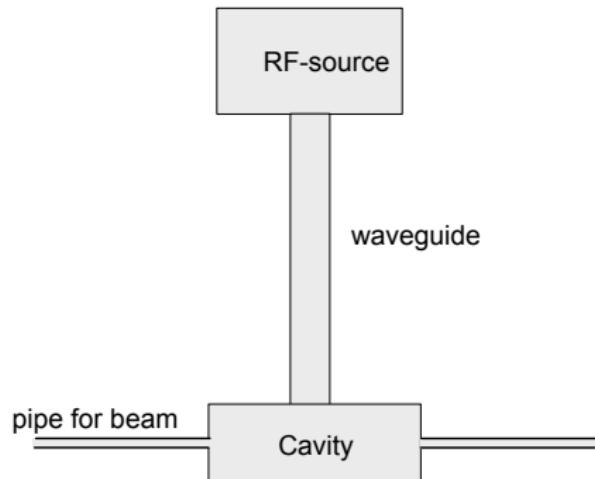
$$(m, n) = (1, 0) f_c = 250 \text{ MHz}$$

$$(m, n) = (0, 1) f_c = 500 \text{ MHz}$$

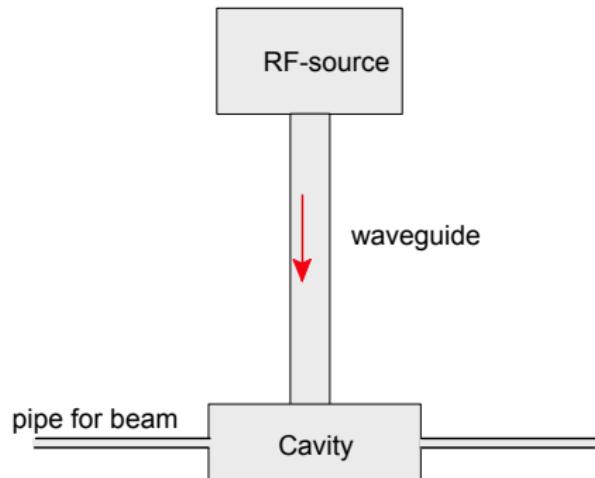
$$(m, n) = (2, 0) f_c = 500 \text{ MHz}$$

$$(m, n) = (1, 1) f_c = 559 \text{ MHz}$$

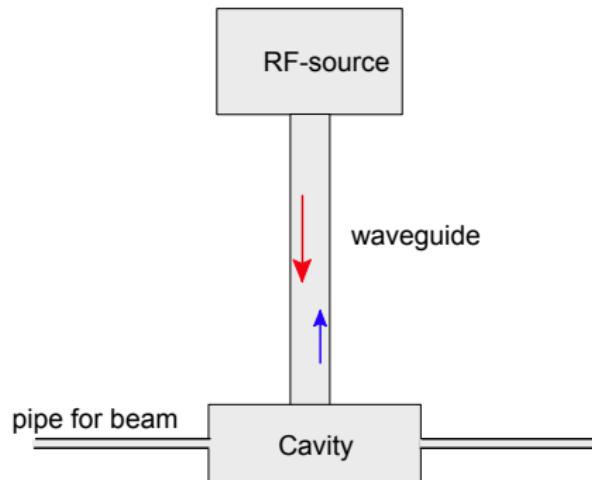
Circulator



Circulator

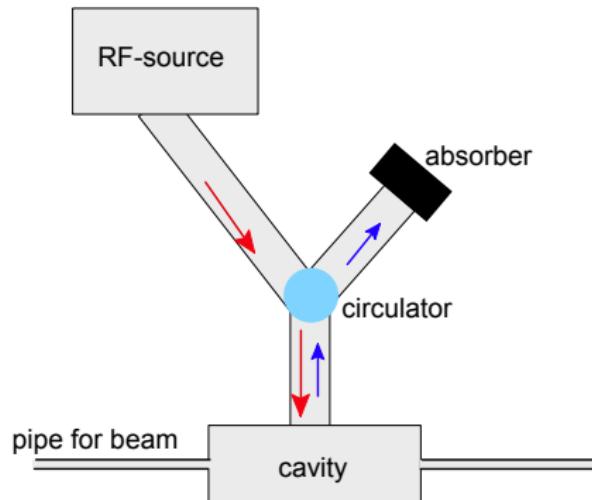


Circulator



Problemo: reflected power.

Circulator



Solution: circulator and matched load.

Conclusions:

- ▶ RF-waves to generate large particle energies (> 30 MeV)
- ▶ RF-cavities make it possible to accelerate in many steps
- ▶ Sophisticated shapes to obtain phase speed=particle speed
- ▶ Standing wave and traveling wave cavities
- ▶ Waveguides to guide the RF-waves from the source to the cavity
- ▶ Coaxial waveguides for short distances or low frequencies (< 200 MHz)
- ▶ Rectangular waveguides for long distances and high frequencies
- ▶ Circulators to avoid reflected power

Questions?